Contents

MORREY SPACES–APPLICATIONS TO INTEGRAL OPERATORS AND PDE, (I AND II), ERROTUM

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This document is the erratum of the book Morrey spaces–applications to integral operators and PDE, Volumes I and II published in 2020.

Note: If the change is too major or too minor, I will not mark the change by red.

⋆ location
  (a) We wrote.
  (b) But we should have written.
  (c) Dates
  (d) Other comments if there are

⋆⋆ location
  (a) Add
  (b) What I want to add actually.
  (c) Dates
  (d) Other comments if there are

⋆⋆⋆ location
   (a) Remove
   (b) What I want to remove actually.
   (c) Dates
   (d) Other comments if there are

(1) p. vii, 4.1.7
   (a) boundednss
   (b) boundedness
   (c) 21 Sept. 2023
(2) p. vii, 4.5.3
   (a) Zgymund
   (b) Zygmund
   (c) 21 Sept. 2023
(3) p. vii, 6.2.2
   (a) Vector-valued bounededness
   (b) Vector-valued boundedness
   (c) 21 Sept. 2023
(4) p. xi, line 11 from below
   (a) Rosental
   (b) Rosenthal
   (c) 21 Sept. 2023
(5) p. xii, line 1 from above
   (a) Stampaccia
   (b) Stampacchia
   (c) 21 Sept. 2023
(6) p. xiii, line 3 from below
   (a) invesitigates
   (b) investigates
   (c) 21 Sept. 2023
(7) p. vx, line 15 from above
   (a) Sakoto
   (b) Satoko
   (c) 9 Nov. 2024
(8) Swap (2) and (3) in page xvii.
(9) p. xvii, (2) the definition of balls
   (a) $\|x-y\|$
   (b) $|x-y|$
   (c) 14 Dec. 2023
(10) p. xviii (6),
   (a) E is integrable over f, Then
   (b) f is integrable over E, then
   (c) 13 Sept. 2024
(11) p. xvii, (11)
   (a) add as (11)
   (b) For simplicity, for a measure μ, we write $\mu(\{\cdots\})=\mu\{\cdots\}$.
   (c) 21 Aug. 2024
(12) p. xix, (7)
   (a) $\displaystyle\int_E f$
   (b) $\displaystyle\int_E f(x)\mathrm{d}x$

   (c) 5 Dec. 2023
(13) p. xix, (8)
   (a) positive
   (b) non-negative
   (c) 5 Dec. 2023
(14) p. xix, (8)
   (a) add
   (b) Generally for a measurable function f, we write $m_E^{(\eta)}(f) = m_E(|f|^\eta)^{\frac{1}{\eta}}$.
   (c) 5 Dec. 2023
(15) p. xix, (9)
   (a) add
   (b) This is a ball-based one. We use the same symbol for the maximal operator generated by cubes.
   (c) 5 Dec. 2023
(16) p. xx (14),
   (a) $m_Q(f)$
   (b) $m_A(f)$
   (c) 5 Dec. 2023
(17) p. xx (20)
   (a) $(j \neq k)$.
   (b) $(j \neq k)$
   (c) 5 Dec. 2023
(18) p. xx, (22)
   (a) Minkovski
   (b) Minkowski
   (c) 21 Sept. 2023

0.1. **Pages 1–9.**

(1) p. 3, Theorem 2
  (a) for all
  (b) at any point
  (c) 15 May 2024
(2) p. 4, Definition 4
  (a) add
  (b) Here the infimum is taken over all $E_j \in \mathcal{B}$ and $F_j \in \mathcal{B}'$.
  (c) 15 May 2024
(3) p. 4, line 14 from below, Theorem 4
  (a) $\mathcal{N}$
  (b) $\mathcal{B}$
  (c) 14 Dec. 2023, 22 May 2024
(4) p. 4, line 14 from below, Theorem 4
  (a) $\mathcal{N}'$
  (b) $\mathcal{B}'$
  (c) 14 Dec. 2023, 22 May 2024
(5) p. 4, line 10 from below, Theorem 4
  (a) $\mathcal{M} \otimes \mathcal{N}$ (twice)
  (b) $\mathcal{B} \otimes \mathcal{B}'$
  (c) 22 May 2024
(6) p. 9, line 16 from above, the proof of Lemma 13
  (a) we have
  (b) using the triangle inequality for $L^{\frac{q}{p}}(\mu)$, we have
  (c) 22 May 2024

0.2. **Pages 10–19.**

(1) p. 10, line 1 from above, Lemma 14
  (a) For all f
  (b) For all g
  (c) 21 Nov. 2024
(2) p. 13, line 5 from below
  (a) $D|B(x,r)|^{\frac{1}{p}-\frac{1}{q}}$
  (b) $D|B(x,r)|^{\frac{1}{q}-\frac{1}{p}}$
  (c) 26 Jul. 2022
(3) p. 15, line 4 from above, Remark 1
  (a) $\|f\|_{\mathcal{L}^q_\lambda} = \|f\|_{\mathcal{M}^p_q} = \|f\|^{\mathcal{B}_0}_{\mathcal{M}^p_q}$.
  (b) $\|f\|_{\mathcal{L}^q_\lambda} = \|f\|^{\mathcal{B}_0}_{\mathcal{M}^p_q} \simeq \|f\|_{\mathcal{M}^p_q}$ for all $f \in L^0(\mathbb{R}^n)$.
  (c) 26 Sept. 2023
(4) p. 15, line 14 from above, Proposition 15
  (a) $p < q$.
  (b) $0 < p < q < \infty$.
  (c) 26 Sept. 2023
(5) p. 17, line 11 from above, Example 6
  (a) Add
  (b) Let $p = \frac{n}{\alpha}$ with $\alpha > 0$.
  (c) 2 Aug. 2023
(6) p. 17, line 9 from above, Example 6
  (a) for all $r > 0$.
  (b) for all $r > 0$. Remark that the right-hand side does not depend on $r > 0$.
  (c) 2 Aug. 2023
  (d) We should have explained why

$$\sup_{r>0} |B(r)|^{\frac{\alpha}{n}-\frac{1}{q}} \|f_\alpha\|_{L^q(B(r))} < \infty.$$

(7) p. 18, line 9 from above, Example 8
  (a) $r = 2\alpha$
  (b) $r = \frac{\alpha}{2N}$
  (c) 2 Aug. 2022
(8) p. 18, line 10 from above, Example 8
  (a) $2r < \alpha$
  (b) $r < \frac{\alpha}{2N}$
  (c) 2 Aug. 2022
(9) p. 18, line 12 from above, Example 8
  (a) is.
  (b) is. Remark the following: $\ell(Q^N_j) = \frac{1}{N}$, $\ell(\alpha Q^N_j) = \alpha^{\frac{p}{q}}$, $\|\chi_{\alpha Q^N_j}\|_{L^p} = \alpha^{\frac{n}{q}}$, $\|f\|_{L^p} = \alpha^{\frac{n}{p}}$, $\|f\|_{L^q} = \alpha^{\frac{n}{q}}$.
  (c) 2 Aug. 2022
(10) p. 18, line 13 from above, Example 8
  (a) $\alpha \le 2r \le 1$
  (b) $\frac{\alpha}{N} \le 2r \le 1$
  (c) 2 Aug. 2022
(11) p. 18, line 17 from above, Example 8
  (a) this R
  (b) R above
  (c) 2 Aug. 2022

(12) p. 18, line 18 from above, Example 8: We should have mentioned that $r > \frac{1}{4N}$, if $r > \frac{\alpha}{2N}$ and $r > \frac{1-\alpha}{2N}$. This observation is useful in the penultipmate line of the proof.
(13) p. 19, line 14 from below, Example 9
   (a) $1, 2, \ldots, n$
   (b) $1, 2, \ldots, j$
   (c) 2 Aug. 2022
(14) p. 19, line 13 from below, Example 9
   (a) any connected component Z
   (b) any connected component Z which intersects Q
   (c) 2 Aug. 2022
(15) p. 19, line 9 from below, Example 9
   (a) a connected component of E_j
   (b) any connected component E_j as long as it intersects
   (c) 2 Aug. 2022

0.3. **Pages 20–29.**

(1) p. 20, line 12 from below, Example 10
  (a) $L^r(Q)$
  (b) $L^r([0,1]^n)$
  (c) 2 Aug. 2022
(2) p. 21, line 1 from above, Example 11
  (a) $\{F_j\}_{j_1}^\infty$
  (b) $\{F_j\}_{j=1}^\infty$
  (c) 2 Aug. 2022
(3) p. 21, line 3 from above, Example 11
  (a) R^k
  (b) $R(1+R)^{k-1}$
  (c) 1 Dec. 2020
(4) p. 21, line 3 from above, Example 11
  (a) $\{a_k\}_{k=1}^j \in \{0,1\}^j$
  (b) $\{a_k\}_{k=1}^n \subset \{0,1\}^n$
  (c) 1 Dec. 2020
(5) p. 21, line 6 from above, Example 11
  (a) R^2
  (b) $R(1+R)$
  (c) 1 Dec. 2020
(6) p. 21, (1.15),
  (a) $k \le ll$
  (b) $k \le l$
  (c) 21 Aug. 2024
(7) p. 24, line 2 from above, Theorem 18
  (a) $\mathcal{M}^p_q(\mathbb{R}^n) \cap L^0_{\rm c}(\mathbb{R}^n) \setminus L^r_{\rm loc}(\mathbb{R}^n)$
  (b) $(\mathcal{M}^p_q(\mathbb{R}^n) \cap L^0_{\rm c}(\mathbb{R}^n)) \setminus L^r_{\rm loc}(\mathbb{R}^n)$
  (c) 2 Aug. 2022
(8) p. 24, line 3 from above, Theorem 18
  (a) We use
  (b) By scaling argument, we may assume $q > 1$, so that we can use the triangle inequality freely. We use
  (c) 2 Aug. 2023, 16 Sept. 2024
  (d) I use this to show that $f \in \mathcal{M}^p_q(\mathbb{R}^n)$ together with the triangle inequality.
(9) p. 24, line 3 from above, Theorem 18
  (a) fixed
  (b) fixed as in Example 8
  (c) 2 Aug. 2022
(10) p. 24, lines 3, 4 and 5 from above, Theorem 18
  (a) f_N (7 times)
  (b) $f_{N!}$ (7 times)
  (c) 2 Aug. 2022
(11) p. 24, line 5 from above, the proof of Theorem 18
  (a) $\mathcal{M}^p_q(\mathbb{R}^n) \cap L^0_{\rm c}(\mathbb{R}^n) \setminus L^r_{\rm loc}(\mathbb{R}^n)$
  (b) $(\mathcal{M}^p_q(\mathbb{R}^n) \cap L^0_{\rm c}(\mathbb{R}^n)) \setminus L^r_{\rm loc}(\mathbb{R}^n)$
  (c) 2 Aug. 2022
(12) p. 25, line 3 from above, the proof of Theorem 20
  (a) of the connected components
  (b) of the connected components of F
  (c) 11 Dec. 2024

(13) p. 25, line 4 from above, the proof of Theorem 20
  (a) $[0, R^j]$
  (b) $[0, (1+R)^j]$
  (c) 2 Aug. 2022
(14) p. 25, line 5 from above (twice), the proof of Theorem 20
  (a) $R^{\frac{jn}{p}-\frac{jn}{q}}$
  (b) $(1+R)^{\frac{jn}{p}-\frac{jn}{q}}$
  (c) 2 Aug. 2022
(15) p. 25, line 9 from below, the proof of Proposition 22(2)
  (a) $f_j \equiv$
  (b) $f^{(j)} \equiv$
  (c) Oct 3, 2023
(16) p. 25, line 9 from below, Proposition 22
  (a) $\|f_j - f_k\|_{\mathcal{M}^p_q} = \|f_1 - f_0\|_{\mathcal{M}^p_q} > 0$
  (b) $\|f^{(j)} - f^{(k)}\|_{\mathcal{M}^p_q} \geq \|f^{(1)} - f^{(0)}\|_{\mathcal{M}^p_q} > 0$
  (c) Oct 3, 2023
(17) p. 26, line 3 from below, the proof of Lemma 23
  (a) $\min(|Q|, \lambda^{-r}\|f\|_{\mathrm{W}\mathcal{M}^p_r}{}^r)$
  (b) $\min(|Q|, \lambda^{-r}|Q|^{1-\frac{r}{p}}\|f\|_{\mathrm{W}\mathcal{M}^p_r}{}^r)$
  (c) Oct 3, 2023
(18) p. 27, I did not need the proof of Theorem 24 since it is a corollary of Lemma 23. Simply let $p = r$ in p. 26 line 3 from below. Oct 3, 2023
(19) In Example 21, (see p. 28 Book I, line 3 from below), We should have said that

$$\sup_{0<a<b<2a}(b-a)^{\frac{1}{2}-\frac{1}{q}}\|f\|_{L^q(a,b)} = \sup_{0<a<b<2a}\sup_{k\in\mathbb{N}}(b-a)^{\frac{1}{2}-\frac{1}{q}}\|a_k{}^{-\frac{1}{2}}\chi_{E_k}\|_{L^q(a,b)}.$$

I noticed this on 26 Jul. 2022.
(20) p. 29, line 18 from below, Example 21
  (a) for any $h \in L^1(\mathbb{R}^n)$
  (b) for any $h \in L^\infty(\mathbb{R}^n)$
  (c) 26 Jul. 2022
(21) p. 29, line 3 from below
  (a) space.
  (b) space. (See Definition 13)
  (c) 21 Aug. 2024

0.4. **Pages 30–39.**

(1) p. 32, line 12 from above, Definition 15,
  (a) $\sup\limits_{\lambda>0} \lambda \|\chi_{(\lambda,\infty]}(|f|)\|_{\mathcal{M}^p_q}$
  (b) $\sup\limits_{\lambda>0} \lambda \|\chi_{(\lambda,\infty]}(|f|)\|_{\mathrm{L}\mathcal{M}^p_q}$
  (c) Oct 3, 2023
(2) p. 32, line 3 from below, Proposition 25(3)
  (a) $1 \le q \le p < \infty$
  (b) $0 < q \le p < \infty$
  (c) Oct 3, 2023
(3) p. 33, line 1 from above, Proposition 25(4)
  (a) $1 \le q \le p < \infty$
  (b) $0 < q \le p < \infty$
  (c) Oct 3, 2023
(4) p. 33, line 14 from below, Example 24(1)
  (a) $1 \le p < \infty$
  (b) $0 < p < \infty$
  (c) Oct 3, 2023
(5) p. 33, line 10 from below, Example 24(1)
  (a) $1 \le q \le p < \infty$
  (b) $0 < q \le p < \infty$
  (c) Oct 3, 2023
(6) p. 33, line 6 from below, Example 24(2)
  (a) $1 \le p < \infty$
  (b) $0 < p < \infty$
  (c) Oct 3, 2023
(7) p. 33 line 1 from below, Example 24(2)
  (a) $1 \le q \le p < \infty$
  (b) $0 < q \le p < \infty$
  (c) 21 Aug. 2024
(8) p. 34, line 1 from above, Example 24(3)
  (a) $1 \le q \le p < \infty$
  (b) $0 < q \le p < \infty$
  (c) Oct 3, 2023
(9) p. 34, line 5 from above, Example 24(4)
  (a) $1 \le q \le p < \infty$
  (b) $0 < q \le p < \infty$
  (c) Oct 3, 2023
(10) p. 34, I did not give the proof of the fact given in Example 25. However, We should have alluded that f is compactly supported and that the support does not contain the origin.
(11) p. 35, line 15 from below, Theorem 26
  (a) $\dot{K}^{\alpha}_{p\infty}(\mathbb{R}^n)$
  (b) $\dot{K}^{\alpha}_{q\infty}(\mathbb{R}^n)$
  (c) Oct 17, 2023
(12) p. 35, line 13 from below (twice), the proof of Theorem 26
  (a) $\dot{K}^{\alpha}_{p\infty}(\mathbb{R}^n)$
  (b) $\dot{K}^{\alpha}_{q\infty}(\mathbb{R}^n)$
  (c) Oct 17, 2023
(13) p. 35, line 10 from below, the proof of Theorem 26
  (a) $\le$

   (b) $\lesssim$
   (c) Oct 17, 2023
   (d) Remark that $|Q(r)| = 2^n r^n$.
(14) p. 35, line 10 from below, the proof of Theorem 26
   (a) $\dot{K}^{\alpha}_{p\infty}$
   (b) $\dot{K}^{\alpha}_{q\infty}$
   (c) Oct 17, 2023
(15) p. 35, line 9 from below, the proof of Theorem 26
   (a) $\dot{K}^{\alpha}_{p\infty}$
   (b) $\dot{K}^{\alpha}_{q\infty}$
   (c) Oct 17, 2023
(16) p. 35, line 5 from below, the proof of Theorem 26
   (a) $u(Q)$
   (b) $|Q|$
   (c) Oct 17, 2023
(17) p. 38, line 1 from above, Example 28
   (a) We should have reminded that v_n stands for the volume of the unit ball.
   (b) Oct 17, 2023
(18) p. 38, line 10 from above, Example 29
   (a) $f^* = \alpha_N \chi_{(0,\mu(E_N))} + \sum_{k=1}^{N-1} \alpha_k \chi_{(\mu(E_N \cup E_{N-1} \cup \cdots \cup E_{k+1}),\mu(E_N \cup E_{N-1} \cup \cdots \cup E_k)}$
   (b) $\lambda_f(t) = \begin{cases} 0 & (t \geq \alpha_N), \\ \mu(E_N \cup E_{N-1} \cup \cdots \cup E_k) & (\alpha_k > t \geq \alpha_{k-1}, k \in \mathbb{N} \cup [2, N]), \\ \mu(E_N \cup E_{N-1} \cup \cdots \cup E_1) & (0 < t \leq \alpha_1). \end{cases}$
   (c) Oct 17, 2023
(19) p. 38, lines 9 and 11 from below (both twice), the proof of Theorem 27
   (a) $\mathbb{R}^n$
   (b) X
   (c) Oct 17, 2023
(20) p. 39, line 1 from above, the proof of Lemma 28
   (a) $\lambda_f(t_2)$
   (b) $\lambda_g(t_2)$
   (c) Oct 17, 2023
(21) p. 39, line 2 from above, the proof of Lemma 28
   (a) $\mu\{|f| > t_2\}$
   (b) $\mu\{|g| > t_2\}$
   (c) Oct 17, 2023
(22) p. 39, line 6 from below, Example 32
   (a) Example 32
   (b) Example 29
   (c) Oct 17, 2023
(23) p. 39, line 5 from below, Example 32
   (a) $\left(\sum_{j=1}^{N} \alpha_j \chi_{E_j} \right)^* (t)$
   (b) f^*
   (c) 7 Nov. 2023

0.5. **Pages 40–49.**

(1) p. 40, Lemma 30,
  (a) $f \in L^0(\mu)$
  (b) $f \in L^0(\mu)$ such that $\mu(\{x \in X : |f(x)| > f^*(t) - \varepsilon\}) < \infty$ for some $\varepsilon > 0$,
  (c) Oct 17, 2023
(2) p. 40, line 8 from above, the proof of Lemma 30
  (a) $f^*(t)$.
  (b) $f^*(t)$. Indeed, since $\lambda_f(u) < \infty$ for some u slighly less than $f^*(t)$, we have
$$f^*(t) = \inf\{u \in [0, \infty) : \lambda_f(u) \le t\}.$$
Since $s < f^*(t)$, this means that $s \notin \{u \in [0, \infty) : \lambda_f(u) \le t\}$, which implies $\lambda_f(s) > t$.
  (c) 21 Aug. 2024
(3) p. 40, line 1 from below, the proof of Proposition 32
  (a) $s \le \lambda_f^{-1}(t)$
  (b) $s \ge \lambda_f^{-1}(t)$
  (c) 7 Nov. 2023
(4) p. 41, line 11 from above, the proof of Lemma 34
  (a) Let
  (b) Let $t > 0$ and
  (c) 7 Nov. 2023
(5) p. 42, line 7 from above, the proof of Proposition 35
  (a) $f^*(t_1) + g^*(t_2) \le (f + g)^*(t_1 + g_2)$
  (b) $f^*(t_1) + g^*(t_2) \ge (f + g)^*(t_1 + g_2)$
  (c) 7 Nov. 2023
(6) p. 43, line 7 from above
  (a) Definition 21
  (b) Definition 20
  (c) 29 Nov. 2024
(7) p. 43, line 7 from above
  (a) $L^0(\mathbb{R}^n)$
  (b) $L^0(\mu)$
  (c) 29 Nov. 2024
(8) p. 43, line 6 from below
  (a) in this book
  (b) in the second book
  (c) 29 Nov. 2024
(9) p. 46, line 9 from below, Example 40
  (a) is bounded
  (b) extends to a bounded linear operator
  (c) 29 Nov. 2024
(10) p. 47, line 4 from above, the proof of Lemma 40
  (a) holds.
  (b) holds. By mollifying μ suitably, we may assume that $\mu \in C^1(0, \infty)$.
  (c) 29 Nov. 2024
(11) p. 47, line 7 from above, the proof of Lemma 40 (1.2)
  (a) $\mu(\rho)$
  (b) $\mu(\rho)^{\frac{1}{q}}$
  (c) 29 Nov. 2024
(12) p. 47, line 9 from above, the proof of Lemma 40
  (a) $\mu(\rho)$

   (b) $\mu(\rho)^{\frac{1}{q}}$
   (c) 29 Nov. 2024
(13) p. 47, line 11 from above, the proof of Lemma 40, (1.3)
   (a) $\frac{K}{\theta}\frac{\mu(\rho)^{\frac{1}{q}}}{\theta}$
   (b) $\frac{K}{\theta}+\frac{\mu(\rho)^{\frac{1}{q}}}{\theta}$
   (c) 29 Nov. 2024
(14) p. 47, line 17 from above, the proof of Lemma 40, (1.5)
   (a) $\sum_{j=0}^{\infty}$
   (b) $\mu(\rho)^{\frac{1}{q}}\sum_{j=0}^{\infty}$
   (c) 29 Nov. 2024
(15) p. 47, line 16 from above, the proof of Lemma 40, (1.5)
   (a) (1.3) and (1.4)
   (b) (1.4)
   (c) 29 Nov. 2024
(16) p. 48, line 8 from above, Exercise 27(1)
   (a) $t \downarrow \infty$
   (b) $t \uparrow \infty$
   (c) 29 Nov. 2024
(17) p. 48, line 11 from above, Exercise 27(2)
   (a) fails
   (b) fail
   (c) 29 Nov. 2024
(18) p. 48, line 11 from above, Exercise 27(2)
   (a) $t \downarrow \infty$
   (b) $t \downarrow 0$
   (c) 29 Nov. 2024
(19) p. 48, line 11 from above, Exercise 27(2)
   (a) $t \downarrow \infty$
   (b) $t \uparrow \infty$
   (c) 29 Nov. 2024
(20) p. 48, line 18 from above, Exercise 27
   (a) (6.25)
   (b) Example 11
   (c) 29 Nov. 2024
(21) p. 49 line 3 from above Take "Work in $\mathbb{R}^2=\mathbb{R}_t\times\mathbb{R}_s$." to (1).
(22) p. 49 line 5 from below
   (a) Section 1.5.3.
   (b) in Section 1.5.3.
   (c) 1 Jan. 2025

0.6. **Pages 50–59.**

(1) p. 50 line 21 from above
  (a) $\Phi(t) = 0$.
  (b) $\Phi(t) = 0$ for all $t \in [0, 1)$.
  (c) 1 Jan 2025
(2) p. 50, one line above Example 42: We should have included the example of $\Phi(t) = \max(t - 1, 0)$, $t > 0$ as examples of non-injective Young functions in this book.
(3) p. 54, Theorem 46
  (a) Add: Assume that Φ^* is finitely valued.
  (b) 5 Dec. 2023
(4) p. 50 line 7 from below, Example 42(4)
  (a) then Φ can be arranged to be a Young function.
  (b) then there exists $\widetilde{\Phi}$ such that $\Phi(r) = \widetilde{\Phi}(r)$ if $0 < r \ll 1$ or $r \gg 1$.
  (c) 1 Jan. 2025
(5) p. 50 line 3 from below, Example 42(5)
  (a) Then e_p can be arranged to be a Young function.
  (b) Then there exists $\widetilde{e_p}$ such that $e_p(r) = \widetilde{e_p}(r)$ if $0 < r \ll 1$ or $r \gg 1$.
  (c) 1 Jan. 2025
(6) p. 50 line 1 from below, Example 42(6)
  (a) can be arranged to be an Orlicz function.
  (b) is an Orlicz function.
  (c) 1 Jan. 2025
(7) p. 57, Lemma 49 (1)
  (a) for all $0 < t \leq s < \infty$
  (b) for all $0 < t \leq s < \infty$,
  (c) 5 Dec. 2023
(8) p. 58, Example 45 (3)
  (a) $\exp(\mu)$
  (b) $L^{\exp}(\mu)$
  (c) 5 Dec. 2023
(9) p. 58, Example 45 (4)
  (a) Let E be a measurable set and let $\Phi : [0, \infty) \to [0, \infty)$ be a Young function.
  (b) Let E be a measurable set of X and let $\Phi : [0, \infty) \to [0, \infty)$ be an injective Young function.
  (c) 5 Dec. 2023
  (d) This book did not define the generalized inverse.
(10) p. 59, line 7 from above, the proof of Theorem 50
  (a) $=$
  (b) $\leq$
  (c) 5 Dec. 2023
(11) p. 59, line 9 from below, Theorem 51
  (a) $\mathrm{d}x$
  (b) $\mathrm{d}\mu(x)$
  (c) 5 Dec. 2023
(12) p. 59, line 9 from below (twice), Theorem 51
  (a) $\|_{L^\Phi}$
  (b) $\|_{L^\Phi(\mu)}$
  (c) 9 Nov. 2024
(13) p. 59, line 8 from below, the proof of Theorem 51
  (a) By the Fatou lemma, we may assume that $f \in L^\infty(\mu)$ and that $\mu\{f \neq 0\} < \infty$.
  (b) If $f \in L^0(\mu) \setminus L^\Phi(\mu)$, then the conclusion is clear.

   (c) 5 Dec. 2023
(14) p. 59, line 7 from below, the proof of Theorem 51
   (a) $\|_{L^\Phi}$
   (b) $\|_{L^\Phi(\mu)}$
   (c) 9 Nov. 2024
(15) p. 59, line 6 from below
   (a) $\mathrm{d}x$
   (b) $\mathrm{d}\mu(x)$
   (c) 5 Dec. 2023
(16) p. 59, Theorem 52
   (a) Add: Assume that Φ^* is finitely valued.
   (b) 5 Dec. 2023

0.7. **Pages 60–69.**

(1) p. 60, Theorem 53
   (a) Add: Assume that Φ^* is finitely valued.
   (b) 5 Dec. 2023
(2) p. 61 line 1 from above
   (a) probabilty
   (b) probability
   (c) 5 Dec. 2024
(3) p. 61, Definition 26,
   (a) $(0,\infty)$
   (b) $[0,\infty)$
   (c) 5 Dec. 2023
(4) p. 61, Definition 26,
   (a) For a cube Q
   (b) For a cube Q and $f \in L^0(Q)$.
   (c) 5 Dec. 2024
(5) p. 61, Definition 26 (1.14)
   (a) $\|f\|_{\Phi;Q} \equiv \inf\left\{\lambda>0\,:\,\frac{1}{|Q|}\int_Q \Phi\left(\frac{|f(x)|}{\lambda}\right){\rm d}x \le 1\right\}$.
   (b) $\|f\|_{\Phi;Q} \equiv \inf\left(\left\{\lambda>0\,:\,\frac{1}{|Q|}\int_Q \Phi\left(\frac{|f(x)|}{\lambda}\right){\rm d}x \le 1\right\}\cup\{\infty\}\right)$.
   (c) 7 Jul. 2021
(6) p. 61, Example 47,
   (a) If $\Phi(t) = {\rm L}\log{\rm L}^\alpha(t) \equiv t(\log(e+t))^\alpha$, then $\|f\|_{{\rm L}\log{\rm L}^\alpha;Q} \equiv \|f\|_{\Phi;Q}$.
   (b) Let $\alpha>0$. If $\Phi(t) = {\rm L}\log{\rm L}^\alpha(t) \equiv t(\log(e+t))^\alpha$, then Φ is a convex function satisfying $\|f\|_{{\rm L}\log{\rm L}^\alpha;Q} \equiv \|f\|_{\Phi;Q}$.
   (c) 5 Dec. 2023
   (d) We note
   $$\frac{{\rm d}^2}{{\rm d}^2t}(t(\log(e+t))^\alpha) = \frac{\alpha(\log(e+t))^{\alpha-2}}{(e+t)^2}((2e+t)\log(e+t)+(a-1)t).$$
(7) p. 61, line 6 from below, the proof of Lemma 54
   (a) $\|f\|_{\Phi;Q} \equiv \inf\left\{\lambda>0\,:\,\frac{1}{|Q|}\int_Q \Phi\left(\frac{|f(x)|}{\lambda}\right){\rm d}x \le 1\right\}$
   (b) $\|f\|_{\Phi;Q} \equiv \inf\left(\left\{\lambda>0\,:\,\frac{1}{|Q|}\int_Q \Phi\left(\frac{|f(x)|}{\lambda}\right){\rm d}x \le 1\right\}\cup\{\infty\}\right)$
   (c) 7 Jul. 2021
(8) p. 61, line 5 from below, the proof of Lemma 54
   (a) $\|f\|_{\Phi;Q} \equiv \inf\left\{\lambda>0\,:\,\frac{1}{|R|}\int_R \Phi\left(\frac{|f(\kappa x)|}{\lambda}\right){\rm d}x \le 1\right\}$
   (b) $\|f\|_{\Phi;Q} \equiv \inf\left(\left\{\lambda>0\,:\,\frac{1}{|R|}\int_R \Phi\left(\frac{|f(\kappa x)|}{\lambda}\right){\rm d}x \le 1\right\}\cup\{\infty\}\right)$
   (c) 7 Jul. 2021
(9) p. 62 line 1 from below, the proof of Lemma 57
   (a) in the left-hand side
   (b) on the left-hand side
   (c) 21 Nov. 2024
(10) p. 64, Definition 27, three times
   (a) ρ_p
   (b) $\rho_{p(\cdot)}$
   (c) 7 Jul. 2021

(11) p. 64, Definition 27
  (a) *variable Lebesgue norm*
  (b) *variable (exponent) Lebesgue norm*
  (c) 7 Jul. 2021
(12) p. 65 line 15 from below
  (a) the trace theorem, the extension theorem
  (b) the trace theorem
  (c) 7 Jul. 2021
(13) p. 65, Definition 28
  (a) Partial derivative
  (b) Weak partial derivative
  (c) 7 Jul. 2021
(14) p. 66, line 12 from above, Example 66,
  (a) of the right-hand
  (b) on the right-hand
  (c) 21 Nov. 2024
(15) p. 69, line 3 from above,
  (a) function.
  (b) function with Lipschitz constant M.
  (c) 1 Dec. 2024
(16) p. 69 line 6 from above
  (a) $y_n > L|y'|$
  (b) $y_n > M|y'|$
  (c) 1 Jan. 2024
(17) p. 69 line 6 from above
  (a) Since
  (b) Let $x, y = (y', y_n) \in \mathbb{R}^n$. $y_n > M|y'|$. Since
  (c) 1 Jan. 2024
(18) p. 69 line 11 from above, the proof of Lemma 67
  (a) $y_n > L|y'|$
  (b) $y_n > M|y'|$
  (c) 1 Jan. 2024
(19) p. 69 line 13 from above, the proof of Lemma 67
  (a) Then
  (b) Then using the matrix notation we have
  (c) 1 Jan. 2024
(20) p. 69 lines 11 from below, Lemma 68
  (a) D^α
  (b) ∂^α
  (c) 1 Jan. 2025
(21) p. 69 lines 11 and 7 from below, Lemma 68 and its proof
  (a) $\tilde{\Omega}_k$
  (b) $\tilde{G}_k$
  (c) 1 Jan. 2025

0.8. **Pages 70–79.**

(1) p. 70, line 1 from above, Lemma 69,
  (a) $a>0$ be fixed.
  (b) $a=2^{\tilde{l}}$ be fixed with $\tilde{l}\in\mathbb{Z}$.
  (c) 1 Dec. 2024
(2) p. 70 line 2 from above, Lemma 69
  (a) $x\in G$ and z
  (b) $x=(x',x_n)\in\tilde{G}_{k-\tilde{l}}$ and $z=(z',z_n)$
  (c) 1 Dec. 2024
(3) p. 70 line 4 and 5 from above, the proof of Lemma 69
  (a) We are now ready for the construction of Burenkov's extension operator. Choose $\omega\in C_{\rm c}^\infty((-a,a)^{n-1}\times(2a,4a))\cap\mathbb{M}^+(\mathbb{R}^n)$ so that $\|\omega\|_{L^1}=1$
  (b) Let $a=2^{\tilde{l}}$ with $\tilde{l}\in\mathbb{Z}$. Choose $\omega\in C_{\rm c}^\infty((-a,a)^{n-1}\times(2a,4a))$ so that
$$\int_{\mathbb{R}^n}x^\alpha\omega(x){\rm d}x=\delta_{\alpha 0}$$
for any α with $|\alpha|\le l$.
  (c) 1 Dec. 2024, 1 Jan. 2025
(4) p. 70 line 6 from above, the proof of Lemma 69
  (a) $x\in G$
  (b) $x\in\tilde{G}_{k-\tilde{l}}$
  (c) 1 Dec. 2024
(5) p. 70 line 10 from above
  (a) ψ_k
  (b) $\psi_{k-\tilde{l}}$
  (c) 1 Dec. 2024
(6) p. 70 line 14 from below
  (a) $\partial^\alpha f\in L^\infty(\Omega)$ for any
  (b) and $\partial^\alpha f$ is bounded uniformly continuous for any
  (c) 1 Dec. 2024
(7) p. 70 line 13 from below
  (a) any order
  (b) order l
  (c) 1 Dec. 2024
(8) p. 70 line 7 from below
  (a) $Z'(x)$
  (b) $Z'(x)=Z'_\beta(x)$
  (c) 1 Dec. 2024
(9) p. 70 line 1 from below, the proof of Theorem 70 (twice)
  (a) ψ_k
  (b) $\psi_{k-\tilde{l}}$
  (c) 1 Jan. 2024
(10) p. 71 line 1 from above, the proof of Theorem 70
  (a) of the Taylor expansion
  (b) of the Taylor expansion of ζ_l
  (c) 1 Dec. 2024
(11) p. 71 line 3 from above, the proof of Theorem 70
  (a) $\omega\in C_{\rm c}^\infty((-a,a)^{n-1}\times(2a,4a))\cap\mathbb{M}^+(\mathbb{R}^n)$ and $\sum\limits_{k=-\infty}^{\infty}\partial^\beta\psi_k\equiv 0$

(b) $\omega \in C_c^\infty((-a,a)^{n-1} \times (2a,4a))$ and $\int_{\mathbb{R}^n} x^\alpha \omega(x) \mathrm{d}x = \delta_{\alpha 0}$ for all $\alpha \in \mathbb{N}_0{}^n$ with $|\alpha| \le l$,
(c) 1 Dec. 2024

(12) p. 71 lines 5 and 6 from above, the proof of Theorem 70
(a) ψ_k
(b) $\psi_{k-\tilde{l}}$
(c) 1 Jan. 2024

(13) p. 71, line 12 from above, the proof of Lemma 71
(a) $x \in G_k$
(b) $x \in \tilde{G}_{k-\tilde{l}}$
(c) 1 Dec. 2024

(14) p. 71, line 12 from above, the proof of Lemma 71
(a) $2^{-k-1} < x_n - \varphi(x') \le 2^{-k}$
(b) $2^{-k-1+\tilde{l}} < x_n - \varphi(x') \le 2^{-k+\tilde{l}}$
(c) 1 Dec. 2024

(15) p. 72 line 5 from above
(a) We move on
(b) Thus, T is a continuous mapping from $W^{l,p}(\Omega)$ to $W^{l,p}(\Omega)$. We move on
(c) 1 Jan. 2024

(16) p. 72, line 14 from above, the proof of Lemma 74
(a) $\left[\frac{1}{2}, \frac{3}{4}\right]$
(b) $\left[\frac{1}{4}, 1\right]$
(c) 1 Dec. 2024

(17) p. 74 line 3 from below, the proof of Lemma 76
(a) $2^{\min(j,k)n-2|j-k|}$
(b) $2^{\min(j,k)\gamma-2|j-k|}$
(c) 1 Dec. 2024

(18) p. 74 line 2 from below, the proof of Lemma 76
(a) $\min(2^{j|\alpha|-k\gamma}, 2^{k|\alpha|-j\gamma})$
(b) $2^{\min(j,k)|\alpha|-\min(j,k)\gamma-2|j-k|}$
(c) 1 Dec. 2024

(19) p. 74 line 2 from below, the proof of Lemma 76
(a) for all
(b) for all $\alpha \in \mathbb{N}_0{}^n$ and
(c) 1 Dec. 2024

(20) p. 75 line 2 from above, Lemma 77
(a) In particular,
(b) In particular, in the topology of $L^\infty(\mathbb{R}^n)$,
(c) 1 Dec. 2024

(21) p. 75 line 16 from above, Lemma 78
(a) is smooth
(b) is smooth if $|\alpha| \ge 2$
(c) 1 Dec. 2024

(22) p. 76 line 14 from below
(a) definitions
(b) definitions of
(c) 1 Dec. 2024

(23) p. 78, lines 8 and 11 from above (three times)
(a) inhomogeneous
(b) nonhomogeneous
(c) 21 Sept. 2023

(d) This is just a matter of unification.

0.9. Pages 80–89.

(1) p. 80, line 16 from above
  (a) Stampaccia
  (b) Stampacchia
  (c) 21 Sept. 2023
(2) p. 87, Theorem 85
  (a) $p : V \to \mathbb{C}$
  (b) $p : V \to [0, \infty)$
  (c) 24 Apr. 2024
(3) p. 87, Theorem 85
  (a) $l_0 : V \to \mathbb{R}$
  (b) $l_0 : V \to \mathbb{C}$
  (c) 24 Apr. 2024
(4) p. 87, Theorem 85
  (a) $l_0(u) \le p(u)$
  (b) $\mathrm{Re}(l_0(u)) \le p(u)$
  (c) 24 Apr. 2024
(5) p. 88, line 1 from below, the proof of Theorem 90
  (a) $\min(\|x\|_{\mathcal{X}}, \|y\|_{\mathcal{X}}$
  (b) $\min(\|x\|_{\mathcal{X}}, \|y\|_{\mathcal{X}})$
  (c) 15 May 2024
(6) p. 90, line 2 from above
  (a) charaterize
  (b) characterize
  (c) 21 Sept. 2023
(7) p. 92, line 15 from below (1)
  (a) Remove
  (b) for all $x \in \mathcal{H}$
  (c) 14 Dec. 2023
(8) p. 92, line 14 from below (2)
  (a) Remove
  (b) for all $x, y \in \mathcal{H}$
  (c) 14 Dec. 2023
(9) p. 92, line 13 from below (3)
  (a) Remove
  (b) for all $x, y, z \in \mathcal{H}$
  (c) 14 Dec. 2023
(10) p. 92, line 8 from below
  (a) Komlos
  (b) Komlós
  (c) 26 June, 2024
(11) p. 92, line 7 from below
  (a) Komlos
  (b) Komlós
  (c) 26 June, 2024
(12) p. 92, line 6 from below
  (a) Komlos
  (b) Komlós
  (c) 26 June, 2024
(13) p. 93, line 3 from above
  (a) $\|g\|_{\mathcal{H}} \in$

   (b) $\|g\|_{\mathcal{H}} : g \in$
   (c) 26 June, 2024
(14) p. 93, line 6 from above
   (a) $\|g\|_{\mathcal{H}} \in$
   (b) $\|g\|_{\mathcal{H}} : g \in$
   (c) 26 June, 2024
(15) p. 94, line 9 from below, the proof of Theorem 94
   (a) A^{2m-1}
   (b) A^{2m}
   (c) 14 Feb. 2021
(16) p. 95, line 9 from above
   (a) $u_0(x) = u_0(x)$
   (b) $u(x,0) = u_0(x)$
   (c) 10 Jul. 2024
(17) p. 95 line 12 from above
   (a) in the right-hand
   (b) on the right-hand
   (c) 21 Nov. 2024
(18) p. 96 line 7 from below, the proof of Theorem 96
   (a) .
   (b) for all $y \in \mathcal{X}$.
(19) p. 98 line 13 from above, Lemma 99
   (a) $\|a_j\|_X$
   (b) $\|a_j\|_{\mathcal{X}}$
   (c) 6 Nov. 2024
(20) p. 98 line 16 from above, the proof of Lemma 99
   (a) $\|b_j\|_X$
   (b) $\|b_j\|_{\mathcal{X}}$
   (c) 6 Nov. 2024
(21) p. 99 line 4 from above, the proof of Lemma 99
   (a) $\|b_j\|_X$
   (b) $\|b_j\|_{\mathcal{X}}$
   (c) 6 Nov. 2024
(22) p. 99 line 5 from above, the proof of Lemma 99
   (a) f
   (b) φ
   (c) 6 Nov. 2024
(23) p. 99 line 6 from above, the proof of Lemma 99
   (a) $\|b_j\|_X$
   (b) $\|b_j\|_{\mathcal{X}}$
   (c) 6 Nov. 2024
(24) p. 99 line 6 from above, the proof of Lemma 99
   (a) f
   (b) φ
   (c) 6 Nov. 2024
(25) p. 99 line 8 from above, the proof of Lemma 99
   (a) $\|b_j\|_X$
   (b) $\|b_j\|_{\mathcal{X}}$
   (c) 6 Nov. 2024
(26) p. 99 line 8 from above, the proof of Lemma 99
   (a) f
   (b) φ

   (c) 6 Nov. 2024

(27) p. 99 line 9 from above, the proof of Lemma 99
   (a) f
   (b) φ
   (c) 6 Nov. 2024

(28) p. 99 (2.8), In Definition 38, delete one of $\int_X \varphi(x)\mathrm{d}\mu(x)$. 11 Nov. 2024

0.10. **Pages 100–109.**

(1) p. 100, line 9 from above, the proof of Lemma 100
  (a) the admissible choice of φ
  (b) the choice of $\{\varphi_j\}_{j=1}^{\infty}$
  (c) 20 Nov. 2024
(2) p. 100, line 17 from above, the proof of Lemma 100
  (a) the admissible choice of φ
  (b) the choice of $\{\varphi_j\}_{j=1}^{\infty}$
  (c) 20 Nov. 2024
(3) p. 101, line 4 from above, the proof of Theorem 102
  (a) the Fatou lemma
  (b) the triangle inequality
  (c) 27 Nov. 2024
(4) p. 101, line 12 from above, the proof of Theorem 102
  (a) we can assume
  (b) we can assume that $\varphi_j(x) \to \varphi(x)$ as $j \to \infty$ for μ-a.e. $x \in X$ and that
  (c) 27 Nov. 2024
(5) p. 102, line 6 from above, Theorem 103
  (a) almost every
  (b) μ-a.e.
  (c) 6 Dec. 2024
(6) p. 102, lines 7,8 from above, Theorem 103
  (a) μ-almost every
  (b) μ-a.e.
  (c) 6 Dec. 2024
(7) p. 102, line 5 from above, Theorem 103: The proof of Theorem 103 uses Theorem 104 since we should have established that φ is an integrable countably simple function. This is actually done in the proof of Theorem 104. We should have mentioned that the proof of Theorem 103 uses Theorem 104.
(8) p. 103 line 12 from above, the proof of Theorem 103
  (a) Since $\bigcup_{j=1}^{\infty} Y_j$ is separable, whenever each Y_j is separable,
  (b) Let $Y_j \subset \mathcal{X}$ be a separable set such that $\mu(X \setminus {\varphi_j}^{-1}(Y_j)) = 0$. Since $\bigcup_{j=1}^{\infty} Y_j$ is separable,
  (c) 6 Dec. 2024
(9) p. 104, line 3 from above, the proof of Theorem 106
  (a) $\int_Y$
  (b) $\iint_{X \times Y}$
  (c) 6 Dec. 2024

0.11. Pages 110–119.

(1) p. 110 line 10 from below, Definition 42
  (a) $\mathcal{P}$
  (b) $\mathcal{P}(\mathbb{R}^n)$
  (c) 1 Nov. 2024
(2) p. 112 line 6 from below, the proof of Theorem 112
  (a) $B(2^j t) \setminus B(t)$
  (b) $B(2^j t) \setminus B(2^{j-1} t)$
  (c) 9 Oct. 2024
(3) p. 113 line 7 from above, Theorem 113
  (a) $\mathcal{P}_{N-1}$
  (b) $\mathcal{P}_{N-1}(\mathbb{R}^n)$
  (c) 1 Nov. 2024
(4) p. 113 line 8 from above, Theorem 113
  (a) differential inequality
  (b) inequality
  (c) 16 Oct. 2024
(5) p. 114 line 3 from below, the proof of Theorem 113
  (a) $\int_{\mathbb{R}^n \setminus B(t^{-1})} \langle x - z \rangle^{-\lambda} |z|^{-\lambda} \mathrm{d}z$
  (b) $\int_{\mathbb{R}^n \setminus B(1)} \langle x - z \rangle^{-\lambda} |z|^{-\lambda} \mathrm{d}z$
  (c) 13 Nov. 2024
(6) p. 115 line 5 from above, the proof of Theorem 113
  (a) $\int_{\mathbb{R}^n \setminus B(t^{-1})} |z|^{N-1-\lambda} \mathrm{d}z$
  (b) $\int_{\mathbb{R}^n \setminus B(1)} |z|^{N-1-\lambda} \mathrm{d}z$
  (c) 13 Nov. 2024
(7) p. 116, line 1 from above, Example 55
  (a) we have
  (b) for all $j, \tilde{j} \in \mathbb{Z}$ and $\nu, \tilde{\nu} \in \mathbb{Z}^n$ we have
  (c) 20 Nov. 2024
(8) p. 116, line 2 from above, Example 55
  (a) $\psi_{\tilde{j}\tilde{\nu}}$
  (b) $\varphi_{\tilde{j}\tilde{\nu}}$
  (c) 20 Nov. 2024
(9) p. 116, line 4 from above, Example 55
  (a) Fix $\tilde{j}$ and $\tilde{\nu}$. Then
  (b)
$$\begin{aligned} &\sim 2^{\frac{1}{2}(j+\tilde{j})n} \int_{\mathbb{R}^n} \chi_{Q_{\tilde{j}\tilde{\nu}}}(x) M\chi_{Q_{j\nu}}(x)^{\frac{n+2}{n}} \mathrm{d}x \\ &+2^{\frac{1}{2}(j+\tilde{j})n} \int_{\mathbb{R}^n} \chi_{Q_{j\nu}}(x) M\chi_{Q_{\tilde{j}\tilde{\nu}}}(x)^{\frac{n+2}{n}} \mathrm{d}x \\ &\lesssim 2^{\frac{1}{2}(j+\tilde{j})n} \int_{\mathbb{R}^n} M\chi_{Q_{\tilde{j}\tilde{\nu}}}(x)^{\frac{n+2}{n}} M\chi_{Q_{j\nu}}(x)^{\frac{n+2}{n}} \mathrm{d}x \end{aligned}$$
  (c) 20 Nov. 2024
(10) Delete lines 5–8 from above in page 116.
(11) p. 116, line 10 from above, Example 55
  (a) $\displaystyle \le 2 \sum_{j \in \mathbb{Z}, \nu \in \mathbb{Z}^n} \sum_{\tilde{j} \in \mathbb{Z}, \tilde{\nu} \in \mathbb{Z}^n} |a_{j\nu}|^2 \left| \int_{\mathbb{R}^n} \varphi_{j\nu}(x) \psi_{\tilde{j}\tilde{\nu}}(x) \mathrm{d}x \right|$
  (b) $\displaystyle \lesssim \int_{\mathbb{R}^n} \left(\sum_{j \in \mathbb{Z}, \nu \in \mathbb{Z}^n} |2^{\frac{jn}{2}} a_{j\nu}| M\chi_{Q_{j\nu}}(x)^{\frac{n+2}{n}} \right)^2 \mathrm{d}x$
  (c) 20 Nov. 2024

(12) p. 116, line 1 from below
   (a) $B(x_0, \rho)$.
   (b) $B(x_0, \rho)$. Here and below $\mathrm{diam}(\Omega)$ stands for the diameter of the set Ω:
$$\mathrm{diam}(\Omega) \equiv \sup_{x,y \in \Omega} |x - y|.$$
   (c) 27 Nov. 2024
(13) p. 118, line 8 from above, Lemma 116
   (a) $L^q_{\mathrm{loc}}(\Omega)$
   (b) $L^q_{\mathrm{loc}}(\overline{\Omega})$
   (c) 6 Dec. 2024
(14) p. 118, line 7 from above
   (a) $B(x_0, \rho)$. Denote by $\mathrm{diam}(\Omega)$ its diameter.
   (b) $B(x_0, \rho)$.
   (c) 27 Nov. 2024
(15) p. 119 line 19 from above, Remark 3
   (a) For any polynomial $P \in \mathcal{P}_k(\mathbb{R}^n)$
   (b) Then
   (c) 18 Dec. 2024
(16) p. 119, Proposition 117(1)
   (a) $P^k_{B(1)} f \left(\frac{\cdot - x_0}{r}\right)$
   (b) $P^k_{B(1)}[f(r \cdot + x_0)] \left(\frac{\cdot - x_0}{r}\right)$
   (c) 11 May, 2021

0.12. **Pages 120–129.**

(1) p. 120, line 11 from below
  (a) For
  (b) Let $\mathcal{P}_{-1}(\mathbb{R}^n) = \{0\}$. For
  (c) 14 Dec. 2023
(2) p. 121, line 6 from above the proof of Lemma 118,
  (a) Since
  (b) Assume $k \ge 2$, since the case where $k = 0, 1$ is trivial. Since
  (c) 6 Jan, 2023
  (d) In the proof We should have assumed $k \ge 2$, since the case where $k = 0, 1$ is trivial.
(3) p. 121, line 8 from above the proof of Lemma 118,
  (a) Since
  (b) Since, for all $k \in \mathbb{N}_0$,
  (c) 6 Jan, 2023
(4) p. 121, line 6 from above the proof of Lemma 118,
  (a) $(a_1x_1 + a_2x_2 + \cdots + a_nx_n)^k$
  (b) $(a_1x_1 + a_2x_2 + \cdots + a_nx_n)^{k-2}$
  (c) 6 Jan, 2023
  (d) We should have considered polynomials of order $k-2$ since the range is $\mathcal{P}_{k-2}(\mathbb{R}^n)$.
(5) p. 121, line 7 from above
  (a) $\mathcal{P}_k(\mathbb{R}^n)$
  (b) $\mathcal{P}_{k-2}(\mathbb{R}^n)$
  (c) 6 Jan, 2023
  (d) We should have considered polynomials of order $k-2$ since the range is $\mathcal{P}_{k-2}(\mathbb{R}^n)$, 21 Aug. 2024.
(6) p. 121, line 9 from above
  (a) $k + 2$
  (b) k
  (c) 21 Aug. 2024
(7) p. 121, line 10 from above
  (a) $(k + 2)(k + 1)({a_1}^2 + {a_2}^2 + \cdots + {a_n}^2)(a_1x_1 + a_2x_2 + \cdots + a_nx_n)^k$
  (b) $k(k-1)({a_1}^2 + {a_2}^2 + \cdots + {a_n}^2)(a_1x_1 + a_2x_2 + \cdots + a_nx_n)^{k-2}$
  (c) 21 Aug. 2024
(8) p. 121, lines 9 and 10 from above
  (a) Since
  (b) Since, for all $k \in \mathbb{N}$,
  (c) 6 Jan, 2023
  (d) We should have mentioned that this is valid for $k \ge 0$.
(9) p. 121, line 5 from below
  (a) $\mathcal{P}_k(\mathbb{R}^n)$
  (b) $\mathcal{H}_k(\mathbb{R}^n)$
  (c) 6 Jan, 2023
(10) p. 121, line 4 from below,
  (a) Consequently,
  (b) Here, we identified constants with polynomials of order zero for penultimate equality. Consequently,
  (c) 6 Jan, 2023
(11) p. 122, Example 59
  (a) $2m$
  (b) m
  (c) 6 Jan, 2023

(12) p. 125, line 1 from above
   (a) Hölder's inequality and the Bessel inequality
   (b) the Cauchy–Schwarz inequality
   (c) 6 Jan, 2023
(13) p. 125, line 7 from below, Example 60
   (a) calcluate
   (b) calculate
   (c) 21 Sept. 2023
(14) p. 126, line 11 from above, Exercise 59
   (a) reexaming
   (b) reexamining
   (c) 21 Sept. 2023

0.13. **Pages 130–139.**

(1) p. 131, Example 62
  (a) $f(t) \geq f(s)$
  (b) $f(t) \leq f(s)$
  (c) 5 June 2024
(2) p. 131, line 1 from below, the proof of Proposition 129
  (a) $M^{\mathcal{Q}}\chi_R$
  (b) $M^{\mathcal{Q}}\chi_R(x)$
  (c) 5 June 2024
(3) p. 134, line 12 from below, one line below Theorem 132
  (a) balls by cubes
  (b) balls by cubes. We have used the word $5r$ but this meaning will be clear after we state Theorem 133.
  (c) 26 sept. 2024
(4) p. 134, line 15 from below, the proof of Theorem 132
  (a) $j > J$
  (b) $j > J + 1$
  (c) 14 Feb. 2021
(5) p. 134, line 13 from below, the proof of Theorem 132
  (a) $J(j) = 2, 3, \ldots, K$
  (b) a mapping $J : \{2, 3, \ldots, J + 1\} \to \{2, 3, \ldots, K\}$
  (c) 14 Feb. 2021
(6) p. 134, line 12 from below, the proof of Theorem 132
  (a) unless
  (b) for each
  (c) 26 sept. 2024
(7) p. 134, line 11 from below, the proof of Theorem 132
  (a) $\iota : \mathbb{N} \to \{1, 2, \ldots, n\}$
  (b) $\iota : \{1, 2, \ldots, J_0 + 1\} \to \{1, 2, \ldots, K\}$
  (c) 17 July 2024
(8) p. 135, line 1 from above, Theorem 133
  (a) $\Lambda_0 \to \Lambda$
  (b) $\Lambda \to \Lambda_0$
  (c) 26 sept. 2024
(9) p. 135 line 3 from below, We should have mentioned that this duplicates Definition 7, 16 Oct. 2024
(10) p. 138, line 15 from above, the proof of Proposition 139
  (a) define
  (b) select
  (c) 27 Nov. 2024
(11) p. 139, line 6 from above, the proof of Proposition 139
  (a) the proof is complete.
  (b) the proof of the first inequality is complete. The second inequality can be proved by reexamining the proof; see the last to two step of this proof.
  (c) 27 Nov. 2024

0.14. **Pages 140–149.**

(1) p. 145, line 9 from above, the proof of Lemma 146
  (a) $= (\|\{f_j\}_{j=1}^{\infty}\|_{\ell^q})^p$
  (b) $= A^{-p'}(\|\{f_j\}_{j=1}^{\infty}\|_{\ell^q})^p$
  (c) 20 Oct. 2021
(2) p. 147, line 7 from above
  (a) Littleowood
  (b) Littlewood
  (c) 21 Sept. 2023

0.15. **Pages 150–159.**

(1) p. 154, line 5 from below, the headder of Section 4.1.7
   (a) boundednss
   (b) boundedness
   (c) 21 Sept. 2023
(2) p. 157 line 1 from below, Proposition 159
   (a) . for all
   (b) for all
   (c) 25 Dec. 2024
(3) p. 159 line 5 from above, the proof of Corollary 162
   (a) in the right-hand
   (b) on the right-hand
   (c) 21 Nov. 2024

0.16. **Pages 160–169.**

(1) p. 169, line 5 from below, Example 79(4)
   (a) famlies
   (b) families
   (c) 21 Sept. 2023

0.17. **Pages 170–179.**

(1) p. 172 line 11 from below, the proof of Lemma 175
  (a) in the right-hand
  (b) on the right-hand
  (c) 21 Nov. 2024
(2) p. 174 line 5 from below
  (a) Before
  (b) Let $u = 1 - \frac{p\alpha}{n}$. Before
  (c) 24 Dec. 2024
(3) p. 175 lines 1 and 2 from above, the proof of Theorem 177
  (a) We choose $u \in (0,1)$ so that $n(1-u) = p\alpha$. Then by Hölder's inequality,
  (b) Then by Hölder's inequality, $\|F\|_{L^{p_\theta}} \le (\|F\|_{L^{p_0}})^{1-\theta}(\|F\|_{L^{p_1}})^{\theta}$, which is valid for $F \in L^0(\mathbb{R}^n)$, $0 < p_0, p_1 < \infty$, $0 < \theta < 1$ and $p_\theta = \left(\dfrac{1-\theta}{p_0} + \dfrac{\theta}{p_1}\right)^{-1}$,
  (c) 17 Oct. 2024
(4) p. 176, line 1 from above, the proof of Theorem 178(1)
  (a) conseqneuce
  (b) consequence
  (c) 21 Sept. 2023
(5) p. 176, line 12 from above, the proof of Theorem 178(1)
  (a) $m^{p_1}_{Q(r)}$
  (b) $m^{(p_1)}_{Q(r)}$
  (c) 21 Sept. 2023
(6) p. 177 line 6 from above, one line above Theorem 179
  (a) of great use.
  (b) of great use. Denote by $\mathcal{D}^\sharp(Q)$ the set of all dyadic cubes containing Q for a dyadic cube Q.
  (c) 17 Sept. 2024
(7) p. 177, line 13 from above
  (a) $\le C$
  (b) $\lesssim$
  (c) 9 Nov. 2024

0.18. **Pages 180–189.**

(1) p. 183 line 10 from above, the proof of Theorem 184
   (a) $\left(|x|^{\frac{n}{p_1}}\log\frac{1}{|x|}\right)$
   (b) $\left(|x|^{\frac{n}{p_1}}\log\frac{1}{|x|}\right)^{-1}$
   (c) 21 Sept. 2023
(2) p. 185 line 14 from above, the proof of Proposition 188
   (a) $\mathcal{D}_\nu(\mathbb{R}^n)$
   (b) $\mathcal{D}_{\nu_0}(\mathbb{R}^n)$
   (c) 21 Sept. 2023
(3) p. 185 line 15 from above, the proof of Proposition 188
   (a) $\mathcal{D}_{\nu+1}(\mathbb{R}^n)$
   (b) $\mathcal{D}_{\nu_0+1}(\mathbb{R}^n)$
   (c) 21 Sept. 2023
(4) p. 187, line 3 from below, the proof of Theorem 192
   (a) the continuity of φ
   (b) the continuity of φ_A
   (c) 21 Sept. 2023
(5) p. 188, line 10 from below
   (a) operartors
   (b) operators
   (c) 21 Sept. 2023
(6) p. 189, line 7 from above, the proof of Lemma 194
   (a) theroem
   (b) theorem
   (c) 21 Sept. 2023
(7) p. 189 line 6 from below, Proposition 196
   (a) Let $s > 0$.
   (b) Let $0 < s < n$.
   (c) 25 Dec. 2024
(8) p. 189 line 6 from below, Proposition 196
   (a) $x \to 0$.
   (b) $x \to 0$. If $s > n$, then G_s is a bounded function.
   (c) 25 Dec. 2024
(9) p. 189 line 5 from below, the proof of Proposition 196
   (a) Let
   (b) Assume $s \in (0, n)$; otherwise modify the proof below. Let
   (c) 25 Dec. 2024

0.19. **Pages 190–199.**

(1) p. 190 line 10 from above
   (a) We remark
   (b) Fix $x \in \mathbb{R}^n \setminus 0$. We remark
   (c) 21 Sept. 2023
(2) p. 190 lines 2 and 1 from below
   (a) dy
   (b) $\mathrm{d}y$
   (c) 29 Dec. 2024
(3) p. 193, line 13 from above
   (a) Zgymund
   (b) Zygmund
   (c) 21 Sept. 2023
(4) p. 199, line 6 from above
   (a) Consequenlty
   (b) Consequently
   (c) 21 Sept. 2023

0.20. **Pages 200–209.**

(1) p. 200, line 1 from above, the headder of §4.5.2
   (a) Zgymund
   (b) Zygmund
   (c) 21 Sept. 2023
(2) p. 202 line 14 from above, the proof of Theorem 206
   (a) nonoverlapping
   (b) nonoverlapping in the sense that $\mathrm{supp}(b_j)$ and $\mathrm{supp}(b_{j'})$ do not have common interior point if j and j' are distinct,
   (c) 25 May, 2021
(3) p. 202 line 15 from above, the proof of Theorem 206
   (a) moment condition
   (b) $L^1(\mathbb{R}^n)$-condition
   (c) 25 May, 2021
(4) p. 206, line 5 from above
   (a) confortable
   (b) comfortable
   (c) 21 Sept. 2023
(5) p. 209, line 4 from above, Proposition 213
   (a) p.v. $\displaystyle\int_{\mathbb{R}^n} \partial_{ij}\Gamma(x)\mathrm{d}x \equiv \lim_{\varepsilon\downarrow 0}\int_{\mathbb{R}^n\setminus B(\varepsilon)} \partial_{ij}\Gamma(x)\mathrm{d}x.$
   (b) $\displaystyle\langle \mathrm{p.v.}\partial_{ij}\Gamma, \varphi\rangle \equiv \lim_{\varepsilon\downarrow 0}\int_{\mathbb{R}^n\setminus B(\varepsilon)} \partial_{ij}\Gamma(x)\varphi(x)\mathrm{d}x$ for $\varphi \in C^\infty_{\mathrm{c}}(\mathbb{R}^n)$.
   (c) 24 Jan. 2024
(6) p. 209, line 6, 8, 9 from above, the proof of Proposition 213
   (a) p.v. $\displaystyle\int_{\mathbb{R}^n} \partial_{ij}\Gamma(x)\mathrm{d}x$
   (b) $\langle \mathrm{p.v.}\partial_{ij}\Gamma, \varphi\rangle$
   (c) 24 Jan. 2024

0.21. **Pages 210–219.**

(1) p. 213 line 8 from below, the proof of Corollary 220
    (a) in the right-hand
    (b) on the right-hand
    (c) 21 Nov. 2024
(2) p. 218, line 16 from above, §4.5
    (a) fundametal
    (b) fundamental
    (c) 21 Sept. 2023

0.22. **Pages 220–229.**

(1) p. 222, line 8 from above
  (a) involoved
  (b) involved
  (c) 21 Sept. 2023
(2) p. 222, line 1 from below, Example 97
  (a) add
  (b) Thus, in this case
$$m_Q(|f - m_Q(f)|) \leq \frac{1}{|Q|^2} \int_{Q \times Q} |f(y) - f(z)| \mathrm{d}y \mathrm{d}z \lesssim 1 < \infty.$$
  Putting these observations together, we see that $f \in \mathrm{BMO}(\mathbb{R}^n)$.
  (c) 21 Sept. 2023
(3) p. 223, line 13 from above, the proof of Proposition 221
  (a) $\leq$
  (b) $=$
  (c) June 26, 2024
(4) p. 223, line 13 from above, the proof of Proposition 221
  (a) $Q(2^j)$
  (b) $Q(2^j) \setminus Q(2^{j-1})$
  (c) June 26, 2024
  (d) Otherwise, we can not go to the next step.
(5) p. 225, line 10 from above, the proof of Theorem 224
  (a) $\lambda = k\, 2^{n+2}\, \|b\|_{\mathrm{BMO}}$
  (b) $\lambda = k\, 2^{n+2}$ in (5.2)
  (c) 17 July 2024
(6) p. 225, line 10 from below, the proof of Theorem 224
  (a) are disjoint
  (b) are not disjoint
  (c) 17 July 2024
(7) p. 226 line 9 from below, the proof of Corollary 225
  (a) p
  (b) $p|Q|$
  (c) 9. Oct. 2024
(8) p. 226 line 8 from below, the proof of Corollary 225
  (a) $\Gamma(p)$
  (b) $\Gamma(p)|Q|$
  (c) 9. Oct. 2024
(9) p. 226 line 7 from below, the proof of Corollary 225
  (a) $\Gamma(p+1)$
  (b) $\Gamma(p+1)|Q|$
  (c) 9. Oct. 2024
(10) p. 227 line 3 from above, the proof of Corollary 226
  (a) $\left(\theta \frac{|b(x) - m_Q(b)|^j}{\|b\|_{\mathrm{BMO}}^j}\right)$
  (b) $\left(\theta^j \frac{|b(x) - m_Q(b)|^j}{\|b\|_{\mathrm{BMO}}^j}\right)$
  (c) 16 Oct. 2024
(11) p. 229, line 5 from above
  (a) (5.1)
  (b) Hölder's inequality
  (c) Dec. 28 2023

0.23. **Pages 230–239.**

(1) p. 230 line 5 from below, the proof of Lemma 230
  (a) $|a - m_Q(a)|$
  (b) $a - m_Q(a)$
  (c) 21 Nov. 2024
(2) p. 230, line 5 from below, the proof of Lemma 230
  (a) $m_Q^{(r)}(|Tf|)$
  (b) $m_Q^{(r)}(Tf)$
  (c) 20 Nov. 2024
(3) p. 230, line 2 from below, the proof of Lemma 230
  (a) $m_Q^{(r)}(|F_2|)$
  (b) $m_Q^{(r)}(F_2)$
  (c) 20 Nov. 2024
(4) p. 231, line 2 from above, the proof of Lemma 230
  (a) $m_{2Q}^{(\sqrt{r})}(|(a - m_Q(a))f|)$
  (b) $m_{2Q}^{(\sqrt{r})}((a - m_Q(a))f)$
  (c) 20 Nov. 2024
(5) p. 231 line 7 from above
  (a) Let $\ell \le 2\ell(Q)$. Then, by
  (b) By
  (c) 29 Dec. 2024
(6) p. 231, line 9 from above, the proof of Lemma 230
  (a) , we have
  (b) for $\ell \le 2\ell(Q)$, we have
  (c) 20 Nov. 2024
(7) p. 232 line 8 from above
  (a) $B(\varepsilon)$
  (b) $B(x, \varepsilon)$
  (c) 27 Nov. 2024
(8) p. 232 line 12 from above, twice
  (a) $B(\varepsilon)$
  (b) $B(x, \varepsilon)$
  (c) 27 Nov. 2024
(9) p. 233 line 10 from above, the proof of Lemma 232
  (a) $M^{(r)}T_*$
  (b) $M^{(r)} \circ M$
  (c) 6 Dec. 2024
(10) p. 233 line 5 from below, the proof of Lemma 232
  (a) $\frac{1}{\varepsilon^n}\varphi$
  (b) φ
  (c) 6 Dec. 2024
(11) p. 234 line 1 from above, the proof of Lemma 232
  (a) L^r
  (b) $L^{\sqrt{r}}$
  (c) 6 Dec. 2024
(12) p. 236, line 2 from above, the proof of Lemma 234
  (a) $\left| \int_{\mathbb{R}^n} \cdots \right|$

   (b) $\frac{1}{|Q|}\left|\int_{\mathbb{R}^n}\cdots\right|\mathrm{d}z$
   (c) 6 Dec. 2024
(13) p. 238, line 4 from above, Exercise 80
   (a) boundedess
   (b) boundedness
   (c) 21 Sept. 2023
(14) p. 239, line 17 from below, Definition 66
   (a) Other variables
   (b) Other variables x_0, ρ, f
   (c) 8 Dec. 2024
(15) p. 239, line 15 from below, Definition 66(2)
   (a) $\alpha \in \mathbb{N}_0$
   (b) $\alpha \in \mathbb{N}_0{}^n$
   (c) Dec. 28 2023
(16) p. 239, line 9 from below, Lemma 239
   (a) for all $x_0 \in \mathbb{R}$ and $\rho > 0$.
   (b) for all $x_0 \in \mathbb{R}^n$, $\rho > 0$ and $\alpha \in \mathbb{N}_0{}^n$ with $|\alpha| \le k$.
   (c) Dec. 28 2023
(17) p. 239 line 2 from below, the proof of Lemma 239
   (a) $-u\|_{L^1(B(x_0,\rho)}$
   (b) $-f\|_{L^1(B(x_0,\rho)}$
   (c) 2 Jan. 2024

0.24. **Pages 240–249.**

(1) p. 240 line 2 from above.
  (a) $1 \le q < \infty$
  (b) $1 \le q < \infty$, $k \in \mathbb{N}_0 \cup \{-1\}$, $\lambda \in \mathbb{R}$
  (c) 8 Dec. 2024
(2) p. 240 line 3 from above.
  (a) $\lambda, 1$
  (b) λ, q
  (c) 8 Dec. 2024
(3) p. 240 line 6 from above, Lemma 240
  (a) $x_0, y_0 \in \mathbb{R}$
  (b) $x_0, y_0 \in \mathbb{R}^n$
  (c) 8 Dec. 2024
(4) p. 240, line 16 from above, the proof of Theorem 241
  (a) $P^{l-1}_{B(x_0,\rho)} f \in \mathcal{P}_l(\mathbb{R}^n)$
  (b) $P \in \mathcal{P}_{l-1}(\mathbb{R}^n)$
  (c) Dec. 28 2023
(5) p. 240 line 17 from above, the proof of Theorem 241
  (a) $P^{l-1}_{B(x_0,\rho)}$
  (b) P
  (c) 8 Dec. 2024
(6) p. 240, line 19 from above, the proof of Theorem 241
  (a) We start with
  (b) Let α be a multiindex with $|\alpha| = l$. We start with
  (c) Dec. 28 2023
(7) p. 240, line 9 from below, the proof of Theorem 241
  (a) $(a_\alpha(x_0;\rho,f) - a_\alpha)$
  (b) $\frac{a_\alpha(x_0;\rho,f)-a_\alpha}{\alpha!}$
  (c) Dec. 28 2023
(8) p. 240, line 9 from below, the proof of Theorem 241
  (a) Delete
  (b) and $P^l_B(x) \equiv Q^l_B(x) - P(x)$
  (c) Dec. 28 2023
(9) p. 240, line 8 from below, the proof of Theorem 241
  (a) $(a_\alpha(x_0;\rho,f) - a_\alpha)$
  (b) $\frac{a_\alpha(x_0;\rho,f)-a_\alpha}{\alpha!}$
  (c) Dec. 28 2023
(10) p. 240, line 8 from below, the proof of Theorem 241
  (a) $P^l_B(x)$
  (b) $P_l(x;x_0,\rho,f)$
  (c) Dec. 28 2023
(11) p. 240, line 7 from below, the proof of Theorem 241
  (a) $P^l_B(x)$
  (b) $P_l(x;x_0,\rho,f)$
  (c) Dec. 28 2023
(12) p. 240, line 7 from below, the proof of Theorem 241
  (a) $a_\alpha(B)$
  (b) $a_\alpha(x_0;\rho,f)$
  (c) Dec. 28 2023
(13) p. 240, line 6 from below, the proof of Theorem 241
  (a) $P = Q^l_B$ with some suitable constant k_α in the above.

(b) $P = Q_B^l - \sum_{|\alpha|=l} \frac{k_\alpha}{\alpha!}(x-x_0)^\alpha$ with some suitable constants k_α that make P belong to $\mathcal{P}_{l-1}(\mathbb{R}^n)$ in the above because f and $f + \sum_{|\alpha|=l} \frac{k_\alpha}{\alpha!}(x-x_0)^\alpha$ are identical in $\mathcal{L}_l^{\lambda,q}(\mathbb{R}^n)$.
(c) Dec. 28 2023

(14) p. 241, line 2 from above, Theorem 242
  (a) $u_\infty = \lim_{R\to\infty} a_0(x_0; R, u)$
  (b) $f_\infty = \lim_{R\to\infty} a_0(x_0; R, f)$
  (c) Dec. 28 2023
(15) p. 241, line 3 from above, Theorem 242
  (a) $a_0(x_0; R, u)$
  (b) $a_0(x_0; R, f)$
  (c) Dec. 28 2023
(16) p. 241, line 4 from above, Theorem 242
  (a) Furthemore u_∞
  (b) Furthermore f_∞
  (c) 21 Sept. 2023, Dec. 28 2023
(17) p. 241, line 5 from above, Theorem 242
  (a) $u - u_\infty$
  (b) $f - f_\infty$
  (c) Dec. 28 2023
(18) p. 241, line 5 from above, Theorem 242
  (a) of u
  (b) of f
  (c) Dec. 28 2023
(19) p. 241, line 6 from above, Theorem 242
  (a) from u
  (b) from f
  (c) Dec. 28 2023
(20) p. 241, line 7 from above, Theorem 242
  (a) $u - u_\infty$
  (b) $f - f_\infty$
  (c) Dec. 28 2023
(21) p. 241, line 8 from above, Theorem 242
  (a) $u - u_\infty$
  (b) $f - f_\infty$
  (c) Dec. 28 2023
(22) p. 241, line 11 from above, the proof of Theorem 242
  (a) $\lambda > 0$
  (b) $\lambda = \frac{n}{p} > 0$
  (c) Dec. 28 2023
(23) p. 241 line 5 from below, the proof of Lemma 243
  (a) 2^{-n}
  (b) ρ'
  (c) 8 Dec. 2024
(24) p. 242 line 11 from above, the proof of Theorem 244 (twice)
  (a) $\|_{L^\infty}$
  (b) $\|_{L^\infty(B)}$
  (c) 2 Jan. 2024
(25) p. 242 line 7 from below, the proof of Theorem 245

(a) be a ball
(b) be a ball with $x_0 = (x_{1,0}, x_{2,0}, \dots, x_{n,0}) \in \mathbb{R}^n$ and $\rho > 0$.
(c) 1 Jan. 2024

(26) p. 242, line 2 from below, the proof of Theorem 245
(a) r
(b) ρ
(c) 8 Dec. 2024

(27) p. 242, line 2 from below, the proof of Theorem 245
(a) $[-\lambda]$
(b) $[-1-\lambda]$
(c) 8 Dec. 2024

(28) p. 242, line 1 from below, the proof of Theorem 245
(a) $r^{-\lambda}$
(b) $\rho^{-\lambda}$
(c) 8 Dec. 2024

(29) p. 243 line 8 from above, the proof of Lemma 246
(a) $x \neq y$
(b) $y \neq 0$
(c) 8 Dec. 2024

(30) p. 243 line 8 from above
(a) $|x-y|$
(b) $|y|$
(c) 8 Dec. 2024

(31) p. 243 lines 11 (twice), 12, 14, 15, 16 from above, the proof of Theorem 246
(a) $B(x, |x-y|)$
(b) $B(x, |y|)$
(c) 8 Dec. 2024

(32) p. 246 lines 13–17 from above: The proof of Theorem 246 (5 times)
(a) $\|f\|_{\mathcal{L}_k^{\lambda,q}}$
(b) $\|f\|_{\mathcal{L}_{[\lambda]}^{-\lambda,q}}$
(c) 1 Jan. 2024

(33) p. 243 line 13 from above
(a) $\lambda < 0$
(b) $\lambda > 0$
(c) 2 Jan. 2025

(34) p. 243 lines 14–18 from above, the proof of Theorem 246
(a) $|x-y|^{-\lambda}$
(b) $|y|^{\lambda}$
(c) 8 Dec. 2024

(35) p. 243 lines 10 and 8 from below, the proof of Theorem 246
(a) $2^{j\lambda}$
(b) $2^{\lambda j}$
(c) 1 Jan. 2024

(36) p. 244 line 6 from above: Exercise 83
(a) $-\dfrac{\lambda}{\|f\|_{\mathcal{L}_k^{\lambda,1}}}$
(b) $-\dfrac{c\lambda}{\|f\|_{\mathcal{L}_k^{\lambda,1}}}$
(c) 1 Jan. 2024

0.25. **Pages 250–259.**

(1) p. 251, line 16 from above
   (a) Next we prove
   (b) Remark that it can be arranged that each $\{B(x_\lambda, kr_\lambda)\}_{\lambda\in L^j}$ be disjoint. In fact, replace $\sqrt{k}-1$ with $2k\sqrt{k}$ and decompose each L^j further. Next we prove
   (c) 9 Dec. 2024
(2) p. 251 line 3 from below, the proof of Theorem 247
   (a) $\left\{B=B(x_\lambda,r_\lambda)\,:\,k^{-\frac{j+1}{2}}<r_\lambda\le k^{-\frac{j}{2}}, B\subset \bigcup_{1\le p\le N, 1\le l\le j}\bigcup_{B'\in\mathcal{B}_{l,p}} kB'\right\}$.
   (b) $\left\{B=B(x_\lambda,r_\lambda)\,:\,k^{-\frac{j+1}{2}}<r_\lambda\le k^{-\frac{j}{2}}, B\setminus \bigcup_{1\le p\le N, 1\le l\le j}\bigcup_{B'\in\mathcal{B}_{l,p}} kB'\neq\emptyset\right\}$.
   (c) 8 Dec. 2024
(3) p. 251, line 2 from below, the proof of Theorem 247
   (a) the Lemma
   (b) Lemma
   (c) 8 Dec. 2024
(4) p. 252, line 4 from above, the proof of Theorem 247
   (a) $\mathcal{X}_j$
   (b) X_j
   (c) 8 Dec. 2024
(5) p. 252, line 5 from above, the proof of Theorem 247
   (a) $B(x_\lambda, r_\lambda)$
   (b) any $B(x_\lambda, r_\lambda)$ in X_j
   (c) Dec. 28 2023
(6) p. 252, line 15 from below, the proof of Theorem 247
   (a) $<$
   (b) $\le$
   (c) Dec. 28 2023
(7) p. 252, line 12 from below, the proof of Theorem 247
   (a) $>$
   (b) $\ge$
   (c) Dec. 28 2023
(8) p. 252 line 11 from below, the proof of Theorem 247
   (a) $k^{-\frac{1}{2}}\log$
   (b) $\log_{\sqrt{k}}$
   (c) 9 Dec. 2024
(9) p. 252, line 6 from below, the proof of Theorem 247
   (a) are bounded. $R=\cdots$
   (b) are bounded: $R=\cdots$
   (c) Dec. 28 2023
(10) p. 253, line 2 from above, the proof of Theorem 249
   (a) We use the $5r$-covering lemma (see Theorem 132)
   (b) We use the remark after Lemma 248.
   (c) Dec. 28 2023
(11) p. 253, line 4 from above, the proof of Theorem 249
   (a) $\{B_\lambda\}_{\lambda\in\Lambda_0}$
   (b) $\{B_\lambda\}_{\lambda\in\Lambda_0}$, where $\Lambda_0\subset\Lambda_0^*$,
   (c) Dec. 28 2023
(12) p. 253, line 6 from above, the proof of Theorem 249

   (a) $\left\{\frac{1}{5}B_\lambda\right\}_{\lambda\in\Lambda_0}$ is disjoint.
   (b) $\{B_\lambda\}_{\lambda\in\Lambda_0}$ is disjoint.
   (c) Dec. 28 2023
(13) p. 253, line 11 from below, the proof of Theorem 249
   (a) from $\{B_\lambda\}_{\Lambda_1^{**}}$
   (b) from $\{B_\lambda\}_{\lambda\in\Lambda_1^{**}}$
   (c) Dec. 28 2023
(14) p. 253, line 10 from below, the proof of Theorem 249
   (a) from $\{B_\lambda\}_{\Lambda_1^{**}}$
   (b) from $\{B_\lambda\}_{\lambda\in\Lambda_1^{**}}$
   (c) Dec. 28 2023
(15) p. 253, line 8 from below, the proof of Theorem 249
   (a) $r(B_\lambda) \geq \dfrac{81}{100}R.$
   (b) $r(B_\lambda) > \dfrac{81}{100}R.$
   (c) Dec. 28 2023
(16) p. 253, line 6 from below, the proof of Theorem 249
   (a) $\dfrac{1}{5}B_j$
   (b) $\dfrac{1}{5}B_\lambda$
   (c) Dec. 28 2023
(17) p. 253, line 6 from below, the proof of Theorem 249
   (a) $\dfrac{1}{5}B_k$
   (b) $\dfrac{1}{5}B_{\lambda'}$
   (c) Dec. 28 2023
(18) p. 253, line 6 from below, the proof of Theorem 249
   (a) $j \in \Lambda_0$
   (b) $\lambda \in \Lambda_0$
   (c) Dec. 28 2023
(19) p. 253, line 6 from below, the proof of Theorem 249
   (a) $k \in \Lambda_1$
   (b) $\lambda' \in \Lambda_1$
   (c) Dec. 28 2023
(20) p. 253, line 6 from below, the proof of Theorem 249
   (a) Indeed, $c(B_k)$
   (b) Indeed, $c(B_{\lambda'})$
   (c) Dec. 28 2023
(21) p. 253, line 5 from below (twice), the proof of Theorem 249
   (a) B_k
   (b) $B_{\lambda'}$
   (c) Dec. 28 2023
(22) p. 253, line 5 from below (three times), the proof of Theorem 249
   (a) B_j
   (b) B_λ
   (c) Dec. 28 2023
(23) p. 253, line 2 from below, the proof of Theorem 249
   (a) or
   (b) for some $j_0 \in \mathbb{N}$ or
   (c) Dec. 28 2023
(24) p. 253, line 1 from below, the proof of Theorem 249

   (a) $r(B_j)$
   (b) $r_j = r(B_j)$
   (c) Dec. 28 2023
(25) p. 253, line 1 from below, the proof of Theorem 249
   (a) as $j \to \infty$,
   (b) as $j \to \infty$, and if $J \neq \mathbb{N}$, the sequence $r_j = r(B_j)$, $j = 1, 2, \ldots, j_0$, is decreasing in j
   (c) Dec. 28 2023
(26) p. 254, line 6 from above, the proof of Theorem 249
   (a) B_j
   (b) B_λ of generation α
   (c) Dec. 28 2023, Nov. 11 2024
(27) p. 254, line 6 from above, the proof of Theorem 249
   (a) B_k
   (b) $B_{\lambda'}$ of generation β
   (c) Dec. 28 2023, Nov. 11 2024
(28) p. 254 line 7 from above, the proof of Theorem 249
   (a) $j > k$ or $k < j$
   (b) $\alpha > \beta$ or $\beta < \alpha$
   (c) 11 Nov. 2024
(29) p. 254 line 7 from above, the proof of Theorem 249
   (a) $k \in \mathbb{N}$
   (b) $k \in J$
   (c) 11 Nov. 2024
(30) p. 254 line 9 from above, the proof of Theorem 249
   (a) $k \in \mathbb{N}$
   (b) $k \in J$
   (c) 11 Nov. 2024
(31) p. 254 line 9 from above, the proof of Theorem 249
   (a) $B_j \cap B_k \neq \emptyset$
   (b) $\frac{1}{2}B_j \cap \frac{1}{2}B_k \neq \emptyset$
   (c) 9. Dec 2024
(32) p. 254, line 9 from above, the proof of Theorem 249
   (a) $j \in I_j$
   (b) $j \in I_k$
   (c) Dec. 28 2023
(33) p. 254 line 9 from above, the proof of Theorem 249
   (a) $r_j \leq 3r_k$
   (b) $r_j \leq 100r_k$
   (c) 9. Dec 2024
(34) p. 254, line 10 from above, the proof of Theorem 249
   (a) $M_n > 0$
   (b) $M_n > 0$ independent of k
   (c) Dec. 28 2023
(35) p. 254 lines 14–15 from above, the proof of Theorem 249
   (a) meet in a point and $c(B_j)$ lies sufficiently close to the boundary of B_{j_1} and B_{j_2}
   (b) satisfy $B_k \subset \frac{2}{3}B_{j_1} \cap \frac{2}{3}B_{j_2}$ and $\frac{r_{j_1}}{r_{j_2}} \in \left[\frac{1}{100}, 100\right]$
   (c) 9. Dec 2024, 11 Nov. 2024
(36) p. 254 line 16 from above, the proof of Theorem 249
   (a) $c(B_j)$
   (b) $c(B_k)$
   (c) 11 Nov. 2024

(37) p. 256 lines 15 and 16 from above
 (a) shows the angle
 (b) shows
 (c) 2 Jan. 2024
(38) p. 254, line 17 from above, the proof of Theorem 249
 (a) B_j
 (b) ∂B_k
 (c) Dec. 28 2023
(39) p. 254 line 19 from above, the proof of Theorem 249
 (a) Even from
 (b) From
 (c) 11 Nov. 2024
(40) p. 254 line 23 from above
 (a) on n.
 (b) on n and the difference of generations.
 (c) 2 Jan. 2024
(41) p. 255 line 5 from above
 (a) The parameter
 (b) Likewise, for a measure ν, we also define

$$M^{\mathcal{B}}_{\kappa,\mathrm{uc}}\nu(x) = M_{\kappa,\mathrm{uc}}\nu(x) \equiv \sup_{B\in\mathcal{B}} \frac{\chi_B(x)\nu(B)}{\mu(\kappa B)} \quad (x \in \mathbb{R}^n).$$

 The parameter
 (c) 5 Oct. 2024
(42) p. 255 line 8 from above, Example 104
 (a) can be
 (b) is
 (c) 2 Jan. 2024
(43) p. 256 line 19 from below, four lines above Theorem 250
 (a) $d\mu$
 (b) $\mathrm{d}\mu$
 (c) 27 Nov. 2024
(44) p. 256 line 17 from below
 (a) k is
 (b) $k > 0$ is
 (c) 2 Jan. 2024
(45) p. 255 lines 11–2 from below should be changed as follows:
 If we take $f \equiv \dfrac{\chi_{B(U)}}{\mu(B(U))}$, $U > 0$ then

$$\mu\{x \in \mathbb{R}^2 \,:\, M_{1,\mathrm{uc}}f(x) > \lambda\} \le \frac{C}{\lambda}$$

 for all $\lambda > 0$. A passage to the limit gives us

$$\mu\{x \in \mathbb{R}^2 \,:\, M_{1,\mathrm{uc}}f(x) \ge \lambda\} \le \frac{C}{\lambda}.$$

 Fix $r > U$. If we take $\lambda = \frac{1}{\mu(B((r-U,0),r))}$, then we have

$$M_{1,\mathrm{uc}}f(x) \ge \frac{\|f\chi_{B((r-U,0),r)}\|_{L^1(\mu)}}{\mu(B((r-U,0),r))} = \frac{1}{\mu(B((r-U,0),r))}$$

 and

$$B(2r-U) \subset \left\{x \in \mathbb{R}^2 \,:\, M_{1,\mathrm{uc}}f(x) \ge \frac{\|f\chi_{B((r-U,0),r)}\|_{L^1(\mu)}}{\mu(B((r-U,0),r))}\right\}.$$

Thus,

$$\mu(B(2r-U)) \le \mu\left\{x \in \mathbb{R}^2 \,:\, M_{1,\mathrm{uc}}f(x) \ge \frac{\|f\chi_{B((r-U,0),r)}\|_{L^1(\mu)}}{\mu(B((r-U,0),r))}\right\} \le C\mu(B((r-U,0),r)).$$

Letting $U \downarrow 0$, we have $\mu(B(2r)) \le C\mu(B((r,0),r))$,

(46) p. 256 line 14 from below, Theorem 250 (twice)
  (a) $d\mu$
  (b) $\mathrm{d}\mu$
  (c) 27 Nov. 2024

(47) p. 256 line 10 from below, (the second line of) the proof of Theorem 250,
  (a) $d\mu$
  (b) $\mathrm{d}\mu$
  (c) 27 Nov. 2024

(48) p. 256 line 5 from below, the proof of Theorem 250, (6.5)
  (a) $d\mu$
  (b) $\mathrm{d}\mu$
  (c) 27 Nov. 2024

(49) p. 257 line 4 from above (twice), the proof of Theorem 250
  (a) $d\mu$
  (b) $\mathrm{d}\mu$
  (c) 27 Nov. 2024

(50) p. 257 line 5 from above, the proof of Theorem 250
  (a) $d\mu$
  (b) $\mathrm{d}\mu$
  (c) 27 Nov. 2024

(51) p. 257 line 6 from above, the proof of Theorem 250
  (a) $d\mu$
  (b) $\mathrm{d}\mu$
  (c) 27 Nov. 2024

(52) p. 257 line 7 from above, the proof of Theorem 250
  (a) $d\mu$
  (b) $\mathrm{d}\mu$
  (c) 27 Nov. 2024

(53) p. 257 line 9 from below, Example 105 (6.8)
  (a) $\displaystyle\int_{\{x \in X \,:\, M_{\kappa,\mathrm{uc}}f(x)>\lambda\}} |g(x)| d\mu(x)$
  (b) $\displaystyle\lambda \int_{\{x \in X \,:\, M_{\kappa,\mathrm{uc}}f(x)>\lambda\}} |g(x)| \mathrm{d}\mu(x)$
  (c) 27 Nov. 2024

(54) p. 258 line 2 from above, Example 105
  (a) M_3
  (b) $M_{3,\mathrm{uc}}$
  (c) 11 Nov. 2024

(55) p. 258 lines 2–8 from above, Example 105 (15 times in total)
  (a) x
  (b) x_1
  (c) 9. Dec 2024

(56) p. 258 lines 2, 5, 6 and 8 from above, Example 105 (6 times in total)
  (a) y
  (b) x_2
  (c) 9. Dec 2024

(57) p. 258 line 5 from above, Example 105
   (a) M_3
   (b) $M_{3,\mathrm{uc}}$
   (c) 11 Nov. 2024
(58) p. 258 line 5 from above, Example 105
   (a) $0 \le x < 2^{-k+1}$
   (b) $0 \le x < 2^{-m+1}$
   (c) 11 Nov. 2024
(59) p. 258 line 6 from above, Example 105
   (a) $(1+s)x/2$
   (b) $3(1+s)x_1/2$
   (c) 11 Nov. 2024
(60) p. 258 line 7 from above, Example 105
   (a) M_3
   (b) $M_{3,\mathrm{uc}}$
   (c) 11 Nov. 2024
(61) p. 258 line 8 from above, Example 105
   (a) $(1+s)$
   (b) $3(1+s)$
   (c) 11 Nov. 2024
(62) p. 258 line 8 from above, Example 105
   (a) R_k
   (b) R_m
   (c) 11 Nov. 2024
(63) p. 258 line 11 from above, Example 105
   (a) supported
   (b) and suppose that φ is supported
   (c) 11 Nov. 2024
(64) p. 258 line 13 from above, Example 105
   (a) $r^2 \cdot - x_j^r$
   (b) $r^{-1}(\cdot - x_j^r)$
   (c) 11 Nov. 2024
(65) p. 258 line 8 from below, Example 105
   (a) $\mu_{m,p}$
   (b) $\mu_{m,\alpha}$
   (c) 11 Nov. 2024
(66) p. 258 line 6 from below, Example 105
   (a) $M_3\mu_{m,\alpha}$
   (b) $M_{3,\mathrm{uc}}\mu_{m,\alpha}$
   (c) 11 Nov. 2024
(67) p. 258 line 5 from below, Example 105
   (a) M_3
   (b) $M_{3,\mathrm{uc}}$
   (c) 11 Nov. 2024
(68) p. 258 line 4 from below, Example 105
   (a) M_3
   (b) $M_{3,\mathrm{uc}}$
   (c) 11 Nov. 2024
(69) p. 258 line 4 from below, Example 105, twice
   (a) 2^{-k+1}
   (b) 2^{-m+1}
   (c) 11 Nov. 2024

(70) p. 258 line 1 from below, Example 105, twice
  (a) 2^{-k+1}
  (b) 2^{-m+1}
  (c) 1 Nov. 2024
(71) p. 259 line 2 from above, Example 105
  (a) $M_3\mu_{k,\alpha}$
  (b) $M_{3,\mathrm{uc}}\mu_{m,\alpha}$
  (c) 11 Nov. 2024
(72) p. 259 line 2 from above, Example 105
  (a) $M_3\mu_{k,p}$
  (b) $M_{3,\mathrm{uc}}\mu_{m,\alpha}$
  (c) 11 Nov. 2024
(73) p. 259 line 3 from above, Example 105
  (a) that $\mu(S_J) \ge \dfrac{C_\alpha}{(m-1)!}$
  (b) that $S_J \ge \dfrac{C_\alpha}{(m-2)!}$
  (c) 11 Nov. 2024
(74) p. 259 line 4 from above, Example 105, twice
  (a) 2^{-k}
  (b) 2^{-m}
  (c) 11 Nov. 2024
(75) p. 259 line 4 from above, Example 105
  (a) $m!$
  (b) $(m-1)!$
  (c) 11 Nov. 2024
(76) p. 259 line 5 from above, Example 105
  (a) $\displaystyle\lim_{k\to\infty} \frac{\mu(B((2^{-k},0),3\cdot 2^{-k}))}{S_J} = 0$
  (b) $\displaystyle\limsup_{m\to\infty} \frac{\mu(B((2^{-m},0),3\cdot 2^{-m}))}{S_J} < \infty$
  (c) 11 Nov. 2024
(77) p. 259 line 5 from above, Example 105
  (a) $S_{\{j\}} \sim \mu(B((2^{-k},0),3\cdot 2^{-k}))$
  (b) $S_{\{j\}} \sim \mu(B((2^{-m},0),3\cdot 2^{-m}))$ for each $j = 1,2,\dots,\alpha$.
  (c) 11 Nov. 2024
(78) p. 259 line 6 from above, Example 105
  (a) $\sim$
  (b) the implicit constant in $\gtrsim$
  (c) 11 Nov. 2024
(79) p. 259 line 6 from above, Example 105
  (a) k
  (b) m
  (c) 11 Nov. 2024
(80) p. 259 line 7 from above, Example 105
  (a) $M_3\mu_{k,p}$
  (b) $\mu(B((2^{-m},0),3\cdot 2^{-m}))M_{3,\mathrm{uc}}\mu_{k,\alpha} \lesssim 1$
  (c) 11 Nov. 2024
(81) p. 259 line 7 from above, Example 105
  (a) on α
  (b) of α and m
  (c) 11 Nov. 2024

0.26. **Pages 260–269.**

(1) Delete n^{-n} in the last line in page 260, the proof of Theorem 253
(2) Delete n^n twice in the second line in page 261, the proof of Theorem 253
(3) p. 261 line 6 from below, the proof of Theorem 255
  (a) , we first obtain
  (b) and use the Layer Cake formula, we first obtain
  (c) 16 Oct. 2024
(4) p. 262 line 6 from above, the proof of Theorem 256
  (a) Then
  (b) Then by Jensen's inequality
  (c) 16 Oct. 2024
(5) p. 265, line 7 from below, the proof of Lemma 259
  (a) $\mathcal{E}^1_0(\mathbb{R})$
  (b) $\mathcal{E}^3_0(\mathbb{R})$
  (c) 1 Nov. 2024
(6) p. 265, line 6 from below, the proof of Lemma 259
  (a) $\mathcal{E}^1_{-1}(\mathbb{R})$
  (b) $\mathcal{E}^3_{-1}(\mathbb{R})$
  (c) 1 Nov. 2024
(7) p. 265, line 5 from below, the proof of Lemma 259
  (a) $\mathcal{E}^1_{-2}(\mathbb{R})$
  (b) $\mathcal{E}^3_{-2}(\mathbb{R})$
  (c) 1 Nov. 2024
(8) p. 265, line 4 from below, the proof of Lemma 259
  (a) $\mathcal{E}^1_{-3}(\mathbb{R})$
  (b) $\mathcal{E}^3_{-3}(\mathbb{R})$
  (c) 1 Nov. 2024
(9) p. 267, line 1 from above, Proposition 260
  (a) c_B
  (b) $c(B)$
  (c) 1 Nov. 2024
(10) p. 267, line 19 from above, the proof of Lemma 261
  (a) $(a+1)M(B)$
  (b) $(a+2)M(B)$
  (c) 1 Nov. 2024
(11) p. 268, line 13 from above, Lemma 263(4)
  (a) $c_{M^{k_0}(B)}$
  (b) $c(M^{k_0}(B))$
  (c) 1 Nov. 2024
(12) p. 269, line 3 from above, the proof of Lemma 263(4)
  (a) $c_{M^{k_0-1}(B)}$
  (b) $c(M^{k_0-1}(B))$
  (c) 1 Nov. 2024
(13) p. 269, line 4 from above, the proof of Lemma 263(4)
  (a) $k \in \{1, 2, \dots, k_0\}$
  (b) $k \in \{0, 1, \dots, k_0 - 1\}$
  (c) 1 Nov. 2024
(14) p. 269, line 6 from above, the proof of Lemma 263(4)
  (a) $c_{M^k(B)}$
  (b) $c(M^k(B))$
  (c) 1 Nov. 2024

(15) p. 269, line 8 from above, the proof of Lemma 263(4)
   (a) $c_{M^{k_0-1}(B)}$
   (b) $c(M^{k_0-1}(B))$
   (c) 1 Nov. 2024

0.27. **Pages 270–279.**

(1) p. 275 line 3 from above,
  (a) $d\mu$
  (b) $\mathrm{d}\mu$
  (c) 27 Nov. 2024
(2) p. 275 line 5 from above,
  (a) $d\mu$
  (b) $\mathrm{d}\mu$
  (c) 27 Nov. 2024,
(3) p. 275 line 14 from above, Theorem 267
  (a) $d\mu$
  (b) $\mathrm{d}\mu$
  (c) 27 Nov. 2024,
(4) p. 276 line 11 from above, the proof of Lemma 268
  (a) $\left\{ j \in \{1, 2, \ldots, N\} \,:\, B(x_j, \delta r_j) \cap \bigcup_{k=j_1}^{j_{q-1}} B(x_j, (2+\delta)r_j) \neq B(x_j, \delta r_j) \right\}$
  (b) $\left\{ j \in \{1, 2, \ldots, N\} \,:\, B(x_j, \delta r_j) \cap \bigcup_{k=j_1}^{j_{q-1}} B(x_k, (2+\delta)r_k) \neq B(x_j, \delta r_j) \right\}$
  (c) 27 Nov. 2024
(5) p. 277 line 8 from above, the proof of Theorem 267
  (a) $d\mu$
  (b) $\mathrm{d}\mu$
  (c) 27 Nov. 2024
(6) p. 277 line 18 from above, the proof of Theorem 267, (6.6)
  (a) $d\mu$
  (b) $\mathrm{d}\mu$
  (c) 27 Nov. 2024
(7) p. 277 line 23 from above, the proof of Theorem 267, (6.7)
  (a) $d\mu$
  (b) $\mathrm{d}\mu$
  (c) 27 Nov. 2024
(8) p. 278 line 8 from below, Proposition 271 (twice),
  (a) $d\mu$
  (b) $\mathrm{d}\mu$
  (c) 27 Nov. 2024
(9) p. 278 line 5 from below, the proof of Proposition 271
  (a) $d\mu$
  (b) $\mathrm{d}\mu$
  (c) 27 Nov. 2024
(10) p. 278 line 1 from below, the proof of Proposition 271 (twice)
  (a) $d\mu$
  (b) $\mathrm{d}\mu$
  (c) 27 Nov. 2024
(11) p. 279 line 5 from above, the proof of Proposition 271
  (a) $\subset$
  (b) $\supset$
  (c) 29 Nov. 2024
(12) p. 279 line 5 from above, the proof of Proposition 271
  (a) $x, (2+a)r_j$
  (b) $x, 3(2+a)r_j$

(c) 29 Nov. 2024

(13) p. 279 line 12 from below the proof of Proposition 271,
  (a) $d\mu$
  (b) $\mathrm{d}\mu$
  (c) 27 Nov. 2024
(14) p. 279 line 11 from below the proof of Proposition 271,
  (a) $d\mu$
  (b) $\mathrm{d}\mu$
  (c) 27 Nov. 2024
(15) p. 279 line 9 from below, the proof of Proposition 271
  (a) (6.13)
  (b) (6.13) and the similar inequality to (4.12), which can be deduced from (6.13),
  (c) 29 Nov. 2024
(16) p. 279 line 8 from below the proof of Proposition 271,
  (a) $d\mu$
  (b) $\mathrm{d}\mu$
  (c) 27 Nov. 2024
(17) p. 279 line 7 from below the proof of Proposition 271, (twice)
  (a) $d\mu$
  (b) $\mathrm{d}\mu$
  (c) 27 Nov. 2024
(18) p. 279 line 6 from below the proof of Proposition 271, (twice)
  (a) $d\mu$
  (b) $\mathrm{d}\mu$
  (c) 27 Nov. 2024
(19) p. 279 line 5 from below the proof of Proposition 271, (twice)
  (a) $d\mu$
  (b) $\mathrm{d}\mu$
  (c) 27 Nov. 2024
(20) p. 279 line 3 from below the proof of Proposition 271, (twice)
  (a) $d\mu$
  (b) $\mathrm{d}\mu$
  (c) 27 Nov. 2024

0.28. **Pages 280–289.**

(1) p. 280 from 2 from above, Corollary 272 (twice)
   (a) $d\mu$
   (b) $\mathrm{d}\mu$
   (c) 27 Nov. 2024
(2) p. 280 from 12 from above, Proposition 273 (a) (twice)
   (a) $d\mu$
   (b) $\mathrm{d}\mu$
   (c) 27 Nov. 2024
(3) p. 280 from 14 from above, Proposition 273 (b) (twice)
   (a) $d\mu$
   (b) $\mathrm{d}\mu$
   (c) 27 Nov. 2024
(4) p. 281 line 7 from below, Definition 69
   (a) (2)
   (b) (3)
   (c) 5 Oct. 2024
(5) p. 281 line 6 from below, Definition 69
   (a) (3)
   (b) (4)
   (c) 5 Oct. 2024
(6) p. 281 line 5 from below, Definition 69
   (a) (4)
   (b) (5)
   (c) 5 Oct. 2024
(7) p. 285, line 10 from above
   (a) geometic
   (b) geometric
   (c) 21 Sept. 2023
(8) p. 287 line 14 from below, Exercise 92 (2-B)
   (a) $d\mu$
   (b) $\mathrm{d}\mu$
   (c) 27 Nov. 2024
(9) p. 287 line 11 from below, Exercise 92 (2-B)
   (a) $d\mu$
   (b) $\mathrm{d}\mu$
   (c) 27 Nov. 2024
(10) p. 287 line 2 from below, Exercise 93(3), (6.27)
   (a) $d\mu$
   (b) $\mathrm{d}\mu$
   (c) 27 Nov. 2024
(11) p. 287 line 1 from below, Exercise 93(3),
   (a) $d\mu$
   (b) $\mathrm{d}\mu$
   (c) 27 Nov. 2024
(12) p. 288 from 2 from above, Exercise 93(3)
   (a) $d\mu$
   (b) $\mathrm{d}\mu$
   (c) 27 Nov. 2024
(13) p. 288 from 4 from above, Exercise 93(4)
   (a) $d\mu$

(b) $\mathrm{d}\mu$
(c) 27 Nov. 2024

(14) p. 288 from 6 from above, Exercise 93(5)
(a) $d\mu$
(b) $\mathrm{d}\mu$
(c) 27 Nov. 2024

(15) p. 288 from 7 from above, Exercise 93(5)
(a) $d\mu$
(b) $\mathrm{d}\mu$
(c) 27 Nov. 2024

(16) p. 288 from 9 from above, Exercise 93(6)
(a) $d\mu$
(b) $\mathrm{d}\mu$
(c) 27 Nov. 2024

(17) p. 288 from 10 from above, Exercise 93(6)
(a) $d\mu$
(b) $\mathrm{d}\mu$
(c) 27 Nov. 2024

(18) p. 288 from 12 from above, Exercise 93(6)
(a) $d\mu$
(b) $\mathrm{d}\mu$
(c) 27 Nov. 2024

(19) p. 288 from 13 from above, Exercise 93(6)
(a) $d\mu$
(b) $\mathrm{d}\mu$
(c) 27 Nov. 2024

(20) p. 288 from 14 from above, Exercise 93(6)
(a) $d\mu$
(b) $\mathrm{d}\mu$
(c) 27 Nov. 2024

(21) p. 289 from 2 from above, Exercise 93(7)
(a) $d\mu$
(b) $\mathrm{d}\mu$
(c) 27 Nov. 2024

(22) p. 289 from 3 from above, Exercise 93(7)
(a) $d\mu$
(b) $\mathrm{d}\mu$
(c) 27 Nov. 2024

(23) p. 289 from 5 from above, Exercise 93(8)
(a) $d\mu$
(b) $\mathrm{d}\mu$
(c) 27 Nov. 2024

(24) p. 289 from 7 from below, Exercise 95 (6.30)
(a) $d\mu$
(b) $\mathrm{d}\mu$
(c) 27 Nov. 2024

(25) p. 289 from 2 from below, Exercise 96 (6.31)
(a) $d\mu$
(b) $\mathrm{d}\mu$
(c) 27 Nov. 2024

0.29. **Pages 290–299.**

(1) p. 290, line 7 from above
   (a) consided
   (b) considered
   (c) 21 Sept. 2023
(2) p. 296 line 14 from below, Example 114
   (a) easily seen from the observation that
   (b) is seen from the rotation argument and the observation that
   (c) 24 Dec. 2024
(3) p. 297 line 7 from above, one line above Theorem 281
   (a) aspects.
   (b) aspects. We recall Example 66, where we defined $M\mu$.
   (c) 10 Nov. 2024
(4) p. 297 line 17 from above, the proof of Theorem 281
   (a) Example 66, the Kolmogorov inequality
   (b) $\mu(\{x \in 10Q \,:\, M\mu_1(x)^\delta > \lambda\}) \lesssim \min(\mu(10Q), \lambda^{-\frac{1}{\delta}}|Q|)$,
   (c) 10 Nov. 2024
(5) p. 298 line 3 from below, Proposition 285
   (a) we decompose
   (b) since

$$M\chi_Q \sim \sum_{j=1}^{\infty} 2^{-jn}\chi_{2^jQ},$$

   we decompose
   (c) 21 Dec. 2023
(6) p. 299, line 15 from above
   (a) inequalites
   (b) inequalities
   (c) 21 Sept. 2023

0.30. **Pages 300–309.**

(1) p. 300 line 11 from below, Example 117
  (a) then
  (b) then a geometric observation shows that $|x| \sim |c(Q)|$ for all $x \in Q$ and that
  (c) 24 Dec. 2024
(2) p. 300 line 11 from below, Example 117
  (a) then
  (b) then take a ball centered at origin of radius $2\sqrt{n}\ell(Q)$ to have
  (c) 24 Dec. 2024
(3) p. 301, line 3 from above, Example 118
  (a) $W(x) \equiv v(x) =$
  (b) $W(x) \equiv$
  (c) 21 Sept. 2023
(4) p. 301 line 4 from above, Example 118
  (a) $\delta^{-p'+1} = \delta^{-\frac{1}{p-1}}$
  (b) $\delta^{-p+1} = \delta^{-\frac{1}{p'-1}}$
  (c) 18 Oct. 2024
(5) p. 301 line 15 from above, Theorem 289
  (a) $([w]_{A_p}\|f\|_{L^p(w)})^p$
  (b) $[w]_{A_p}(\|f\|_{L^p(w)})^p$
  (c) 18 Oct. 2024
(6) p. 301 line 18 from above, Theorem 289
  (a) for all $f \in L^0(\mathbb{R}^n)$
  (b) for all $f \in L^0(\mathbb{R}^n)$ and $w \in A_p$
  (c) 18 Oct. 2024
(7) p. 301 line 8 from below, Example 119
  (a) if we choose $\kappa > 0$ appropriately
  (b) if we let M be uncentered maximal operator
  (c) 18 Oct. 2024
(8) p. 301 line 7 from below, Example 119
  (a) $\{w \in \mathbb{R}^n : Mf(x) > \delta^{-1}\} = w(B(1)) \sim 1$
  (b) $\left\{w \in \mathbb{R}^n : Mf(x) > \frac{c_n}{\delta}\right\} \geq w\left(B\left(\frac{1}{2}\right)\right) \sim 1$, where c_n is a constant chosen appropriately
  (c) 18 Oct. 2024
(9) p. 301 line 6 from below, Example 119
  (a) $w\{x \in \mathbb{R}^n : Mf(x) > \lambda\} \leq$
  (b) $(w\{x \in \mathbb{R}^n : Mf(x) > \lambda\})^{\frac{1}{p}} \leq$
  (c) 18 Oct. 2024
(10) p. 301 line 5 from below, Example 119
  (a) $\delta^{-1} \leq$
  (b) $\delta^{-1} \lesssim$
  (c) 18 Oct. 2024
(11) p. 301 line 4 from below, Example 119
  (a) Suppose
  (b) Fix $A > 0$ once again. Suppose
  (c) 18 Oct. 2024
(12) p. 302 line 3 from above
  (a) Going
  (b) This forces $A \geq 1$. Alternatively going
  (c) 18 Oct. 2024
(13) p. 303 line 11 from above

(a) $\|f \cdot \sigma^{-1}\|_{L^p(\sigma)}$
(b) $\left\|\frac{f}{\sigma}\right\|_{L^p(\sigma)}$
(c) 10 Nov. 2024
(d) This is because of the marginal typo.

(14) p. 303 line 10 from below
(a) $a = \frac{1}{p-1}$.
(b) $a = \frac{1}{p-1}$ since this bound together with Example 118 forces $\delta^{-1} \lesssim \delta^{-a(p-1)}$ for all $0 < \delta < n$.
(c) 10 Nov. 2024

(15) p. 305 line 2 from above, the proof of Lemma 292
(a) $\inf_{x \in Q_k^j} M_\sigma^{\mathfrak{D}_{\mathbf{a}}}[f \cdot w](x) \cdot \inf_{x \in Q_k^j} M_w^{\mathfrak{D}_{\mathbf{a}}}[g \cdot \sigma](x)$
(b) $\left(\inf_{x \in Q_k^j} M_\sigma^{\mathfrak{D}_{\mathbf{a}}}[f \cdot w](x)\right) \cdot \left(\inf_{x \in Q_k^j} M_w^{\mathfrak{D}_{\mathbf{a}}}[g \cdot \sigma](x)\right)$
(c) 25 Dec. 2024

(16) p. 305 line 3 from above, the proof of Lemma 292
(a) If we insert this estimate into (7.6), then $\mathrm{I} \lesssim [w]_{A_2} \|M_\sigma^{\mathfrak{D}_{\mathbf{a}}}[f \cdot w]\|_{L^2(\sigma)} \|M_w^{\mathfrak{D}_{\mathbf{a}}}[g \cdot \sigma]\|_{L^2(w)}$
(b) If we insert this estimate and the disjointness of $\{E_k^j\}_{j \in \mathbb{N}, k \in K_j}$ into (7.6), then

$$\begin{aligned}
\mathrm{I} &\lesssim [w]_{A_2} \|M_\sigma^{\mathfrak{D}_{\mathbf{a}}}[f \cdot w]\|_{L^2(\sigma)} \|M_w^{\mathfrak{D}_{\mathbf{a}}}[g \cdot \sigma]\|_{L^2(w)} \\
&\lesssim [w]_{A_2} \|f \cdot w\|_{L^2(\sigma)} \|g \cdot \sigma\|_{L^2(w)} \\
&= \|f\|_{L^2(w)} \|g\|_{L^2(\sigma)}.
\end{aligned}$$

(c) 25 Dec. 2024

(17) p. 306 line 9 from above, Example 122
(a) $\in A_1$
(b) $\in A_1 \subset A_q$ due to Theorem 281 and Remark 13
(c) 25 Dec. 2024

(18) p. 306 line 10 from above, Example 122
(a) .
(b) due to Theorem 288.
(c) 25 Dec. 2024

(19) p. 306 line 7 from below, Definition 72
(a) $\exp(-m_Q(\log w))$
(b) $\exp(m_Q(\log w^{-1}))$
(c) 25 Dec. 2024

(20) p. 306 line 2 from below, Example 123
(a) by Hölder's inequality
(b) by Jensen's inequality
(c) 25 Dec. 2024

(21) p. 307 line 15 from above, the proof of Theorem 295
(a) $\mathrm{d}\lambda$.
(b) $\mathrm{d}\lambda \geq w(Q)$.
(c) 25 Dec. 2024

(22) p. 307 line 15 from above
(a) Thus
(b) Thus by Fubini's theorem
(c) 25 Dec. 2024

(23) p. 308 line 1 from above
(a) we have

   (b) $M_{\log}^{\mathcal{D}(Q)}[\chi_Q w](x) \ge w(x)$ for almost all $x \in \mathbb{R}^n$ and
   (c) 25 Dec. 2024
(24) p. 308 line 8 from below, the proof of Lemma 296, (7.8)
   (a) $\frac{2^{n+3}[w]_{A_\infty}}{1+2^{n+4}[w]_{A_\infty}}$
   (b) $\frac{2^{n+3}[w]_{A_\infty}}{1+2^{n+3}[w]_{A_\infty}}$
   (c) 11 Nov. 2024
(25) p. 309 line 14 from above, the proof of Theorem 297
   (a) reverse Hölder's inequality,
   (b) Hölder's inequality and reverse Hölder's inequality, $w(E) \le \left(\int_Q w(x)^{1+\varepsilon}\mathrm{d}x\right)^{\frac{1}{1+\varepsilon}} |E|^{\frac{\varepsilon}{1+\varepsilon}}$ and
   (c) 25 Dec. 2024

0.31. **Pages 310–319.**

(1) p. 312, two lines below (7.12)
   (a) sufficient
   (b) necessary
   (c) 21 Aug. 2024
(2) p. 313, line 11 from below
   (a) inequalites
   (b) inequalities
   (c) 21 Sept. 2023

0.32. **Pages 320–329.**

(1) p. 325 line 11 from above, 4 lines above Thoerem 312
  (a) $L^p(t^p \mathrm{d}\mu)$
  (b) $L^q(t^q \mathrm{d}\mu)$
  (c) 21 Nov. 2024
(2) p. 328 line 7 from above, the proof of Theorem 314
  (a) $\displaystyle\int_r^\infty w(t)^{-p'}\mathrm{d}t$
  (b) $\displaystyle\int_0^{r^{-1}} t^{-2}w(t^{-1})^{-p'}\mathrm{d}t$
  (c) 21 Nov. 2024
(3) p. 328 line 9 from above, the proof of Lemma 323
  (a) $t^{\frac{2}{p'}}$
  (b) $t^{-\frac{2}{p}}$
  (c) 21 Nov. 2024
(4) p. 328, line 14 from below
  (a) charecterization
  (b) characterization
  (c) 21 Sept. 2023
(5) p. 328, line 10 from below
  (a) boundendness
  (b) boundedness
  (c) 21 Sept. 2023
(6) p. 329 line 1 from above
  (a) two lemmas
  (b) a lemma
  (c) 21 Sept. 2023
(7) p. 329 line 12 from above, the proof of Lemma 316
  (a) in the right-hand side
  (b) on the right-hand side
  (c) 21 Nov. 2024
(8) p. 329, line 1 from below
  (a) montone
  (b) monotone
  (c) 21 Sept. 2023

0.33. **Pages 330–339.**

(1) p. 331 the left-hand side of (7.3), twice
   (a) θ_1
   (b) θ_2
   (c) 21 Nov. 2024
(2) p. 331 the left-hand side of (7.4)
   (a) $[0, 1]$
   (b) $[0, t]$
   (c) 21 Nov. 2024
(3) p. 331 line 6 from below
   (a) condition (7.4)
   (b) the inequality $\mathcal{A} < \infty$
   (c) 21 Nov. 2024
(4) p. 332 line 4 from above twice
   (a) g
   (b) f
   (c) 21 Nov. 2024
(5) p. 332 line 6 from above
   (a) $g = {v_1}^{\theta_1}$
   (b) $f = 1$
   (c) 21 Nov. 2024
(6) p. 337, line 2 from below
   (a) measue
   (b) measure
   (c) 21 Sept. 2023

0.34. **Pages 340–349.**

(1) p. 345 line 12 from above, the proof of Proposition 321
   (a) Since $L^\infty_{\rm c}(\mathbb{R}^n)$ is dense in $L^p(\mathbb{R}^n)$,
   (b) By the use of the almost-everywhere inequality $|g(x)| \le \|g\|_{L^\infty}$
   (c) 19 Oct. 2024

0.35. **Pages 350–359.**

(1) p. 350, Example 133
  (a) $< \infty$.
  (b) $< \infty$. Consider $\mathcal{M}^p_q(\mathbb{R}^n)$.
  (c) 21 Aug. 2024
(2) p. 351, line 13 from below, the proof of Proposition 331
  (a) and
  (b) and that
  (c) 25 Oct. 2024
(3) p. 351, line 12 from below, the proof of Proposition 331
  (a) and that
  (b) and
  (c) 25 Oct. 2024
(4) See Theorem 322 for Example 135(7) in p. 354, 25 Oct. 2024
(5) See Theorem 322 for Example 136(7) in p. 354, 25 Oct. 2024
(6) p. 355, line 2 from below, Example 139
  (a) as in Example 12
  (b) as above
  (c) 25 Oct. 2024
(7) p. 355, line 1 from below, Example 139
  (a) According
  (b) Similar
  (c) 25 Oct. 2024
(8) p. 357, line 3 from above, Example 140(7)
  (a) d
  (b) d
  (c) 7 Dec. 2024
(9) p. 359, Exercise 108 (1)(c)
  (a) operator.
  (b) operator with $\|e^{s\Delta}\|_{\mathcal{M}^p_q \to \mathcal{M}^p_q} \le 1$.
  (c) 25 Oct. 2024
(10) p. 359, Exercise 108 (2)
  (a) C^∞
  (b) $C^\infty(\mathbb{R}^n)$
  (c) 25 Oct. 2024
(11) p. 359, Exercise 108 (2)(c)
  (a) which
  (b) for all $t \in (0, T(k))$, which
  (c) 25 Oct. 2024

0.36. **Pages 360–369.**

(1) p. 360, line 9 from above, §8.1.1
  (a) subpsaces
  (b) subspaces
  (c) 21 Sept. 2023
(2) p. 360, line 14 from above, §8.1.1
  (a) intergrals
  (b) integrals
  (c) 21 Sept. 2023
(3) p. 361, line 4 from above, §8.2.2
  (a) charecterized
  (b) characterized
  (c) 21 Sept. 2023
(4) p. 363, line 16 from above
  (a) isometrc
  (b) isometric
  (c) 21 Sept. 2023
(5) p. 367 line 3 from below (twice), the proof of Lemma 342
  (a) $\frac{|f(x)|}{g(x)}$
  (b) $\frac{|f(x)|}{g(x)}\chi_{\{y\,:\,g(y)\neq 0\}}(x)$
  (c) 6 Nov. 2024
(6) p. 367 line 2 from below, the proof of Lemma 342
  (a) $(f/g)b'_k$
  (b) $(f/g)\chi_{\{y\,:\,g(y)\neq 0\}}b'_k$
  (c) 6 Nov. 2024
(7) p. 368 line 13 from above, the proof of Lemma 343
  (a) block
  (b) block supported in a cube Q_k
  (c) 6 Nov. 2024
(8) p. 368 line 19 from above, the proof of Lemma 343
  (a) $\mathrm{supp}(b_k)$
  (b) Q_k
  (c) 6 Nov. 2024

0.37. **Pages 370–379.**

(1) p. 370 line 9 from below, the proof of Theorem 345
  (a) b_j.
  (b) $b_j \in L^{q'}_{\rm c}(\mathbb{R}^n)$.
  (c) 6 Nov. 2024
(2) p. 373 line 3 from above, Example 145
  (a) dense
  (b) dense in $\mathcal{H}^{p'}_{q'}(\mathbb{R}^n)$ thanks to Theorem 345
  (c) 21 Sept. 2023
(3) p. 373, Theorem 348
  (a) $f_k \in$
  (b) f_k belongs to
  (c) 21 Aug. 2024
(4) p. 374 line 11 from above, Erase the expression of f_{k_j}. 21 Aug. 2024
(5) p. 374 line 1 from below, the proof of Theorem 348
  (a) L^1
  (b) $L^1(Q_0 \cap Q)$
  (c) 6 Nov. 2024
(6) p. 374 line 1 from below, the proof of Theorem 348
  (a) $|Q|^{\frac{1}{q}}$
  (b) $|Q \cap Q_0|^{\frac{1}{q}}$
  (c) 6 Nov. 2024
(7) p. 375 line 1 from above, the proof of Theorem 348
  (a) contained in
  (b) intersecting
  (c) 6 Nov. 2024
(8) p. 375 line 2 from above, the proof of Theorem 348
  (a) $\displaystyle\sum_{l=-\infty}^{\infty} |Q|^{\frac{1}{p}-\frac{1}{q}}|Q \cap Q_0|^{\frac{1}{q}}$
  (b) $\displaystyle\sum_{l=-\infty}^{\infty} (2^{ln})^{\frac{1}{p}-\frac{1}{q}} \min(1, 2^l)^{\frac{n}{q}}$
  (c) 6 Nov. 2024
(9) p. 376 line 6 from above: Example 146
  (a) $= L^p(\mathbb{R}^n)$
  (b) $\approx L^p(\mathbb{R}^n)$
  (c) 2 Jan. 2024

0.38. **Pages 380–389.**

(1) p. 382 line 2 from above, Example 148
  (a) Recall
  (b) Let $n = 1$ for simplicity. Recall
  (c) 6 Nov. 2024
(2) p. 382 line 6 from above, Example 148
  (a) $\mathbb{R}^n$
  (b) $\mathbb{R}$
  (c) 6 Nov. 2024
(3) p. 382 line 12 from below, Example 148
  (a) obtains
  (b) would obtain
  (c) 6 Nov. 2024
(4) p. 383, line 10 from above
  (a) $P^L_{Q(1)} f\left(\frac{\cdot - x_0}{r}\right)$
  (b) $P^L_{Q(1)}[f(r \cdot + x_0)]\left(\frac{\cdot - x_0}{r}\right)$
  (c) 11 May, 2021
(5) p. 383, line 13 from above
  (a) g
  (b) g^k
  (c) 11 May, 2021
(6) p. 383, line 18 from above (2),
  (a) $P_{Q^k_j}(f)\chi_{Q^k_j}$.
  (b) $P^L_{Q^k_j}(f)\chi_{Q^k_j}, \quad b^k_j = \chi_{Q^k_j}(f - P^L_{Q^k_j}(f))$.
  (c) 11 May, 2021
(7) p. 383, (3), The L^∞-condition
  (a) $2^n A^k$
  (b) CA^k
  (c) 11 May, 2021
(8) p. 383, (5), The moment condition
  (a) $b^k_l \perp \mathcal{P}_l(\mathbb{R}^n)$
  (b) $b^k_l \perp \mathcal{P}_L(\mathbb{R}^n)$
  (c) 7 Jul, 2021
(9) p. 384, Corollary 360
  (a) any $\lambda > 0$.
  (b) any $\lambda > 0$. Assume that $M^{\mathcal{D}} f < \infty$ almost everywhere.
  (c) 11 May, 2021
  (d) Otherwise there is a counterexample of $f \equiv 1$.
(10) p. 384, the first line of the proof of Proposition 361(2)
  (a) We calculate
  (b) Let $\beta \in \mathbb{N}_0$. We calculate
  (c) 11 May, 2021
(11) p. 388 line 6 from below, Exercise 115
  (a) $F(x) - I_\alpha f(x) = \lim_{j\to\infty} \varepsilon^\alpha g_\varepsilon * (f_j - f)(x)$
  (b) $F(x) - I_\alpha f(x) = \lim_{\varepsilon \downarrow 0} \lim_{j\to\infty} \varepsilon^\alpha g_\varepsilon * (f_j - f)(x)$
  (c) 6 Nov. 2024
(12) p. 389 line 6 from below, Definition 88
  (a) $0 \le d$
  (b) $0 < d$

(c) 6 Nov. 2024

0.39. **Pages 380–389.**

(1) p. 390 line 4 from below
  (a) $\inf \cdots$
  (b) $\inf(\{\cdots \cup \{\infty\}\})$
  (c) 6 Nov. 2024
(2) p. 393, line 11 from below, the proof of Theorem 367
  (a) Thus,
  (b) Since $\{K_j\}_{j=1}^{\infty}$ is decreasing,
  (c) 10 Dec. 2024
(3) p. 394 line 4 from above, the proof of Theorem 368
  (a) and
  (b) for each $j \in \mathbb{N}$ and
  (c) 10 Dec. 2024
(4) p. 395 line 11 from below, two lines above (9.15), the proof of Theorem 368
  (a) $O_{1,m} \equiv O_m \cap \operatorname{Int}\left(\bigcup_{i \in I_1} Q_i^{(1)}\right) \quad (m \in \mathbb{N}).$
  (b) $O_{1,m} \equiv O_m \cap \left(\bigcup_{i \in I_1} Q_i^{(1)}\right) \quad (m \in \mathbb{N}).$
  (c) 30 Dec. 2024
(5) p. 395 line 2 from above, (9.18), the proof of Theorem 368
  (a) $O_1 = \left(O_1 \cap O_m \cap \operatorname{Int}\left(\bigcup_{k \in I_1} Q_i^{(1)}\right)\right) \cup \left(O_1 \cap \bigcup_{k \in J_1} Q_{1,k,m}^*\right),$
  (b) $O_1 \subset \operatorname{Int}\left(O_{1,m} \cup \bigcup_{k \in J_1} Q_{1,k,m}^*\right),$
  (c) 30 Dec. 2024
(6) p. 395 line 4 from above: (9.19), the proof of Lemma 368
  (a) $O_1 \subset \bigcup_{k \in K_{1,m}} Q_{m,k}^{(1)} \cup \bigcup_{k \in J_1} Q_{1,k,m}^*.$
  (b) $O_1 \subset \operatorname{Int}\left(\bigcup_{k \in K_{1,m}} Q_{m,k}^{(1)} \cup \bigcup_{k \in J_1} Q_{1,k,m}^*\right).$
  (c) 2 Jan. 2024
(7) p. 395 line 4 from below, the proof of Lemma 368
  (a) $Q_{2,k}^*$
  (b) $Q_{2,k,m}^*$
  (c) 2 Jan. 2024
(8) p. 395 line 3 from below, the proof of Lemma 368
  (a) $Q_{2,k}^*$
  (b) $Q_{2,k,m}^*$
  (c) 2 Jan. 2024
(9) p. 396 line 2 from above
  (a) we can find a collection $\{Q_{m,k}^{(j)}\}_{k \in I_{j,m}}$
  (b) we can find collections $\{Q_{m,k}^{(j)}\}_{k \in I_{j,m}}$ and $\{Q_{j,k,m}^*\}_{k \in J_j}$ of dyadic cubes
  (c) 2 Jan. 2024
(10) p. 396 line 3 from above, the proof of Lemma 368
  (a) $Q_{j,k}^*$
  (b) $Q_{j,k,m}^*$

(c) 2 Jan. 2024

0.40. **Pages 400–409.**

(1) p. 403 line 7 from below, the proof of Theorem 377
   (a) $\mathbb{M}^p_1$
   (b) $\mathbb{M}^{\frac{p}{q}}_1$
   (c) 6 Nov. 2024
(2) p. 404 line 5 from below, the proof of Lemma 378
   (a) $)^v$
   (b) $)^q$
   (c) 6 Nov. 2024
(3) p. 404 line 5 from below, the proof of Lemma 378
   (a) $)^{q-1}$
   (b) $)^{v-1}$
   (c) 6 Nov. 2024
(4) p. 404 line 5 from below, the proof of Lemma 378
   (a) $|g_j| {w_j}^{1-q}$
   (b) $|g_j|^v {w_j}^{1-v}$
   (c) 6 Nov. 2024
(5) p. 404 line 3 from below, the proof of Lemma 378
   (a) $)^q$
   (b) $)^v$
   (c) 6 Nov. 2024
(6) p. 404 line 3 from below, the proof of Lemma 378
   (a) W^{1-q}
   (b) W^{1-v}
   (c) 6 Nov. 2024
(7) p. 404 line 3 from below, the proof of Lemma 378
   (a) $|g_j| {w_j}^{1-q}$
   (b) $|g_j|^v {w_j}^{1-v}$
   (c) 6 Nov. 2024
(8) p. 404 line from 1 below, the proof of Lemma 378
   (a) $)^q$
   (b) $)^v$
   (c) 6 Nov. 2024
(9) p. 404 line 1 from below, the proof of Lemma 378
   (a) $W(x)^{1-q}$
   (b) $W(x)^{1-v}$
   (c) 6 Nov. 2024
(10) p. 405 line 1 from above, the proof of Lemma 378
   (a) ≤ 1
   (b) $\le \sum_{j=1}^\infty \lambda_j$
   (c) 6 Nov. 2024
(11) p. 405 line 3 from above, the proof of Lemma 378
   (a) $|^q$
   (b) $|^v$
   (c) 6 Nov. 2024
(12) p. 405 line 3 from above, the proof of Lemma 378
   (a) $W^k(x)^{1-q}$
   (b) $W^k(x)^{1-v}$
   (c) 6 Nov. 2024
(13) p. 405 line 8 from below, the proof of Proposition 379
   (a) w

   (b) w satisfying $\|w\|_{L^1(\tilde{H}_0^d)} \leq 1$ with $d = n - \frac{nq}{p}$
   (c) 6 Nov. 2024
(14) p. 405 line 7 from below, the proof of Proposition 379
   (a) inequality
   (b) inequality $\lesssim$
   (c) 6 Nov. 2024
(15) Chapter 9, Remark, see p. 406, §9.1.
   (a) 20 Sep. 2020
   (b) The definition of the predual of Morrey spaces by way of the blocks is due to Long [1].

0.41. **Pages 410–419.**

(1) p. 419 line 15 from below, the proof of Theorem 391
    (a) in the right-hand
    (b) on the right-hand
    (c) 21 Nov. 2024

0.42. **Pages 420–429.**

(1) p. 420 line 9 from above, the proof of Proposition 393
  (a) $\frac{t}{s} = \frac{q}{p}$
  (b) $\frac{t}{s} \le \frac{q}{p}$
  (c) 6 Nov. 2024
(2) p. 420, Example 157
  (a) $1 < p < \infty$
  (b) $1 < p < \infty$, $1 < t \le s < \infty$
  (c) 11 May, 2021
(3) p. 425, line 11 from below, Definition 95
  (a) Morrrey
  (b) Morrey
  (c) 21 Sept. 2023
(4) p. 427, line 1 from above, Definition 96
  (a) Morrrey
  (b) Morrey
  (c) 21 Sept. 2023
(5) p. 427 line 1 from below
  (a) Theorem 356
  (b) Theorem 356 and Proposition 321
  (c) 6 Nov. 2024
(6) p. 428 line 1 from above
  (a) thanks to Proposition 321.
  (b) .
  (c) 6 Nov. 2024
(7) p. 428, line 15 from above, Definition 97
  (a) Morrrey
  (b) Morrey
  (c) 21 Sept. 2023
(8) p. 428 line 7 from below, Example 160
  (a) $f \in \mathcal{M}^p_q(\mathbb{R}^n)$
  (b) $f \in \mathcal{H}^{p'}_{q'}(\mathbb{R}^n)$
  (c) 6 Nov. 2024

0.43. **Pages 420–429.**

(1) p. 430 line 4 from above, Example 162
   (a) Let
   (b) Let $1 \le q \le p < \infty$. Let
   (c) 6 Nov. 2024
(2) p. 430 line 12 from above, Example 162 twice
   (a) $\|_{L^p}$
   (b) $\|_{L^q}$
   (c) 6 Nov. 2024
(3) p. 430 line 12 from above, Example 162
   (a) C_p
   (b) C_q
   (c) 6 Nov. 2024
(4) p. 430 line 13 from above, Example 162
   (a) $f \in L^1_{\rm c}(\mathbb{R}^n) = L^1(\mathbb{R}^n) \cap L^0_{\rm c}(\mathbb{R}^n)$
   (b) $f \in L^\infty_{\rm c}(\mathbb{R}^n)$
   (c) 6 Nov. 2024
(5) p. 431, line 1 from below, one line above the headder of §10.5.1
   (a) respecitvely
   (b) respectively
   (c) 21 Sept. 2023
(6) p. 433 line 5 from above,
   (a) such that
   (b) such that $\lim\limits_{\varepsilon \downarrow 0}[a,T]_\varepsilon f = [a,T]f$ almost everywhere and that
   (c) 6 Nov. 2024
(7) p 433 line 5 from above, the proof of Theorem 409.
   (a) 236
   (b) 236 except for the existence of the limit
   (c) 6 Nov. 2024
(8) p. 435, line 3 from above
   (a) specilizes
   (b) specializes
   (c) 21 Sept. 2023
(9) p. 437, line 8 from below
   (a) boundendness
   (b) boundedness
   (c) 21 Sept. 2023

0.44. **Pages 440–last.**

(1) p. 450, [122]
  (a) inequalites
  (b) inequalites
  (c) 21 Sept. 2023
(2) p. 451, [142]
  (a) assosiated
  (b) associated
  (c) 21 Sept. 2023
(3) p. 458, [237]
  (a) Kolmogorov A.
  (b) A. Kolmogorov
  (c) 21 Aug. 2024
(4) p. 462, [297], [298]
  (a) Shimomua
  (b) Shimomura
  (c) 21 Sept. 2023
(5) p. 470, [413]
  (a) Stampaccia
  (b) Stampacchia
  (c) 21 Sept. 2023
(6) p. 478, index
  (a) Minkovski
  (b) Minkowski
  (c) 21 Sept. 2023

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A [2] J.O. Strömberg and A. Torchinsky, Weighted Hardy Spaces, Lecture Notes in Math., 1381, Springer-Verlag, 1989.