BΓ School

Homotopy Theory of Foliations

- 日程・場所

  2023 年 9 月 4 日 (月) – 9 月 7 日 (木) 中央大学理工学部，５号館１階５１３４号室

  112-8551 東京都文京区春日 1-13-27

- 講演予定者

  Elmar Vogt (Freie U., Berlin), Gaël Meigniez (U. Aix-Marseille), Sam Nariman (Purdue U.), 坪井俊（武蔵野大学），森田茂之（東京大学），森吉仁志（名古屋大学），三松佳彦 (中央大学), 北野晃朗（創価大学）

  Filip Samuelsen（Stony Brook U.), Shuhei Maruyama (Chuo U.), KyeongRo Kim (Seoul National U.)

- **Date and Venue**

  September 4th (Mon.) – September 7th (Thur.), 2023
  Room: 5134, Ground floor, No. 5 Building,
  Faculty of Science and Engineering, Chuo University, 1-13-27
  Kasuga, Bunkyo-ku, Tokyo, 112-8551, Japan

- **Speakers**

  Elmar Vogt (Freie U., Berlin), Gaël Meigniez (U. Aix-Marseille), Sam Nariman (Purdue U.),
  Takashi Tsuboi (Musashino U.), Shigeyuki Morita (U. Tokyo), Hitoshi Moriyoshi (Nagoya U.),
  Yoshihiko Mitsumatsu (Chuo U.), Teruaki Kitano (Soka U.)

  Filip Samuelsen（Stony Brook U.), Shuhei Maruyama (Chuo U.), KyeongRo Kim (Seoul National U.)

- **Time Schedule:**

9/4 (Mon)
11:00-12:20 Gaël Meigniez -1
14:00-15:20 Takashi Tsuboi
15:50-17:10 Elmar Vogt -1

9/5 (Tues)
10:00-11:20 Teruaki Kitano
11:40-12:40 Shigeyuki Morita
14:30-15:30 Short communications 1, 2, 3

9/6 (Wed)
10:00-11:00 Elmar Vogt -2
11:20-12:20 Yoshihiko Mitsumatsu -1
14:00-15:20 Gaël Meigniez -2
15:50-17:10 Sam Nariman -1

9/7 (Thur)
10:00-11:00 Yoshihiko Mitsumatsu -2
11:20-12:20 Hitoshi Moriyoshi
14:00-15:20 Sam Nariman -2
15:50-17:10 Gaël Meigniez -3

17:30 Banquet

- **Title and Abstract**

■ Gaël Meigniez (Université de Aix-Marseille):

1. BΓ basics

Abstract: this will be an elementary, introductory microcourse on the classical homotopy theory of foliations:
Bott's obstructions, Godbillon-Vey, BΓ, the h-principle for foliations, the Mather-Thurston theory, and the Haefliger-Thurston conjecture.

2. Homotopy types of foliations spaces by surfaces

Abstract: It will be shown that on every closed manifold of dimension at least 4, the space of the smooth foliations of dimension 2 has the same weak homotopy type as the space of the distributions of 2-planes.

■ Takashi Tsuboi (Musashino University / RIKEN iTHEMS):

$B\bar{\Gamma}^{\omega}$ and $H_1(\mathrm{Diff}^{\omega}(M)_0)$ for M with circle action

Abstract: In the first part of my talk, we review some of the works of André Haefliger on the classifying spaces for real-analytic $\bar{\Gamma}$-structures. The current situation looks almost the same as at the time when Haefliger treated them.
In the second part, we explain that the identity component of the group of real-analytic diffeomorphisms of a closed manifold is perfect if the manifold admits a semi-free circle action. This follows from that there is a non-trivial equivariant map from manifolds with semi-free circle action to the complex line.

- Elmar Vogt (Freie Universität Berlin, Emeritus):

The Mather-Thurston Theorem: Mather's 1976 write-up of Thurston's third proof. An Overview

Abstract: We begin with a brief introduction to Haefliger structures and their relationship to foliations and their homotopy (concordance) classification. Roughly the Mather-Thurston theorem relates the group of diffeomorphism with compact support of an n-manifold M with the space of compactly supported sections of a fibration over M with fibre the classifying space of Haefliger structures of codimension n with trivialized normal bundle.
The proof of Mather is based on talks given by Thurston and is quite involved. We will outline the key steps and describe the many spaces and maps between them which figure in the proof.

- Teruaki Kitano (Soka University):

Remarks on flat S^1-bundles, C^∞ vs C^ω

Abstract: We describe low dimensional homology groups of the real analytic, orientation preserving diffeomorphism group of S^1 in terms of $B\overline{\Gamma_1}$ by applying a theorem of Thurston. It is an open problem whether some power of the rational Euler class vanishes for real analytic flat S^1-bundles. In this talk we discuss that if it occurs, then the homology group should contain many torsion classes which vanish in the smooth case. This talk is based on a joint work with Shigeyuki Morita and Yoshihiko Mitsumatsu.

- Shigeyuki Morita (University of Tokyo, Emeritus):

Questions on $B\overline{\Gamma_1}$

Abstract: We discuss a few questions about the homotopy type and the homology group of Haefliger's classifying space for foliations of codimension one. We consider differences between consequences of two extremal models for it, one is the Eilenberg-MacLane space $K(\boldsymbol{R},3)$ and the other is the Moore space $M(\boldsymbol{R},3)$. This talk is based on joint work with Teruaki Kitano and Yoshihiko Mitsumatsu.

- Yoshihiko Mitsumatsu (Chuo University):

Mather-Thurston maps for the flat real analytic circle bundles

Abstract: Under the real analytic setting, there is no reason for the Mather-Thurston map to induce isomorphism on the homology. In the case of codimension 1, and for flat circle bundles, we show that a difference from and a similarity to being isomorphic in homology or in homotopy of the Mather-Thurston map. One of the key ingredient is a fine analysis on the 1-dimensional real analytic diffeo germs by 1-dimensional holomorphic dynamics, namely the theory of parabolic linearization. This is based on a joint work with Shigeyuki Morita and Teruaki Kitano.

■ Sam Nariman (Purdue University):

1. Some applications of Mather-Thurston's theorem

Abstract: I will talk about two applications of the Mather-Thurston theorem. The first one is about PL homeomorphisms of surfaces. We discuss that Greenberg's work on PL foliations can be used to prove the Haefliger-Thurston conjecture for codimension 2 PL foliations and as a result, we could answer the case of surfaces of a question posed by Epstein in 1970 about the simplicity of PL homeomorphisms that are isotopic to the identity. If time permits, I will talk about another application of using the Mather-Thurston theorem in an equivariant setting. And I will answer a question posed by Haefliger about the powers of the Euler class for flat odd-dimensional sphere bundles.

2. Bounded and unbounded cohomology of diffeomorphism groups

Abstract: Bounded cohomology for groups and spaces was originally defined by Gromov in the 80s and it is intimately related to the geometric and dynamical properties of the groups. For example, Ghys used the bounded Euler class to classify certain group actions on the circle up to (semi)conjugacy. However, unlike group cohomology, it is notoriously difficult to calculate the bounded cohomology of groups. And in fact, there is no countably generated group known for which we can completely calculate the bounded cohomology unless it is trivial in all positive degrees like the case of amenable groups. In this talk, I will talk about joint work with Nicolas Monod on the bounded cohomology of certain homeomorphisms and diffeomorphism groups. In particular, we show that the bounded cohomology of Diff(S^1) and Diff(D^2) are polynomial rings generated by the Euler class.
If time permits, I also discuss our solution to Ghys' question about generalizing Milnor-Wood inequality to flat S^3-bundles. In particular, we show that the Euler class for flat S^3-bundles is an unbounded class.

■ Hitoshi Moriyoshi (Nagoya University):

Geometry on the circle diffeomorphism group and the space of equicentroaffine curves

Abstract: A plane curve γ: $S^1 \to \boldsymbol{R}^2$ is called equi-centro-affine if the position vector γ and the velocity vector γ' makes a triangle of constant area with respect to the origin. In other words, the determinant of 2 by 2 matrix ($\gamma\ \gamma'$) is constant. Even though the space M of all equicentroaffine curves is infinite dimensional, M admits a transitive action by the circle diffeomorphism group due to Pinkall. It is also known that there exists an invariant pre-symplectic form on M, called the Fujioka-Kurose 2-form. In this talk we shall manifest an intriguing interaction between Geometry and Analysis, namely a beautiful relationship among curvature of equicentroaffine curves, moment map, the Bott-Virasoro group and the KdV equation. This is a joint work with A. Fujioka and T. Kurose.

■ Short Communications:

Filip Samuelsen（Stony Brook U.）Complex structures on open manifolds

Abstract: On open almost complex manifolds, Haefliger's framework reduces the problem of constructing complex structures to a homotopy lifting problem. This short communication presents Masahisa Adachi's theorem, which asserts that the relevant homotopy fiber is at least n-connected. As a notable corollary, we establish that any open almost complex manifold with a real dimension of at most 6 admits a complex structure.

Shuhei Maruyama (Chuo U.) On the boundedness of characteristic classes for foliated symplectic and contact fibrations

Abstract: In this talk, we will discuss the relationship between the boundedness of characteristic classes for foliated bundles and the descendibility of quasimorphisms. We will also present two specific characteristic classes for foliated symplectic and contact fibrations: one is a bounded class, and the other is an unbounded class.

KyeongRo Kim (Seoul National U.) Invitation to laminar group theory

Abstract : Thurston tried to combine the study of geometric structures and the study of tautly foliated 3–manifolds to solve the geometrization conjecture. He first observed that the fundamental group of a three manifold admitting a R-covered uniform taut foliation acts on the circle preserving a pair of circle laminations. In this talk, I briefly introduce some notions related to circle laminations and I explain the interaction between the dynamics on the circle and the topology of hyperbolic manifolds via circle laminations.

- **Access**

About a 15-minute walk from JR Soubu Line Suidoubashi Station

About a 5-minute walk from Tokyo Metro Marunouchi Line or Nanboku Line Kourakuen Station

About a 7-minute walk from Tokyo Metro Mita Line or O-edo Line Kasuga Station

Click here for a map (in Japanese) Click here for a map (in English)

Organizers:
Yoshihiko Mitsumatsu (Chuo Univ.), Teruaki Kitano (Soka Univ.)
E-mail: yoshi(-AT-)math.chuo-u.ac.jp, kitano(-AT-)soka.ac.jp