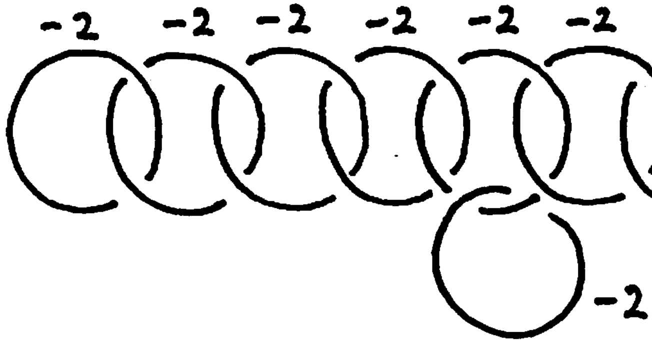
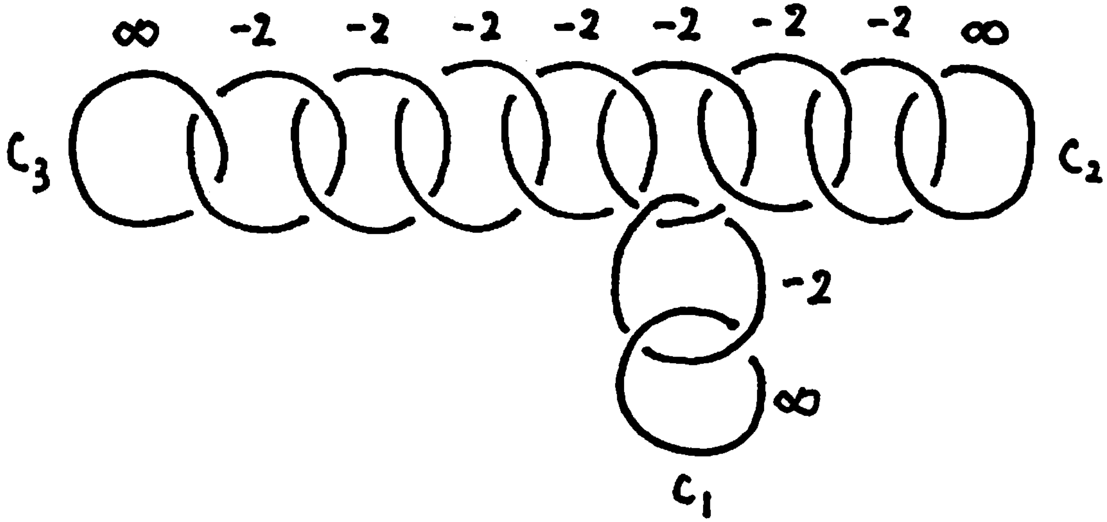
McKay Correspondence

PH is obtained by surgery on E_8-link

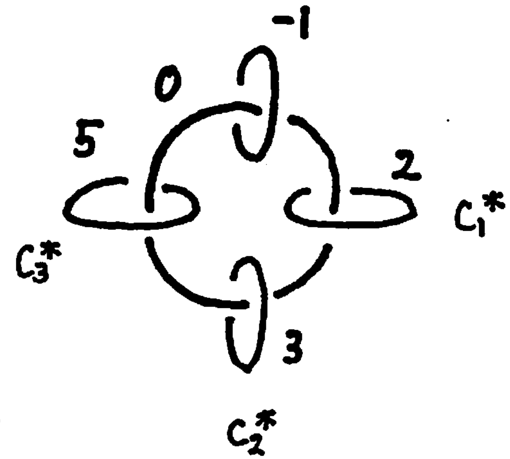


$(\because)$ Observe that E_8-surgery is a Seifert fibered space with 3 singular fibers C_1, C_2, C_3



$\rightsquigarrow$

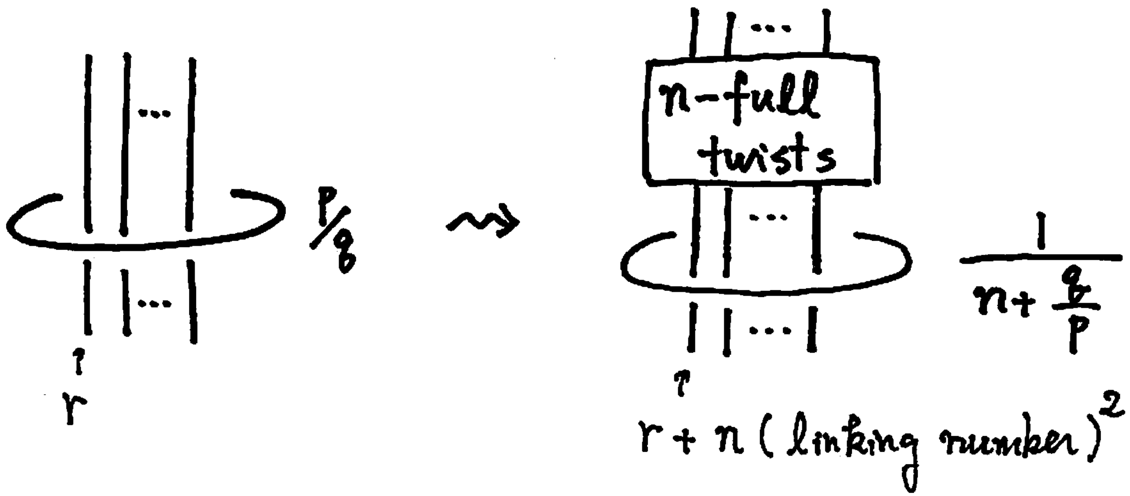
Kirby - Rolfsen move



$\cong$

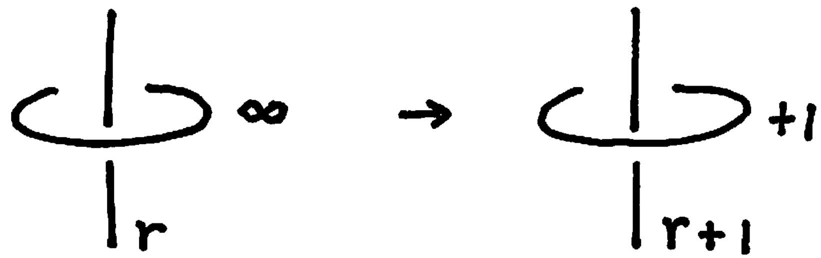
PH

Kirby - Rolfsen move

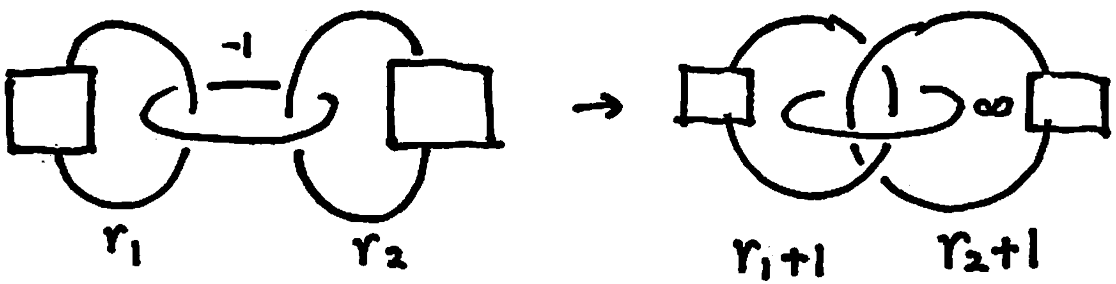


特に

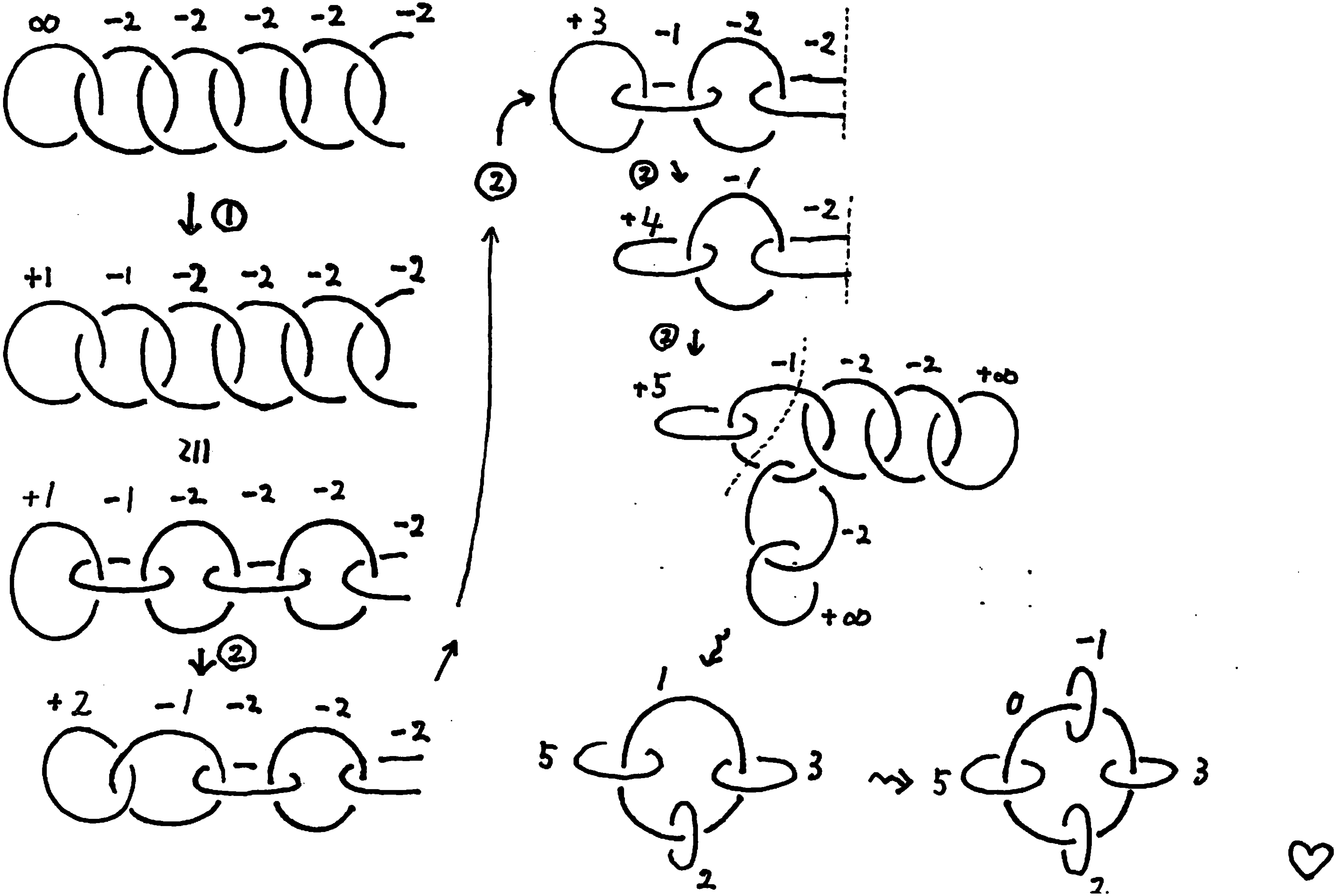
①



②



(Proof of PH = E_8-surgery)



- Integral surgery is realized by a 2-handle addition

$$D_1^2 \times D_2^2 \supset \partial D_1^2 \times D_2^2 \xrightarrow[\cong]{\Phi} N(K) \subset S^3 = \partial B^4 \subset B^4$$

$$[\partial D_1^2] \longmapsto [l] + p[m]$$

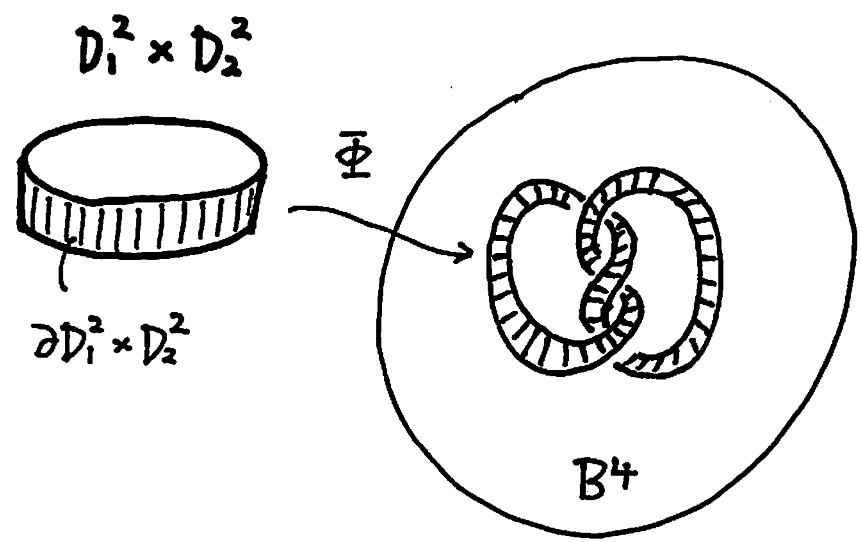
$$[\partial D_2^2] \longmapsto [m]$$

$$W^4 := D_1^2 \times D_2^2 \underset{\Phi}{\cup} B^4$$

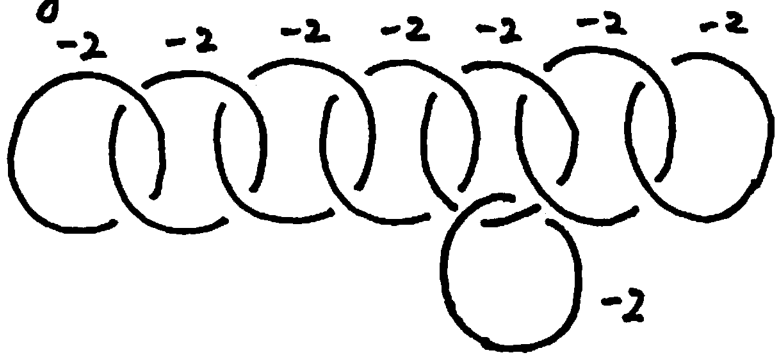
$$\cup$$

$$\partial W^4 = D_1^2 \times \partial D_1^2 \underset{\varphi}{\cup} E(K)$$

$$= K(p/1)$$



Cor $PH = \partial W^4$, where W^4 is the 4-mfd obtained from B^4 by attaching 2-handles along E_8-link.



Plumbing of disk bundles

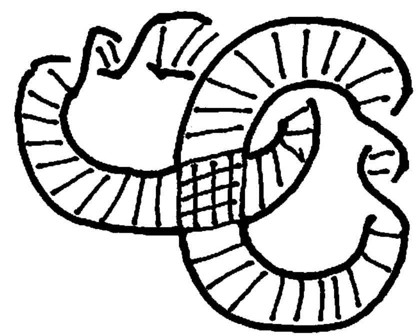
- $p : T^4 \to S^2$ D^2-bundle with Euler number -2

$$\begin{tikzcd}
T^4 \arrow[r, "p"] \arrow[d, phantom, "\cup" description] & S^2 \arrow[d, phantom, "\cup" description] \\
p^{-1}(B^2) \arrow[r] \arrow[d, "\varphi"', "\cong"] & B^2 \\
B^2 \times D^2 \arrow[ur, "\mathrm{Proj}_1"'] &
\end{tikzcd}$$

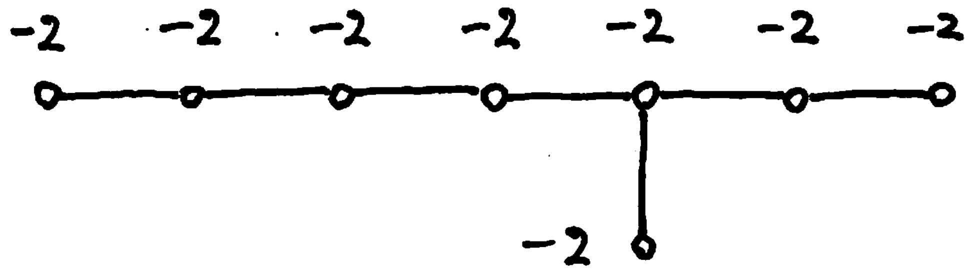
- T_1^4, T_2^4 : two copies of T^4

Plumbing of T_1^4 and T_2^4

$$= T_1^4 \sqcup T_2^4 \Big/ \varphi_1^{-1}(x, y) \sim \varphi_2^{-1}(y, x)$$



Fact The 4-mfd W^4 is E_8-plumbing of 8 copies of T^4



Fact The intersection form on $H_2(W^4)$ is represented by the $(-E_8)$-matrix

$$-E_8 = -\begin{bmatrix} 2 & & & 1 & & & & \\ & 2 & 1 & & & & & \\ & 1 & 2 & 1 & & & & \\ 1 & & 1 & 2 & 1 & & & \\ & & & 1 & 2 & 1 & & \\ & & & & 1 & 2 & 1 & \\ & & & & & 1 & 2 & 1 \\ & & & & & & 1 & 2 \end{bmatrix}$$

<u>Note</u> The intersection form on $H_2(W^4)$ is

- even ie $a \cdot a \equiv 0 \ (2) \quad \forall a \in H_2(W^4)$
- unimodular ie $\det(-E_8) = 1$
- negative definite ie $\operatorname{sgn}(W^4) = \operatorname{sgn}(-E_8) = -8$

<u>Algebraic Fact</u>

The signature of even unimodular form is divisible by 8

<u>Rokhlin's Theorem</u>

The signature of a closed oriented $\overbrace{\text{4-manifold}}^{\text{smooth, Spin}}$ is divisible by 16.

[Freedman]

- $\exists$ topological compact contractible 4-mfd Δ^4 with $\partial\Delta^4 = PH$

The topological closed mfd. $W^4 \cup -\Delta^4$

does not admit a smooth structure.

$(\because)$ $\operatorname{sgn}(W^4 \cup -\Delta^4) = \operatorname{sgn}(W^4) = -8$

is not divisible by 16.

- Exotic sphere (Milnor, Brieskorn)

  W^{4n} : E_8-plumbing of D^{2n}-bundles

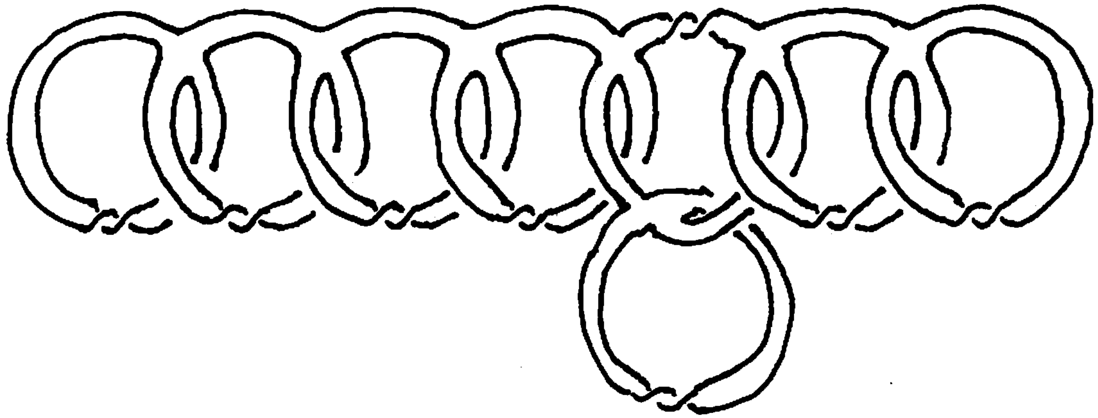
  $PH^{4n-1} := \partial W^{4n}$

  If $n \geqq 2$, then PH^{4n-1} is homeomorphic to S^{4n-1},

  but not diffeomorphic to S^{4n-1}.

  Key Hirzebruch's signature theorem

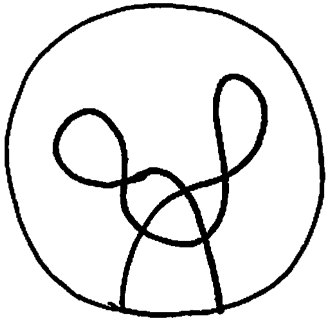
Seifert surface F of (3.5) torus knot is E_8 - Hopf plumbing



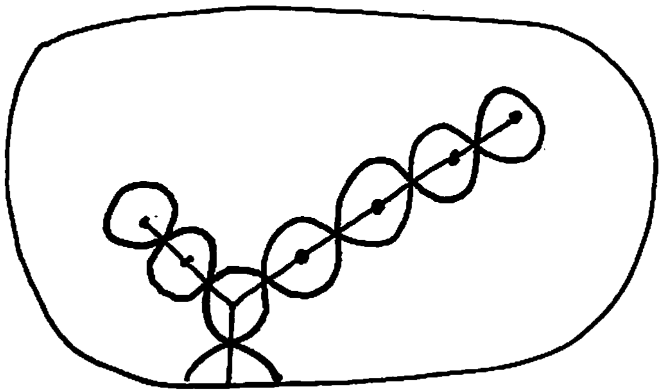
$\hat{F} \subset B^4$ $\underbrace{\text{surface, pushing int } F}_{\text{obtained from } F}$ into int B^4

<u>Prop</u> The double branched cover of B^4 branched over $\hat{F}$ is diffeomorphic to W^4.

A' Campo's theory of divide

 $\rightsquigarrow$ Fibered knot in S^3

E_8 - slalom knot is (3, 5) torus knot



By 平澤's visualization (3, 5) torus knot is

E_8 - plumbing of Hopf bands

Knot Floer homology の応用

[Ghiggini]

Conjecture $\hat{\mathrm{I}}$ is true.

If $K(r) = PH$, then $(K, r) =$  $^{-1}$

$\uparrow$

Ozsvath - Szabó conjecture holds for $g = 1$

Conj (Ozsvath - Szabó) K : Knot of genus g

Then K is fibered $\Leftrightarrow \widehat{HFK}(K, g) \cong \mathbb{Z}$

[Ni] The above conjecture holds for $\forall g \in \mathbb{N}$

([村杉] An alternating knot is fibered
iff the Alexander polynomial is monic.)

Heegaard Floer Homology Poincare Conjecture

$$\widetilde{PH} := (-PH) \# PH$$

$$P(n) := \overset{n}{\#} \widetilde{PH}$$

この時

$$\widehat{HF}(P(n)) = \widehat{HF}(S^3) = (\mathbb{Z})_0$$

(注) $\widehat{HF}(PH) \cong \mathbb{Z}$ だが grading が異なる.

<u>Conj.</u> Y : homology 3-sphere

$$\widehat{HF}(Y) = \widehat{HF}(S^3) \Rightarrow Y \cong S^3 \text{ or } P(n)$$

[Fetekhary]

Conj is valid if Y is a Seifert fibered space.

正12面体の特殊性

$\mathbb{P}$: 正多面体

$V \subset S^2$ 頂点集合

$$\mathrm{Aut}(\mathbb{P}) \curvearrowright T_V S^2 = \bigoplus_{v \in V} T_v(S^2)$$

<u>Observation</u>

- $P \neq$ dodecahedron

  $\Rightarrow$ 既約分解における各既約成分の重複度は 1

- $P =$ dodecahedron $\Rightarrow$ 重複度2の既約成分あり

次の方々に感謝します.

三松 佳彦 様　歴史的経緯

佐藤 肇 様　ポアンカレに関する資料

市原 一裕 様　ヘガード フレア ホモロジー 情報

Alexander Mednykh 様　Hyperbolic dodecahedron 情報

円山 憲子 様　不変量関係情報

ありがとうございました.

研究集会

Branched Coverings, Degenerations, and Related Topic

2010年 3月8日(月)-11日(木) 広島大学

Guest Speaker : Norbert A'Campo
Alexander Mednykh

組織委員: 足利正, 作間誠, 島田伊知朗
徳永浩雄, 松本幸夫