Contents

MORREY SPACES–APPLICATIONS TO INTEGRAL OPERATORS AND PDE, II, ERROTUM

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This document is the erratum of the book Morrey spaces–applications to integral operators and PDE, Volumes II published in 2020.

Note: If the change is too major or too minor, I will not mark the change by red.

$\star$ location
- (a) We wrote.
- (b) But we should have written.
- (c) Dates
- (d) Other comments if there are

$\star\star$ location
- (a) Add
- (b) What I want to add actually.
- (c) Dates
- (d) Other comments if there are

$\star\star\star$ location
- (a) Remove
- (b) What I want to remove actually.
- (c) Dates
- (d) Other comments if there are

1. Book II

(1) p. vi, 12.3.2
   (a) Morrey Campanato
   (b) Morrey–Campanato
(2) xii, line 7 from above
   (a) type
   (b) -type
   (c) 30 Sept. 2024
(3) p. xiv, line 6 from above
   (a) type
   (b) -type
   (c) 30 Sept. 2024
(4) p. xiv, line 7 from above
   (a) type
   (b) -type
   (c) 30 Sept. 2024
(5) p. xiv, line 4 from below
   (a) type
   (b) -type
   (c) 30 Sept. 2024
(6) Acknowledgement
   (a) Sakoto Sugano
   (b) Satoko Sugano
(7) Acknowledgement
   (a) Lorenzo Tuccari Yohei Tsutsui
   (b) Lorenzo Tuccari, Yohei Tsutsui
(8) p. xv, swap (1) and (2), since we consider k-times expansion of cubes and then balls.
(9) p. xv, (2) the definition of balls
   (a) $\|x-y\|$
   (b) $|x-y|$
   (c) 14 Dec. 2023
(10) p. xv (5)
   (a) E is integrable over f
   (b) f is integrable over E
(11) p. xvii, swap (1) and (2), since we consider k-times expansion of cubes and then balls.
(12) p. xvii, (7)
   (a) $\displaystyle\int_E f$
   (b) $\displaystyle\int_E f(x)\mathrm{d}x$
   (c) 5 Dec. 2023
(13) p. xvii, (8)
   (a) positive
   (b) non-negative
   (c) 5 Dec. 2023
(14) p. xvii, (8)
   (a) add
   (b) Generally for a measurable function f, we write $m_E^{(\eta)}(f) = m_E(|f|^\eta)^{\frac{1}{\eta}}$.
   (c) 5 Dec. 2023
(15) p. xvii, (9)
   (a) add

   (b) This is a ball-based one. We use the same symbol for the maximal operator generated by cubes.
   (c) 5 Dec. 2023
(16) p. xvii (14),
   (a) $m_Q(f)$
   (b) $m_A(f)$
(17) p. xviii (20)
   (a) $(j \neq k)$.
   (b) $(j \neq k)$
(18) p. xviii (22)
   (a) Minkovski
   (b) Minkowski

1.1. **Pages 1–9.**

(1) p. 1, line 6 from below
  (a) We define
  (b) We define their tensor product
  (c) Oct 3, 2023

(2) p. 2, Example 1
  (a) $\dfrac{1}{p}-\displaystyle\sum_{j=1}^{m}\dfrac{1}{q_j}$
  (b) $\displaystyle\sum_{j=1}^{m}\dfrac{1}{q_j}-\dfrac{1}{p}$

(3) p. 5, line 8 from above, (11.2)
  (a) $|K(y_1,y_2,\ldots,y_m)| \lesssim \frac{1}{(|y_1|+|y_2|+\cdots+|y_m|)^n}$
  (b) $|K(x,y_1,y_2,\ldots,y_m)| \lesssim \frac{1}{(|x-y_1|+|x-y_2|+\cdots+|x-y_m|)^{mn}}$
  (c) 31 Oct. 2023

(4) p. 5, line 10 from above, (11.3)
  (a) $|\nabla K(y_1,y_2,\ldots,y_m)| \lesssim \frac{1}{(|y_1|+|y_2|+\cdots+|y_m|)^{n+1}}$
  (b) $|\nabla K(x,y_1,y_2,\ldots,y_m)| \lesssim \frac{1}{(|x-y_1|+|x-y_2|+\cdots+|x-y_m|)^{mn+1}}$
  (c) 31 Oct. 2023

(5) p. 5, line 17 from above, (11.4)
  (a) $2^{jn}\tau$
  (b) $2^{jn}\Delta\tau$
  (c) 31 Oct. 2023

(6) p. 5, line 13 from above, (11.4)
  (a) $K(x-y_1,x-y_2,\ldots,x-y_m)$
  (b) $K(x,y_1,y_2,\ldots,y_m)$
  (c) 31 Oct. 2023

(7) p. 5, line 8 from below, Example 3
  (a) $\displaystyle\sum_{Q\in\mathcal{D}}\ell(Q)^{-a-mn}\chi_{3Q}(x)\prod_{j=1}^{n}\chi_{3Q}(y_j)\sim\left(\sum_{j=1}^{m}|x-y_j|\right)^{-n-a}$ for $a>0$
  (b) $\displaystyle\sum_{Q\in\mathcal{D}}\ell(Q)^{-a}\chi_{DQ}(x)\prod_{j=1}^{n}\chi_{DQ}(y_j)\sim\left(\sum_{j=1}^{m}|x-y_j|\right)^{-a}$ for $a>0$ and $D\in\mathbb{N}\cap[2,\infty)$
  (c) 31 Oct. 2023

(8) p. 5, line 5 from below, Example 3
  (a) $\langle f_k,\tau_{j\nu}\rangle_{L^2}$ is a slight abuse of notation, since it may happen that $f_k\notin L^2(\mathbb{R}^n)$. However, we tolerate this abuse by considering $f_k=f_k\chi_{\mathrm{supp}(\tau_{j\nu})}$.
  (b) 31 Oct. 2023

(9) p. 6, Example 4
  (a) We should have defined $\widehat{f}$:

$$\widehat{f}(\xi)=\int_{\mathbb{R}^n}f(x)e^{-2\pi ix\cdot\xi}\mathrm{d}x\quad(\xi\in\mathbb{R}^n).$$

  (b) 7 Nov. 2023

(10) p. 6, Example 4
  (a) We should have cited a reference showing that $\mathcal{T}$ is actually a singular integral operator.
  (b) 7 Nov. 2023

(11) p. 7 line 9 from above, the proof of Theorem 7

   (a) T
   (b) $|T$
(12) p. 7, (11.7), the proof of Theorem 7
   (a) g_s, b_s, b_{s+1}
   (b) g_s, b_{s+1}, b_{s+2}
(13) p. 7, line 5 from below, Example 3
   (a) Chapter 3
   (b) Example 55 in Chapter 3
   (c) 31 Oct. 2023
   (d) Note: In the online version, we did not use the citation command.
(14) p. 7 line 2 from below, the proof of Theorem 7
   (a) g_s, b_s, b_{s+1}
   (b) g_s, b_{s+1}, b_{s+2}
   (c) 29 Nov. 2024
(15) p. 8 lines 3 and 8 from above, the proof of Theorem 7
   (a) g_s, b_s, b_{s+1}
   (b) g_s, b_{s+1}, b_{s+2}
   (c) 29 Nov. 2024
(16) p. 9, line 11 from below, Corollary 9
   (a) Then if
   (b) If
   (c) 19 Dec. 2024

1.2. **Pages 10–19.**

(1) p. 10, Exercise 2. x_n (twice) must be replaced by x_m.
(2) p. 12 Example 5. remove: Here, $j = 1, 2, ..., m.$, 18 Dec. 2024
(3) p. 12 line 4 from below, the title of §11.2.2
  (a) Grafakos type
  (b) Grafakos-type
  (c) 18 Dec. 2024
(4) p. 13, Definition 4, twice
  (a) type
  (b) -type
  (c) 30 Sept. 2024
(5) p. 13, line 3 from below, Theorem 14.
  (a) Define p, q, s, t by
  (b) Define p, q, s and t by
(6) p. 13, line 1 from below, Theorem 14.
  (a) Assume $q > 1$.
  (b) Assume $q > 1$ and $0 < t \le s < \infty$.
(7) p. 14 line 1 from above, the proof of Theorem 14
  (a) $\frac{1}{u_1} + \frac{1}{u_2} < 1$
  (b) $\frac{1}{u_1} + \frac{1}{u_2} = 1$
(8) p. 18 line 4 from above,
  (a) 12.2.4.
  (b) 12.2.4, respectively.
(9) We should have taken lines 1–7 from above in page 18 to the last line of page 33, since it concerns the description of Section 2.2.
(10) p. 19, Example 7 (2)
  (a) In fact, we have
  (b) In fact,+ Replace = with $\simeq$.
  (c) 10 Dec. 2021
(11) p. 19 line 7 from below, Example 7(3)
  (a) $\varphi(t_2)$
  (b) $\varphi_2(t_2)$

1.3. **Pages 20–29.**

(1) p. 20, Example 7 (8) (see the range of the supremum)
   (a) $(x,r) \in \mathbb{R}^n \times (0,1)$
   (b) $(x,r) \in \mathbb{R}^n \times (0,\infty)$
   (c) 10 Dec. 2021
(2) p. 21, Example 8
   (a) Let $x \in \mathbb{R}^n$ and $r > 0$.
   (b) Let $x \in \mathbb{R}^n$ and $r > 0$. Also let $\varphi \in \mathcal{G}_q$ with $1 \le q < \infty$. That is, suppose $\varphi \in \mathbb{M}^+(0,\infty)$ and that $t \mapsto t^{-\frac{n}{q}}\varphi(t)$ is decreasing.
   (c) 10 Dec. 2021
(3) p. 22 Example 9. (3)
   (a) By the Lebesgue differentiation theorem, $\|f\|_{L^\infty} \le 0$ if $\beta_1 < 0$, so that $f = 0$ a.e.
   (b) Let $h \in \mathcal{M}^\varphi_q(\mathbb{R}^n)$. Remark that $\|h\|_{L^\infty} \le 0$ if $\beta_1 < 0$, so that $h = 0$ a.e. since

$$\lim_{r\downarrow 0}\ell^{\mathbb{B}}(r) = 0, \lim_{r\downarrow 0}\left(\frac{1}{|Q(x,r)|}\int_{Q(x,r)}|f(y)|^q \mathrm{d}y\right)^{\frac{1}{q}} = |f(x)|$$

   for almost everywhere by the Lebesgue differentiation theorem.
   (c) 19 Dec. 2024
(4) p. 22 line 2 from below, Lemma 19
   (a) Furthemore
   (b) Furthermore
(5) We should have deleted the nesting numbering in Example 9.
(6) p. 23, Lemma 20
   (a) $1 \hookrightarrow$
   (b) $\overset{1}{\hookrightarrow}$
(7) p. 24 line 10 from above, the proof of Proposition 22
   (a) $\varphi_1(t') \equiv \inf_{t \ge t'} \varphi(t)$ then
   (b) $\varphi_1(t') \equiv \inf_{t \ge t'} \varphi(t)$, then
(8) p. 25 line 11 from above, Proposition 23
   (a) decreasing
   (b) increasing
(9) p. 25, the proof of Proposition 23(2)
   (a) Thus, $\psi \in \mathcal{G}_q$.
   (b) Thus, $\psi \in \mathcal{G}_q$. Remark that

$$\psi(t) + \varphi(t) = 2\varphi(t)$$

   if $0 < t < 1$ and that

$$\varphi(t) \le \psi(t) + \varphi(t) \sim \left(\frac{1-e^{-t}}{1-e^{-1}}\right)^{\frac{n}{q}} + \varphi(t) \le \left\{1 + \left(\frac{1-e^{-t}}{1-e^{-1}}\right)^{\frac{n}{q}}\right\}\varphi(t)$$

   if $1 \le t < \infty$.
(10) p. 25 line 9 from below, the proof of Proposition 23
   (a) $\psi + \varphi \simeq \varphi$.
   (b) $\psi + \varphi \sim \varphi$.
   (c) 25 Dec. 2024
(11) p. 26, Example 11.
   (a) $q < \infty$
   (b) $q < p < \infty$
(12) p. 26, Example 11.(1)
   (a) Let $\varphi(t) \equiv \max(t^{\frac{n}{p}}, 1)$

   (b) Let $\varphi(t) \equiv \min(t^{\frac{n}{p}}, 1)$
(13) p. 26, Example 11.(2)
   (a) Let $\varphi(t) \equiv \max(t^a, 1)$ with $a \geq \frac{n}{q}$
   (b) Let $\varphi(t) \equiv \min(t^a, 1)$ with $a = \frac{n}{q}$
(14) p. 26 Definition 9. (1)–(4) (4 times)
   (a) $0 \leq t$
   (b) $0 < t$
(15) p. 27, Example 12(5)
   (a) Lebegsgue
   (b) Lebesgue
(16) p. 27, Example 12(6),
   (a) The
   (b) Then
(17) p. 28, line 11 from above, the proof of Lemma 25
   (a) $\varphi(R)$.
   (b) $\varphi(R)$,
(18) p. 28, Corollary 26
   (a) $> \frac{n}{q}$
   (b) $> 1 + \frac{n}{q}$
   (c) As it stands, we have a counterexample $\varphi = const.$, $q = 2n$, $N_0 = 1$.
(19) p. 28, line 10 from below, the proof of Corollary 26
   (a) we have
   (b) We have

1.4. **Pages 30–39.**

(1) p. 31, one line below (12.8)
   (a) as $k \to \infty$
   (b) for all $k \in \mathbb{N}$
(2) p. 31, two lines above (12.9), Example 14
   (a) Remove
   (b) .
(3) p. 31 line 8 from below, Example 14
   (a) $j \in {F_{l'}}^n$
   (b) $j \in F_{l'}$
(4) p. 31 line 7 from below, Example 14
   (a) $l' = 1, 2 \ldots, 3^n$
   (b) $l' = 1, 2, \ldots, 3^n$
(5) p. 31 line 5 from below, Example 14
   (a) $\varphi((\ell_k m_k)^{-1})$
   (b) $(\varphi((\ell_k m_k)^{-1}))^{-1}$
(6) p. 32 line 12 from above, the proof of Corollary 30
   (a) $l' = 1, 2 \ldots, 3^n$
   (b) $l' = 1, 2, \ldots, 3^n$
(7) p. 32 lines 12 from above, the proof of Corollary 30
   (a) $\varphi((\ell_k m_k)^{-1})$
   (b) $\varphi_1((\ell_k m_k)^{-1})$
(8) p. 32 lines 12 from above, the proof of Corollary 30
   (a) $\dfrac{n}{q_2} - \dfrac{n}{q}$
   (b) $\dfrac{n}{q_2} - \dfrac{n}{q_1}$
(9) p. 32 lines 13 from above, the proof of Corollary 30
   (a) $\frac{1}{\varphi((\ell_k m_k)^{-1})}$
   (b) $\varphi_1((\ell_k m_k)^{-1})$
(10) p. 32 lines 13 from above, the proof of Corollary 30
   (a) $\dfrac{n}{q_2} - \dfrac{n}{q}$
   (b) $\dfrac{n}{q_1} - \dfrac{n}{q_2}$
(11) p. 32 line 4 from below
   (a) $\overline{\mathcal{M}}_q^{\varphi}$
   (b) $\overline{\mathcal{M}}_q^{\varphi}$
   (c) 17 Dec. 2024
(12) p. 33 line 1 from above, the proof of Lemma 32
   (a) $\overline{\mathcal{M}}_q^p(\mathbb{R}^n)$.
   (b) $\overline{\mathcal{M}}_q^p(\mathbb{R}^n)$.
   (c) 11 Nov. 2024
(13) p. 33 line 13 from above, Exercise 6
   (a) $=$
   (b) $\approx$
   (c) 19 Dec. 2024
(14) p. 36 line 3 from above, the proof of Theorem 37
   (a) $\log(m_0')$
   (b) $(m_0' - 1)\log 2$
(15) p. 36 line 3 from above , the proof of Theorem 37
   (a) $\frac{1}{\varphi(r)}$

(b) $\frac{1}{\varphi(r)}$.

(16) p. 36 line 18 from above, the proof of Proposition 38
(a) $\chi_{[1,j]}(m)$
(b) $\chi_{[1,m]}(j)$

(17) p. 36 line 1 from below, the proof of Proposition 38
(a) $\sum_{j=1}^{m}\left(\frac{1}{4^n}\right)^u$
(b) $\left(\sum_{j=1}^{m}\left(\frac{1}{4^n}\right)^u\right)^{\frac{1}{u}}$

(18) p. 38 line 12 from below, the proof of Theorem 40
(a) $\int_0^r \psi(t)\frac{\mathrm{d}t}{t}$
(b) $\int_r^\infty \eta(t)\frac{\mathrm{d}t}{t}$
(c) 30 Sept. 2024
(d) This was not a mistake in itself. But the expression is too abrupt.

(19) p. 38 line 6 from below, Proposition 41
(a) $\frac{r}{2}$
(b) $\frac{r}{s}$

(20) p. 39 line 10 from below, Example 17
(a) $\int_1^\infty$
(b) $\int_0^1$

1.5. Pages 40–49.

(1) p. 42 line 11 from below
  (a) the predual and the predual
  (b) the predual and its predual
(2) p. 43, line 9 from above, the proof of Proposition 44
  (a) suppported
  (b) supported
  (c) 10 Sept. 2023
(3) p. 44, line 2 from below, the proof of Proposition 49
  (a) for all $Q_0 \in \mathcal{D}(\mathbb{R}^n)$.
  (b) for all $Q_0 \in \mathcal{D}(\mathbb{R}^n)$ with $|Q_0| \le 1$.
  (c) 4 Jan. 2024
(4) p. 45, line 1 from above, the proof of Proposition 49
  (a) By
  (b) Denote by $\mathcal{D}^\sharp(Q)$ the set of all dyadic cubes containing Q for a dyadic cube Q. By
  (c) 17 Sept. 2024
(5) p. 45 line 8 from above, the proof of Proposition 49
  (a) $l \in \mathbb{N}$
  (b) $N \in \mathbb{N}$
(6) p. 45 line 9 from above, the proof of Proposition 49
  (a) $Q \in \mathcal{D}_(\mathbb{R}^n)$
  (b) $Q \in \mathcal{D}(\mathbb{R}^n)$
(7) p. 46, lines 13 and 14 from above, the proof of Proposition 52
  (a) If $y \in V \cap B(2^{m-1}r_m) \setminus B(2r_m)$, then $x - y \in V$ and $r_m \le |x-y| \le 2^m r_m$.
  (b) If $y \in \mathbb{R}^n$ satisfies $-y \in V \cap B(2^{m-1}r_m) \setminus B(2r_m)$, then $x - y \in V$ and $x_1 - y_1 \sim -y_1 \sim |x-y| \sim |y|$.
(8) p. 46, line 16 from above, the proof of Proposition 52
  (a) $=$
  (b) $\sim$
(9) p. 47 line 11 from above
  (a) characterization
  (b) characterization without proof. See Lemma 315 below, whose proof is given, for a related assertion.
(10) p. 47 (12.25), Lemma 53
  (a) $= 0$
  (b) $= f$
(11) p. 47, line 1 from below
  (a) $I_\rho f(x)$
  (b) $I_\rho f(x) = T_\rho f(x)$
  (c) We should have used both notation.
(12) p. 48, line 2 from above
  (a) Generalized
  (b) Here, we will use both I_ρ and T_ρ but this is just a matter of taste. Generalized
(13) p. 49, line 14 from above, one line above Lemma 56.
  (a) estimate.
  (b) estimate. For convenience, write $\tilde{\rho}(r) \equiv \displaystyle\int_0^r \frac{\rho(t)}{t} \mathrm{d}t$ for $r > 0$.
(14) p. 49, line 13 from above, Lemma 56.
  (a) $\tilde{\rho}\left(\frac{R}{2}\right) \lesssim I_\rho \chi_{B(R)}(x)$
  (b) $\tilde{\rho}\left(\frac{R}{2}\right) \equiv \int_0^{\frac{R}{2}} \frac{\rho(s)}{s} \mathrm{d}s \lesssim I_\rho \chi_{B(R)}(x)$

1.6. **Pages 50–59.**

(1) p. 50 line 6 from above. Replace the integration domains with and $\mathbb{R}^n \setminus B(2R/3)$. This concerns the proof of Lemma 57. 12 Dec. 2024
(2) p. 50, line 10 from above, Take the sentence
For convenience, write $\tilde{\rho}(r) \equiv \int_0^r \frac{\rho(t)}{t} \mathrm{d}t$ for $r > 0$.
to one line above the statement of Lemma 56. 12 Dec. 2024
(3) Delete the second term in line 6 from above in p. 51 since it duplicates, the proof of Theorem 59.
(4) We should have defined $\tilde{\rho}$ earlier than p. 50 line 10 from above. 8 Oct. 2024
(5) p. 51 line 6 from above, the proof of Lemma 57
  (a) $\leq \frac{1}{r^n} \int_{B(r/2)} I_\rho \chi_{B(r)}(x) \mathrm{d}x \leq$
  (b) $\leq$
  (c) 23 Dec. 2024
(6) p. 51 line 8 from below, the proof of Theorem 59
  (a) $\mathcal{M}^\varphi_q$
  (b) $\mathcal{M}^\varphi_p$
(7) p. 51 line 7 from below, the proof of Theorem 59
  (a) the doubling condition of ψ
  (b) the doubling condition of φ and ψ
(8) p. 52 line 5 from below, Example 28
  (a) $\ell_{-1,0}$
  (b) $\ell^{(-1,0)}$
(9) p. 53 line 2 from above, Example 28
  (a) extending Proposition 26.
  (b) extending Example 26.
(10) p. 53, line 6 from above
  (a) type
  (b) -type
  (c) 30 Sept. 2024
(11) p. 53 line 13 from below, Theorem 60
  (a) simplies
  (b) simplifies
(12) p. 54 line 9 from below, the proof of Lemma 61
  (a) $B(z; r)$
  (b) $B(z, r)$
(13) p. 55 line 6 from below
  (a) $2\varphi(R)$
  (b) $\frac{1}{2\varphi(R)}$
(14) p. 56, line 6 from below, Example 29:
  (a) Let $\lambda < 0$ satisfy $0 < \left(\frac{p}{q} - 1\right) \lambda < n$ and $-\frac{n}{p} < \lambda$
  (b) Let $1 < p < q < \infty$ and $\lambda < 0$ satsfy $-\frac{n}{p} < \lambda$.
  (c) 4 Jan. 2025
(15) p. 56 line 5 from below, Example 29
  (a) $\ell^{(-\mu_1-,\mu_2)}$
  (b) $\ell^{(-\mu_1,-\mu_2)}$
(16) Delete the definition of β_i in Example 30. 11 Nov. 2024
(17) p. 57, line 1 from above, Example 30:
  (a) Let $\mu_1, \mu_2 \geq 0$. Set

(b) Let $1 < p < q < \infty$ and $\mu_1, \mu_2 \geq 0$. Set
(c) 4 Jan. 2025

(18) p. 57 line 7 from above, Example 31
(a) $\beta_2 \equiv \left(\frac{p}{q} - 1\right) \mu_2 - 1 \in (-1, \infty)$
(b) $\beta_2 \equiv -\left(\frac{p}{q} - 1\right) \mu_2 - 1 \in (-\infty, 1)$
(c) 11 Nov. 2024

(19) p. 57 line 9 from above, Example 31
(a) $\varphi \notin$
(b) $\varphi \in$
(c) 11 Nov. 2024

(20) p. 57 line 11 from above, Example 31
(a) $(\beta_1 - 1, \beta_2 + 1)$
(b) $(\beta_1 + 1, \beta_2 + 1)$
(c) 11 Nov. 2024

(21) p. 57 line 11 from above, Example 31
(a) $\beta_1 > 1$
(b) $\beta_1 < -1$
(c) 11 Nov. 2024

(22) p. 57 line 11 from above, Example 31
(a) $(\beta_1 - \mu_1 - 1, \beta_2 - \mu_2 + 1)$
(b) $(\beta_1 - \mu_1 + 1, \beta_2 - \mu_2 + 1)$
(c) 11 Nov. 2024

(23) p. 57, line 14 from above, Example 32:
(a) Let $1 < p, q < \infty$.
(b) Let $1 < p < q < \infty$.
(c) 4 Jan. 2025

(24) p. 57, line 17 from above, Example 32:
(a) $\max(1, r^{-\alpha})$
(b) $\min(1, r^{\alpha})$
(c) 4 Jan. 2025

(25) p. 57 line 15 from below, Example 32.
(a) $\mathbb{Z}_0$
(b) $\mathbb{Z}_0$.

(26) p. 57 line 15 from below, Example 32
(a) Theorems 60–59.
(b) Theorems 59 and 60.

(27) p. 58 line 6 from above, Example 33
(a) $(x \in \mathbb{R}^n)$
(b) $(x \in \mathbb{R}^n)$.

(28) p. 58, line 9 from below
(a) type
(b) -type
(c) 30 Sept. 2024

(29) p. 59 lines 12 and 13 from above
(a) generalized Orlicz–Morrey spaces
(b) generalized Morrey spaces

(30) p. 59, Theorem 68(2)
(a) Moreover, under the assumption that $\rho \sim \varphi/\psi$
(b) Moreover, under the assumption that $\rho \sim \varphi/\psi$ and that ρ is almost increasing

1.7. **Pages 60–69.**

(1) p. 60 line 6 from above, the proof of Theorem 68
  (a) Since $\theta \in \mathcal{G}_n$ and $\rho \sim \varphi/\psi = \varphi/\theta(\varphi)$,
  (b) By assumption,
(2) p. 60 line 8 from above, the proof of Theorem 68
  (a) $\|\{\rho(2^j r_m)\}_{j=1}^m\|_{\ell^u}\chi_{B(r_m)}\|_{\mathcal{M}_1^\psi}$,
  (b) $\|\|\{\rho(2^j r_m)\}_{j=1}^m\|_{\ell^u}\chi_{B(r_m)}\|_{\mathcal{M}_1^\psi}$,
(3) p. 61, line 1 from below, Definition 13
  (a) $\sup\limits_{a\ge 0, r>0}$
  (b) $\|f\|_{\mathrm{BMO}_\varphi(0,\infty)} \equiv \sup\limits_{a\ge 0, r>0}$
  (c) 7 Dec. 2024
(4) p. 62, line 2 from above, (12.1)
  (a) $\equiv$
  (b) $\sim$
  (c) 7 Dec. 2024
(5) p. 62, line 8 from above
  (a) The function
  (b) Note that the function
  (c) 7 Dec. 2024
(6) p. 62, line 13 from above, Lemma 69
  (a) add
  (b) $\varphi \in \mathbb{M}^{\downarrow}$
(7) p. 62, line 14, the proof of Lemma 69
  (a) $\mathbb{M}^{\uparrow}$
  (b) $\mathbb{M}^{\downarrow}$
(8) p. 62, line 15 from above, the proof of Lemma 69
  (a) d
  (b) d
  (c) 7 Dec. 2024
(9) p. 62, lines 16–18, the proof of Lemma 69
  (a) Thus, we conclude

$$\int_0^r \frac{\mathrm{d}t}{\Phi^*(t)} \lesssim \int_0^r \frac{\mathrm{d}t}{\log\max(t,2)} \lesssim 1 + \int_0^r \left(\frac{\mathrm{d}}{\mathrm{d}t}\frac{\max(2,t)}{\log\max(2,t)}\right)\mathrm{d}t \sim \frac{r}{\log r}$$

  for $r \ge 2$.
  (b) Hence if $t \ge 3$

$$\frac{\mathrm{d}}{\mathrm{d}t}\frac{\max(2,t)}{\log\max(2,t)} \ge \frac{1}{\log\max(2,t)} - \frac{1}{\log 3 \times \log\max(2,t)} \sim \frac{1}{\log\max(2,t)}.$$

  Thus, we conclude

$$\begin{aligned}\int_0^r \frac{\mathrm{d}t}{\log\max(t,2)} &= \int_0^3 \frac{\mathrm{d}t}{\log\max(t,2)} + \int_3^r \frac{\mathrm{d}t}{\log\max(t,2)} \\ &\lesssim 1 + \int_0^r \left(\frac{\mathrm{d}}{\mathrm{d}t}\frac{\max(2,t)}{\log\max(2,t)}\right)\mathrm{d}t \sim \frac{r}{\log r}.\end{aligned}$$

  If $2 \le r \le 3$, then we still have

$$\int_0^r \frac{\mathrm{d}t}{\log\max(t,2)} \sim 1 \sim \frac{r}{\log r}.$$

(10) Replace the statement of Lemma 70 and its proof as follows:

Let $\varphi \in \mathbb{M}^{\downarrow}(0,\infty)$ satisfy $\frac{1}{\varphi} \in \mathcal{G}_n$. Then for all $r \geq 0$ and $h > 0$, $\displaystyle\frac{1}{h}\int_r^{r+h} |\Phi^*(s) - \Phi^*(r+h)|\mathrm{d}s \lesssim \frac{1}{\varphi(h)}$. In particular, Φ^* is an element in $\mathrm{BMO}_\varphi(0,\infty)$.

Proof. We calculate

$$|\Phi^*(s) - \Phi^*(r+h)| = \int_s^{r+h} \frac{\mathrm{d}t}{\varphi(t)t}.$$

If we integrate this equality over $s \in [r, r+h]$, then

$$\frac{1}{h}\int_0^h |\Phi^*(s) - \Phi^*(r+h)|\mathrm{d}h = \frac{1}{h}\int_r^{r+h} \frac{(t-r)\mathrm{d}t}{\varphi(t)t} \leq \frac{1}{h}\int_0^h \frac{\mathrm{d}t}{\varphi(t)}.$$

Here in the last step, we have used the fact that $t \mapsto t\varphi(t)$ is increasing. Since φ is decreasing, it follows that

$$\frac{1}{h}\int_0^h |\Phi^*(s) - \Phi^*(r+h)|\mathrm{d}h \leq \frac{1}{h}\int_0^h \frac{\mathrm{d}t}{\varphi(h)} = \frac{1}{\varphi(h)}.$$

□

(11) p. 63, header of § 12.3.2
  (a) Morrey Campanato
  (b) Morrey–Campanato
(12) p. 64 line 10 from below, the proof of Theorem 72
  (a) $D \equiv \{r\omega \,:\, r \in (|x|-r, |x|+r), \omega \in \Omega\}$.
  (b) $D \equiv \{s\omega \,:\, s \in (|x|-r, |x|+r), \omega \in \Omega\}$.
(13) p. 65, line 6 from above, the proof of Lemma 73(1)
  (a) $\Phi^*(6|a|)$
  (b) $(\Phi^*(6r) + \Phi^*(6|a|))$
(14) p. 65, line 7 from above, the proof of Lemma 73(1)
  (a) $\Phi^*(|a|)$
  (b) $(\Phi^*(r) + \Phi^*(|a|))$
(15) p. 65, lines 6 and 7 from above, the proof of Lemma 73(1)
  (a) BMO_φ^+
  (b) BMO_φ
(16) p. 65, line 10 from below, Proposition 74.
  (a) decreasing
  (b) increasing
(17) p. 65, line 8 from below, Proposition 74.
  (a) $Q(a,r) \setminus B(r) = \emptyset$
  (b) $Q(a,r) \cap B(r) = \emptyset$
(18) p. 66, Lemma 75
  (a) $r > 0$
  (b) $r > 0$,
(19) p. 66 line 15 from above, the proof of Lemma 75
  (a) Hölder's inequality
  (b) $\Phi_*(r) = \Phi_*(1)$
(20) p. 66 Exercise 18.(1)
  (a) 2^{-n}
  (b) 2^{-1}
  (c) 1 Oct. 2024
(21) p. 66 Exercise 18.(1)
  (a) $0 < t < s < 2t$

   (b) $0 < s < t < 2s$
   (c) 1 Oct. 2024
(22) p. 68 line 24 from above, §12.2.1
   (a) Gadiyev
   (b) Gadjiev
(23) p. 69 line 7 from above, §12.1.2
   (a) see 10.
   (b) see Example 10.

1.8. **Pages 70–79.**

(1) p. 70 line 18 from below, §12.2.2
   (a) boundendness
   (b) boundedness
(2) p. 70, line 16 from below, §12.2.2
   (a) type
   (b) -type
   (c) 30 Sept. 2024
(3) p. 71 line 6 from above, §12.2.3
   (a) [426, Theorem 5.1], [101, Example 5.1], [426, Theorem 5.1] and [101, Example 5.1]
   (b) [426, Theorem 5.1] and [101, Examples 4 and 5]
(4) p. 71 line 9 from above
   (a) Corollaries 62
   (b) Corollary 63
   (c) 18 Dec. 2024
(5) p. 71, line 17 from above, §12.2.4
   (a) type
   (b) -type
   (c) 30 Sept. 2024
(6) p. 71, line 20 from above, §12.2.4
   (a) type
   (b) -type
   (c) 30 Sept. 2024
(7) p. 72, Take out the whole sentence in Section 12.3.2 to p. 67, line 11 from above 18 Dec. 2024
(8) p. 72 line 5 from below, the header
   (a) Section 12.3.1
   (b) Section 12.3.2
(9) p. 74, Definition 15
   (a) $\inf\left\{\lambda>0\,:\,\frac{\varphi(\ell(Q))}{|Q|}\int_Q \Phi\left(\frac{|f(x)|}{\lambda}\right)\mathrm{d}x \le 1\right\}$.
   (b) $\inf\left(\left\{\lambda>0\,:\,\frac{\varphi(\ell(Q))}{|Q|}\int_Q \Phi\left(\frac{|f(x)|}{\lambda}\right)\mathrm{d}x \le 1\right\}\cup\{\infty\}\right)$.
   (c) 7 Jul. 2021
(10) p. 75, line 2 from above, (13.2)
   (a) $\inf\left\{\lambda>0\,:\,\left\|\Phi\left(\frac{|f|}{\lambda}\right)\right\|_{\mathcal{M}^{\varphi}_1} \le D\right\}$
   (b) $\inf\left(\left\{\lambda>0\,:\,\left\|\Phi\left(\frac{|f|}{\lambda}\right)\right\|_{\mathcal{M}^{\varphi}_1} \le D\right\}\cup\{\infty\}\right)$
   (c) 7 Jul. 2021
(11) p. 75, line 4 from below, the proof of Proposition 76
   (a) $\inf\left\{\lambda>0\,:\,\left\|\Phi\left(\frac{|f|}{\lambda}\right)\right\|_{\mathcal{M}^{\varphi^*}_1} \le D\right\}$
   (b) $\inf\left(\left\{\lambda>0\,:\,\left\|\Phi\left(\frac{|f|}{\lambda}\right)\right\|_{\mathcal{M}^{\varphi^*}_1} \le D\right\}\cup\{\infty\}\right)$
   (c) 7 Jul. 2021
(12) p. 75, line 3 from below, the proof of Proposition 76

(a) $\inf\left\{\lambda>0\,:\,\left\|\Phi\left(\frac{|f|}{\lambda}\right)\right\|_{\mathcal{M}^{\varphi^*}_1}\le D\right\}$
(b) $\inf\left(\left\{\lambda>0\,:\,\left\|\Phi\left(\frac{|f|}{D\lambda}\right)\right\|_{\mathcal{M}^{\varphi^*}_1}\le 1\right\}\cup\{\infty\}\right)$
(c) 7 Jul. 2021

(13) p. 76, line 2 from above, Lemma 77
  (a) add
  (b) there exists $D>0$ such that
(14) p. 76 line 12 from above
  (a) generalized Morrey spaces of the first kind
  (b) generalized Orlicz–Morrey spaces of the first kind
(15) p. 77 line 6 from above, the proof of Proposition 79
  (a) delete
  (b) for all $r>0$
(16) p. 77 line 10 from below, Example 41
  (a) $L^0(4r)$
  (b) $L^0(Q(4r))$
  (c) 19 Dec. 2024
(17) p. 77 line 7 from below, Example 41
  (a) Lr
  (b) r
  (c) 19 Dec. 2024
(18) p. 77 line 2 from below
  (a) $\{R_j\}$
  (b) $\{R_j\}_{j=1}^{(\kappa+1)^n}$
  (c) 19 Dec. 2024
(19) p. 78 line 6 from above, Example 41
  (a) ≤ 1
  (b) $\lesssim 1$
(20) p. 78 line 7 from above, Example 41
  (a) geometical
  (b) geometrical
(21) p. 78 line 10 from above, Example 41
  (a) $\frac{|Q|kt^n\varphi(R)}{r^n\varphi(t)}\lesssim_n\frac{\varphi(R)}{\varphi(r)}$
  (b) $\frac{kt^n\varphi(R)}{r^n\varphi(t)}\sim\frac{\varphi(R)}{\varphi(r)}$
(22) p. 78 line 11 from above, Example 41
  (a) (2)
  (b) (3)
(23) p. 78 line 12 from above, Example 41
  (a) $\frac{\varphi(R)}{|Q|}\int_Q\Phi(|f(x)|)\mathrm{d}x\le\frac{\varphi(4R)}{|Q_0|}\int_{Q_0}\Phi(|f(x)|)\mathrm{d}x$
  (b) $\frac{\varphi(R)}{|Q|}\int_Q\Phi(|f(x)|)\mathrm{d}x\lesssim_n\frac{\varphi(r)}{|Q_0|}\int_{Q_0}\Phi(|f(x)|)\mathrm{d}x$
(24) p. 79 line 11 from above, Lemma 82
  (a) $\frac{\Phi^{-1}(\varphi(r)^{-1})}{\Psi^{-1}(r)}$
  (b) $\frac{\Phi^{-1}(\varphi(r)^{-1})}{\Psi^{-1}(\psi(r)^{-1})}$
(25) p. 79 line 9 from below, the proof of Lemma 82

(a) $|Q(r_k)|$
(b) $\psi(r_k)^{-1}|Q(r_k)|$

1.9. **Pages 80–89.**

(1) p. 80, lines 12, 10, 3 from below, the proof of Theorem 83
  (a) $\displaystyle\int_{Q(x,r)} \Psi\left(\frac{Mf_1(x)}{\lambda}\right) \mathrm{d}x$
  (b) $\displaystyle\int_{Q(x,r)} \Psi\left(\frac{Mf_1(y)}{\lambda}\right) \mathrm{d}y$

(2) p. 80 line 9 from below, the proof of Theorem 83
  (a) $\leq$
  (b) $>$

(3) p. 80 line 7 from below, the proof of Theorem 83
  (a) $2\Psi^{-1}(\psi(r)^{-1})$
  (b) $\Psi^{-1}(\psi(r)^{-1})/2$

(4) p. 81, lines 2, 7 from above, the proof of Theorem 83
  (a) $\displaystyle\int_{Q(x,r)} \Psi\left(\frac{Mf_1(x)}{\lambda}\right) \mathrm{d}x$
  (b) $\displaystyle\int_{Q(x,r)} \Psi\left(\frac{Mf_1(y)}{\lambda}\right) \mathrm{d}y$

(5) p. 81, line 9 from above, the proof of Theorem 83
  (a) $\displaystyle\int_{Q(x,r)} \Psi\left(\frac{Mf_2(x)}{\lambda}\right) \mathrm{d}x$
  (b) $\displaystyle\int_{Q(x,r)} \Psi\left(\frac{Mf_2(y)}{\lambda}\right) \mathrm{d}y$
  (c) 30 Sept. 2024

(6) p. 82 (13.15), the proof of Theorem 83
  (a) $\mathrm{supp}(f)$
  (b) $\mathrm{supp}(f_k)$

(7) p. 84 line 6 from above, the proof of Theorem 84
  (a) $0 < a \ll 0$
  (b) $0 < a \ll 1$

(8) p. 84 line 6 from above, the proof of Theorem 84
  (a) $2k\Phi(kr) \leq \Phi(r)$
  (b) $2k\Phi(r) \leq \Phi(kr)$

(9) p. 85, line 17 from below, the proof of Theorem 86
  (a) (12.23) and
  (b) $\|M^{\left(\frac{1}{2}\right)}\chi_{V\cap B(r_m)}\|_{\mathcal{M}_1^{\psi}} = (\|M\chi_{V\cap B(r_m)}\|_{\mathcal{M}_2^{\psi^{\frac{1}{2}}}})^2$ and

(10) p. 85, line 17 from below, the proof of Theorem 86
  (a) $V \cap B(r_m)$
  (b) $V \cap B(r_m)$ (see (12.23)).

(11) p. 89 lines 3 and 4 from above
  (a) generalized Morrey spaces
  (b) generalized Orlicz–Morrey spaces

(12) p. 89 Definition 18. (2)
  (a) $\chi_{(0,\lambda)}$
  (b) $\chi_{(\lambda,\infty]}$

(13) p. 89 Definition 18(2)
  (a) The function space $\mathrm{W}\mathcal{M}_{\Phi}^{\varphi}(\mathbb{R}^n)$ is defined as the weak generalized Orlicz–Morrey space
  (b) The weak generalized Orlicz–Morrey space $\mathrm{W}\mathcal{M}_{\Phi}^{\varphi}(\mathbb{R}^n)$ of the second kind
  (c) 30 Dec. 2024

1.10. **Pages 90–99.**

(1) p. 90 Example 47
  (a) Remark 4. Since
  (b) Remark 4. Since Φ_1 is convex,
  $\Phi_1(t) \geq \Phi_1(1)t$
  for all $t \geq 1$. Since this inequality yields
(2) p. 90 line 2 from below, Example 47. Take off the period
(3) p. 91 line 11 from above
  (a) Orlicz–Morrey spaces
  (b) generalized Orlicz–Morrey spaces
(4) p. 91, Proposition 91
  (a) and for all $f \in \mathcal{M}^p_{\mathrm{L}\log\mathrm{L}}(\mathbb{R}^n)$. Moreover,
  $\|Mf\|_{\mathcal{M}^p_1} \sim \|f\|_{\mathcal{M}^p_{\mathrm{L}\log\mathrm{L}}}$.
  (b) . Moreover,
  $\|Mf\|_{\mathcal{M}^p_1} \sim \|f\|_{\mathcal{M}^p_{\mathrm{L}\log\mathrm{L}}}$
  for all $f \in \mathcal{M}^p_{\mathrm{L}\log\mathrm{L}}(\mathbb{R}^n)$.
(5) p. 91 line 3 from below
  (a) Orlicz–Morrey spaces
  (b) generalized Orlicz–Morrey spaces
(6) p. 92 line 5 from above, Example 48
  (a) Theorem 83
  (b) Theorem 83 and Proposition 92
  (c) 23 Jan. 2025
(7) p. 92 line 6 from above, Example 48 (13.1)
  (a) φ
  (b) ψ
  (c) 23 Jan. 2025
(8) p. 92 line 7 from above, Example 48
  (a) Theorem 95 below will prove that equality fails in (13.1).
  (b) fails
  (c) 23 Jan. 2025
  (d) Also delete the period in the above line.
(9) p. 92 Example 49
  (a) and let $\varphi \in \mathcal{G}_1 \cap \mathbb{Z}_0$.
  (b) and let $\varphi \in \mathcal{G}_1$. Assume $\frac{1}{\varphi} \in \mathbb{Z}_0$.
  (c) 23 Jan. 2025
(10) p. 93, Theorem 94
  (a) Set
  (b) Assume that ρ satisfies the doubling condition: $\rho(s) \sim \rho(r)$ if $0 < r < s < 2r$. Set
(11) p. 93 line 3 from below, the proof of Theorem 94
  (a) $\displaystyle\sum_{l=1}^{\infty} \frac{\rho(\ell(Q^0))}{\varphi(\ell(Q^0))}$
  (b) $\displaystyle\sum_{l=1}^{\infty} \frac{\rho(2^l\ell(Q^0))}{\varphi(2^l\ell(Q^0))}$
  (c) 30 Sept. 2024
(12) p. 94 line 9 from above, add the period. the proof of Theorem 94
(13) p. 94 line 4 from below, the proof of Theorem 94
  (a) $R \in \mathcal{D}$
  (b) $R \in \mathcal{D}(Q^0)$
  (c) 30 Sept. 2024

(14) p. 95 lines 2 (twice), 3 and 6 from above, the proof of Theorem 94
   (a) Q
   (b) Q^0
   (c) 30 Sept. 2024
(15) p. 96 lines 1, 2, 4 and 8 from above
   (a) Orlicz–Morrey spaces
   (b) generalized Orlicz–Morrey spaces
   (c) 30 Sept. 2024
(16) p. 96 line 13 from above, Theorem 95
   (a) $E_j \equiv \bigcup_{e \in \{0,1\}^n} \{(1-\kappa)e + \kappa E_{j-1}\}$
   (b) $E_j \equiv \bigcup_{\vec{e} \in \{0,1\}^n} \{(1-\kappa)\vec{e} + \kappa E_{j-1}\}$
   (c) 30 Sept. 2024
(17) p. 96 line 12 from below, the proof of Theorem 96
   (a) Then for all $t > 0$.
   (b) Then for all $t > 0$,
   (c) 30 Sept. 2024
(18) p. 96 line 11 from below, the proof of Theorem 96
   (a) $\varphi(t^n)^{-1}$
   (b) $\varphi(t^n)$
   (c) 2 Oct. 2024
(19) p. 96 line 95 from below, the proof of Theorem 96
   (a) $\varphi(\kappa^{-nj})^{-1}$
   (b) $\varphi(\kappa^{jn})$
   (c) 2 Oct. 2024
(20) p. 96 line 5 from below, the proof of Theorem 96
   (a) $\varphi(\kappa^{-nk})^{-1}$
   (b) $\varphi(\kappa^{-nk})$
   (c) 2 Oct. 2024
(21) p. 96 line 4 from below, the proof of Theorem 96
   (a) $\varphi(\kappa^{-nk_j})^{-1}$
   (b) $\varphi(\kappa^{-nk_j})$
   (c) 2 Oct. 2024
(22) p. 96 line 2 from below, the proof of Theorem 96
   (a) $\leq \varphi(\kappa^{-nk_j})^{-1}$
   (b) $= \varphi(\kappa^{-nk_j})$
   (c) 2 Oct. 2024, 7 Dec. 2024
   (d) Remove -1 twice.
(23) p. 97 line 2 from above, the proof of Theorem 96
   (a) j_k
   (b) k_j
   (c) 30 Sept. 2024
(24) p. 97 (13.6), the proof of Theorem 96
   (a) $\varphi(\kappa^{-nj})$
   (b) $\varphi(\kappa^{-nj})^{-1}$
   (c) 30 Sept. 2024
(25) p. 97 line 4 from above, the proof of Theorem 96
   (a) $nj/2$
   (b) $jn/2$
   (c) 30 Sept. 2024
(26) p. 97 line 10 from above, Example 50

  (a) if
  (b) the most right inclusion is non-trivial. If
  (c) 30 Sept. 2024
(27) p. 97 line 12 from above, Example 50
  (a) then
  (b) then since $\varphi(\ell(Q)) \sim \ell(Q)^{\frac{n}{3}}$ and $\Phi(t) \sim t^3$ for $t \geq 1$,
  (c) 30 Sept. 2024
(28) p. 97 line 14 from above, Example 50
  (a) then
  (b) then since $\varphi(\ell(Q)) \sim \ell(Q)^{\frac{n}{2}}$ and $\Phi(t) \sim t^3$ for $t \geq 1$,
  (c) 30 Sept. 2024
(29) p. 97 line 5 from below, the proof of Theorem 96
  (a) $\|\chi_{[0,t]^n}\|_{\mathcal{L}^{\varphi}_{\Phi}} \sim$
  (b) $\|\chi_{[0,t]^n}\|_{\mathcal{L}^{\varphi}_{\Phi}} =$
  (c) 30 Sept. 2024
(30) p. 98 line 8 from above, the proof of Theorem 96
  (a) $\sim$
  (b) $\lesssim$
  (c) 18 Dec. 2024
(31) p. 98 (13.12), the proof of Theorem 96
  (a) add
  (b) for some constant $K \geq 1$ independent of t
  (c) 30 Sept. 2024
(32) p. 99 line 12 from above, §13.1
  (a) Kokilashvil
  (b) Kokilashvili
  (c) 30 Sept. 2024

1.11. **Pages 100–109.**

(1) p. 100 line 11, 9 and 5 from below, §13.2.3, §13.2.4
  (a) Orlicz–Morrey spaces
  (b) generalized Orlicz–Morrey spaces
  (c) 30 Sept. 2024
(2) p. 100 lines 2 and 3 from below, §13.2.4
  (a) generalized Morrey spaces
  (b) generalized Orlicz–Morrey spaces
  (c) 30 Sept. 2024
(3) p. 100 line 2 from below, §13.2.4
  (a) Theorem
  (b) Theorem 14
  (c) 30 Sept. 2024
(4) p. 101 line 7, 8 and 15 from above
  (a) Orlicz–Morrey spaces
  (b) generalized Orlicz–Morrey spaces
  (c) 30 Sept. 2024
(5) Swap lines 7–9 from above in p. 101 and the header "Section 13.3.2". Therefore, Section 13.3.2 starts with "Using generalized Orlicz–Morrey spaces of the second kind, we can describe the action of the iterated maximal operator M^j, whose range is $\mathcal{M}^p_1(\mathbb{R}^n)$." 30 Sept. 2024
(6) p. 103 Definition 19
  (a) $\mathcal{M}^p_q(\mathbb{R}^n)$
  (b) $\mathcal{M}^p_q(\mathbb{R}^n_+)$
  (c) 30 Sept. 2024
(7) p. 105 line 6 from below, the proof of Theorem 99
  (a) $L^q(\mathbb{R}^n)$
  (b) $L^q(\mathbb{R}^n_+)$
  (c) 30 Sept. 2024
(8) p. 107 line 18 from above, Example 51
  (a) regareded
  (b) regarded
  (c) 10 Sept. 2023
(9) p. 107 line 3 from below, Definition 22
  (a) $\mathcal{M}^p_q$
  (b) $\mathrm{W}\mathcal{M}^p_q(\Omega)$
  (c) 30 Sept. 2024
(10) p. 107 line 2 from below, Definition 22
  (a) $\mathcal{M}^p_q(\Omega)$
  (b) $\mathrm{W}\mathcal{M}^p_q(\Omega)$
  (c) 30 Sept. 2024
(11) p. 107 line 1 from below, Definition 22
  (a) $\mathrm{W}\mathcal{M}^p_q$
  (b) $\mathrm{W}\mathcal{M}^p_q(\Omega)$
  (c) 30 Sept. 2024
(12) p. 109 line 10 from below, the definition of M
  (a) $x, y \in \mathbb{R}^n$
  (b) $x, y \in \mathbb{R}^{n-1}$
  (c) 30 Sept. 2024
(13) p. 109 line 9 from below
  (a) $\Omega_\varphi \equiv$

(b) $\Omega = \Omega_\varphi \equiv$
(c) 30 Sept. 2024

1.12. **Pages 110–119.**

(1) p. 111, line 16 from below, the proof of Theorem 108
  (a) immediatly
  (b) immediately
  (c) 26 Sept. 2023
(2) p. 112 line 1 from above, the proof of Theorem 108
  (a) Θ_j
  (b) Θ_h
  (c) 30 Sept. 2024
(3) p. 112 line 13 from above
  (a) $F \in W^l \mathcal{M}^p_q(\Omega)$
  (b) $F \in W^l \mathcal{M}^p_q(\mathbb{R}^n)$
  (c) 30 Sept. 2024
(4) p. 114 Example 54. (1)
  (a) $\mathcal{L}_k^{\frac{1}{p},q}(\Omega)$
  (b) $\mathcal{L}_k^{\frac{n}{p},q}(\Omega)$
  (c) 30 Sept. 2024
(5) p. 114 line 15 from below
  (a) Taylor expansion of u
  (b) Taylor expansion of f
  (c) 30 Sept. 2024
(6) p. 114 line 9 from below (Definition 28),
  (a) $L^q(\Omega(x_0; \rho))$
  (b) $L^q(\Omega(x_0, \rho))$
  (c) 30 Sept. 2024
(7) p. 114 line 4 from below, Example 55. (1)
  (a) $m_{\Omega(x_0,\rho)}(u)$
  (b) $m_{\Omega(x_0,\rho)}(f)$
  (c) 30 Sept. 2024
(8) p. 114 line 3 from below, Example 55. (2)
  (a) $u - m_{\Omega(x_0,\rho)}(u)$
  (b) $f - m_{\Omega(x_0,\rho)}(f)$
  (c) 30 Sept. 2024
(9) p. 114 line 3 from below (Example 55),
  (a) $L^q(\Omega(x_0; \rho))$
  (b) $L^q(\Omega(x_0, \rho))$
  (c) 30 Sept. 2024
(10) p. 114 line 1 from below, Example 55. (2)
  (a) $u - m_{\Omega(x_0,\rho)}(u)$
  (b) $f - m_{\Omega(x_0,\rho)}(f)$
  (c) 30 Sept. 2024
(11) p. 115 line 4 from above, Lemma 110, twice
  (a) P_k
  (b) $\partial^\alpha P_k$
  (c) 11 Nov. 2024
(12) p. 115 line 4 from above, Lemma 110
  (a) L^q
  (b) L^∞
  (c) 11 Nov. 2024
(13) p. 115 line 4 from above, Lemma 110

   (a) $\frac{n}{q} - \lambda$
   (b) $-|\alpha| - \lambda$
   (c) 11 Nov. 2024
(14) p. 115 line 8 from above, Lemma 111
   (a) $\mathbb{N}_0 \setminus \{-\lambda\}$
   (b) $\mathbb{N}_0$
   (c) 11 Nov. 2024
(15) p. 115 line 9 from above, Lemma 111
   (a) with $|\alpha| \le k$
   (b) with $|\alpha| \le k$ and $|\alpha| + \lambda \ne 0$
   (c) 11 Nov. 2024
(16) p. 115 line 10 from above (twice), Lemma 111
   (a) $-k - \lambda$
   (b) $-|\alpha| - \lambda$
   (c) 11 Nov. 2024
(17) p. 115 line 13 from above, the proof of Lemma 111
   (a) right-hand side
   (b) left-hand side
   (c) 30 Sept. 2024
(18) p. 115 line 17 from above, the proof of Lemma 111
   (a) $\partial^\alpha P_k(x; x_0, 2^{-j}\rho, f)$
   (b) $\partial^\alpha P_k(x_0; x_0, 2^{-j}\rho, f)$
   (c) 30 Sept. 2024
(19) p. 115 line 17 from above, the proof of Lemma 111
   (a) $\partial^\alpha P_k(x; x_0, 2^{-j-1}\rho; f)$
   (b) $\partial^\alpha P_k(x_0; x_0, 2^{-j-1}\rho, f)$
   (c) 30 Sept. 2024
(20) p. 115 line 17 from above, the proof of Lemma 111
   (a) remove
   (b) $(x \in \mathbb{R}^n)$
   (c) 30 Sept. 2024
(21) p. 115 line 20 from above, the proof of Lemma 111
   (a) $\partial^\alpha P_k(x_0; x_0, 2^{-j-1}\rho; f)$
   (b) $\partial^\alpha P_k(x_0; x_0, 2^{-j-1}\rho, f)$
   (c) 30 Sept. 2024
(22) p. 115 line 6 from below, the proof of Lemma 111
   (a) delete
   (b) with $x_0 = y_0$
   (c) 30 Sept. 2024
(23) p. 115 line 5 from below, the proof of Lemma 111
   (a) $-k - \lambda$
   (b) $-|\alpha| - \lambda$
   (c) 11 Nov. 2024
(24) p. 115 line 4 from below twice, the proof of Lemma 111,
   (a) $-k - \lambda$
   (b) $-|\alpha| - \lambda$
   (c) 11 Nov. 2024
(25) p. 116 line 2 from above, Lemma 112
   (a) $k \in \mathbb{N}_0$
   (b) $k \in \mathbb{N}_0$ satisfy $\lambda + k < 0$
   (c) 11 Nov. 2024
(26) p. 116 line 6 from above,

   (a) $L^q(\Omega(x_0;\rho))$
   (b) $L^q(\Omega(x_0,\rho))$
   (c) 30 Sept. 2024
(27) p. 116 line 7 from above,
   (a) $L^q(\Omega(x_0;\rho))$
   (b) $L^q(\Omega(x_0,\rho))$
   (c) 30 Sept. 2024
(28) p. 116 line 8 from above (twice),
   (a) $L^q(\Omega(x_0;\rho))$
   (b) $L^q(\Omega(x_0,\rho))$
   (c) 30 Sept. 2024
(29) p. 116 line 9 from above (twice), the proof of Lemma 112
   (a) $L^q(\Omega(x_0;2\rho))$
   (b) $L^q(\Omega(x_0,2\rho))$
   (c) 30 Sept. 2024
(30) p. 116 line 17 from above (the proof of Lemma 112), (14.5)
   (a) $L^q(\Omega(x_0;\rho))$
   (b) $L^q(\Omega(x_0,\rho))$
   (c) 30 Sept. 2024
(31) p. 117, one line above (14.7), Corollary 114
   (a) $x \in \Omega$
   (b) $x_0 \in \Omega$
   (c) 30 Sept. 2024
(32) p. 118 line 11 from above (twice) (Exercise 31),
   (a) $L^q(\Omega(x_0;\rho))$
   (b) $L^q(\Omega(x_0,\rho))$
   (c) 30 Sept. 2024
(33) p. 118 line 1 from below (Exercise 31)
   (a) $-u+u$
   (b) $-f+f$
   (c) 30 Sept. 2024

1.13. **Pages 120–129.**

(1) p. 120 line 4 from below, Example 56
  (a) delete
  (b) of this proposition
  (c) 30 Sept. 2024
(2) p. 124 line 11 from above, the proof of Lemma 122
  (a) K_j
  (b) J_k
  (c) 30 Sept. 2024
(3) p. 124 line 13 from above (twice), the proof of Lemma 122
  (a) $z($
  (b) $c($
  (c) 30 Sept. 2024
(4) p. 125 line 8 from above, the proof of Theorem 121
  (a) in the right-hand
  (b) on the right-hand
  (c) 21 Nov. 2024
(5) p. 125 line 4 from below, the proof of Theorem 121
  (a) 2^{i-1}
  (b) 2^{l-1}
  (c) 30 Sept. 2024
(6) p. 125 line 4 from below, the proof of Theorem 121
  (a) $\leq$
  (b) $\leq \sum_{l=1}^{\infty}$
  (c) 30 Sept. 2024
(7) p. 125 line 3 from below, the proof of Theorem 121
  (a) $=$
  (b) $= \sum_{l=1}^{\infty}$
  (c) 30 Sept. 2024
(8) p. 125 line 1 from below, the proof of Theorem 121
  (a) $\mu(Q_0)^{\frac{1}{p}}$
  (b) $\mu(Q_0)^{\frac{1}{p}} \sum_{l=1}^{\infty}$
  (c) 30 Sept. 2024
(9) p. 126 line 2 from above, the proof of Theorem 121
  (a) $\leq$
  (b) $\leq \sum_{l=1}^{\infty}$
  (c) 30 Sept. 2024
(10) p. 126 line 9 from above, Exercise 32
  (a) $\mathcal{M}^p_q(\mu)$
  (b) $\mathcal{M}^s_t(\mu)$
  (c) 30 Sept. 2024
(11) p. 126 line 7 from below, Exercise 32
  (a) torelate
  (b) tolerate
  (c) 30 Sept. 2024

(12) p. 126 line 6 from below, Exercise 33
 (a) $\mathcal{H}^{p'}_{q'}(2,\mu)$
 (b) $\mathcal{H}^{p}_{q}(2,\mu)$ with $1 < p \le q < \infty$
 (c) 30 Sept. 2024
(13) p. 127, line 13 from below
 (a) Eucledean
 (b) Euclidean
 (c) 26 Sept. 2023
(14) p. 128 line 9 from below, the proof of Proposition 123
 (a) satifying
 (b) satisfying
 (c) 30 Sept. 2024
(15) p. 129 line 9 from below
 (a) . when
 (b) when
 (c) 5 Oct. 2024
(16) p. 129 line 4 from below
 (a) $\mathcal{B}_a$
 (b) $\mathcal{B}_A$
 (c) 30 Sept. 2024
(17) p. 129 line 4 from below
 (a) (μ)
 (b) (γ)
 (c) 30 Sept. 2024
(18) p. 129 line 3 from below
 (a) (μ)
 (b) (γ)
 (c) 30 Sept. 2024

1.14. **Pages 130–139.**

(1) p. 130 line 1 from above
  (a) μ
  (b) γ
  (c) 30 Sept. 2024
(2) p. 130 line 1 from above
  (a) $\mathcal{B}_a$
  (b) $\mathcal{B}_A$
  (c) 30 Sept. 2024
(3) p. 131 line 7 from above, the proof of Proposition 125
  (a) (i)–(iv)
  (b) (A)–(D)
  (c) 30 Sept. 2024
(4) p. 130 line 12 from above, the proof of Theorem 124
  (a) $8^{n+\frac{1}{p}-\frac{1}{q}}$
  (b) $8^{n-\frac{1}{p}+\frac{1}{q}}$
  (c) 30 Sept. 2024
(5) p. 130 line 14 from above, the proof of Theorem 124
  (a) $8^{n+\frac{1}{p}-\frac{1}{q}}$
  (b) $8^{n-\frac{1}{p}+\frac{1}{q}}$
  (c) 30 Sept. 2024
(6) p. 131 line 18 from above, Example 57
  (a) Then if
  (b) If
  (c) 19 Dec. 2024
(7) p. 131 line 18 from above, Example 57
  (a) $-\frac{1}{p}$
  (b) $\frac{1}{p}$
  (c) 5 Oct. 2024
(8) p. 131 line 10 from above, the proof of Proposition 125
  (a) Applying Lemma 264(iii) in the first book together with (13.12),
  (b) Thus,
  (c) 5 Oct. 2024
(9) p. 135 line 2 from above, the proof of Lemma 129
  (a) $N(3^{k-1})$
  (b) $N(3^{-k+1})$
  (c) 30 Sept. 2024
(10) p. 135 line 11 from above, the proof of Lemma 129
  (a) $N(-3^{(2k-1)a+1})$
  (b) $N(3^{-(2k-1)a+1})$
  (c) 30 Sept. 2024
(11) p. 135 line 12 from above, the proof of Lemma 129
  (a) $N(3^{-(2k-1)a}+1)$
  (b) $N(3^{-(2k-1)a+1})$
  (c) 30 Sept. 2024
(12) p. 135 line 3 from below (twice), the proof of Lemma 129
  (a) $\mathbf{o}$
  (b) $\mathbf{O}$
  (c) 30 Sept. 2024
(13) p. 135 line 2 from below, the proof of Lemma 129, twice

   (a) **o**
   (b) **O**
   (c) 30 Sept. 2024
(14) p. 136 line 12 from above, the proof of Lemma 129
   (a) delete
   (b) $\Delta(x_{N(3^{-k})}, r)\times$
   (c) 30 Sept. 2024
(15) p. 138 line 9 from above, §14.1
   (a) Calderon
   (b) Calderón
   (c) 30 Sept. 2024
(16) p. 138 line 13 from above, §14.1
   (a) Calderon
   (b) Calderón
   (c) 30 Sept. 2024
(17) p. 138 line 18 from above, §14.1
   (a) Camapanto
   (b) Campanato
   (c) 26 Sept. 2023
(18) Take out line 7 from below in p. 138 to the comments about §14.3.1.
   (a) 5 Oct. 2024
(19) Take out lines 1–3 from above in p. 139 to the comments about §14.2.1.
   (a) 5 Oct. 2024
(20) p. 139 line 4 from below, §14.3.2
   (a) Gorka
   (b) Górka
   (c) 30 Sept. 2024

1.15. **Pages 140–149.**

(1) p. 140 line 3 from above. Take out
    See the textbook *** for the Gauss measure space.
  to Section 14.5 in p. 141. 5 Oct. 2024
(2) p. 141, line 10 from below
  (a) singular operatrs
  (b) singular integral operators
  (c) 12 Dec. 2024
(3) p. 142, line 2 from below
  (a) type
  (b) -type
  (c) 30 Sept. 2024
(4) p. 143 (15.1)–(15.3),
  (a) $\mathrm{d}x$
  (b) $\mathrm{d}y$
  (c) 30 Sept. 2024
(5) Remove the unnecessary space in p. 144 line 1 from above.
(6) p. 144 line 2 from above
  (a) $v(x)$
  (b) $w(x)$
  (c) 30 Sept. 2024
(7) p. 144, line 6 from above
  (a) local weighted Morrey
  (b) weighted local Morrey
  (c) 30 Sept. 2024
(8) p. 144 line 4 from below
  (a) a_{ij}
  (b) a_{jk}
  (c) 30 Sept. 2024
(9) p. 144 line 1 from below,
  (a) $B(x, 3r)$
  (b) $B(x_0, 3r)$
  (c) 30 Sept. 2024
(10) p. 145 line 3 from above
  (a) $a_{ij}(x)\partial_i u(x)\partial_j \kappa$
  (b) $\sum_{j,k=1}^{n} a_{jk}(x)\partial_j u(x)\partial_k \kappa$
  (c) 30 Sept. 2024
(11) p. 146, line 13 from below
  (a) local weighted Morrey
  (b) weighted local Morrey
  (c) 30 Sept. 2024
(12) p. 147 line 4 from below,
  (a) We borrowed from the idea of [89] for the proof of
  (b) We drew from the concept presented in [89] to support our proof of
  (c) 18 Dec. 2024
(13) p. 147 line 4 from below,
  (a) [107]
  (b) [89]
  (c) 18 Dec. 2024
(14) p. 149 line 1 from above, the proof of Proposition 131

(a) in the right-hand
(b) on the right-hand
(c) 21 Nov. 2024

(15) p. 149 (15.2), Take out $\ell(Q)^{n-\frac{n+\beta}{p}-\frac{\varepsilon}{q}}$ to the right-hand side.

1.16. **Pages 150–159.**

(1) p. 150, line 10 from above, the proof of Proposition 131
  (a) $\mathrm{L}\mathcal{M}^p_q$
  (b) $\mathcal{M}^p_q$
  (c) 31 Jul. 2024
(2) p. 151 line 9 from below, the proof of Proposition 133
  (a) Example and 58
  (b) Example 58
  (c) 30 Sept. 2024
(3) p. 152 lines 12, 10 and 6 from below, Proposition 135 and its proof
  (a) $\mathcal{M}^p_q(|x|^{tq},|x|^{tq})$
  (b) $\mathcal{M}^s{}_t(|x|^{tq},|x|^{tq})$
  (c) 10 Dec. 2021, 5 Oct. 2024
(4) p. 152, Proposition 135
  (a) Remove
  (b) "The following are equivalent".
  (c) 10 Dec. 2021
  (d) 30 Sept. 2024
(5) p. 152 line 5 from below, the proof of Proposition 135
  (a) $\chi_{B(1)} \in \mathcal{M}^p_q(|x|^{t\beta},|x|^{t\beta})$ if and only if $0 \ge \dfrac{t\beta+n}{p}$
  (b) $\chi_{B(1)} \in \mathcal{M}^s_t(|x|^{t\beta},|x|^{t\beta})$ if and only if $0 \ge -\dfrac{t\beta+n}{s}$
  (c) 22 Dec. 2024
(6) p. 152 line 4 from below, the proof of Proposition 135
  (a) $\in$
  (b) $\notin$
  (c) 30 Sept. 2024
(7) p. 152 line 4 from below, the proof of Proposition 135
  (a) $\ge$
  (b) $<$
  (c) 30 Sept. 2024
(8) p. 152 line 3 from below, the proof of Proposition 135
  (a) In
  (b) (Remark that $M_\alpha[|\cdot|^{-n}\chi_{B(1)}]=\infty$.) In
  (c) 30 Sept. 2024
(9) p. 153 lines 2 and 3 from above, the proof of Proposition 135
  (a) $\mathcal{M}^p_q(|x|^{t\beta},|x|^{t\beta})$
  (b) $\mathcal{M}^s{}_t(|x|^{t\beta},|x|^{t\beta})$
  (c) 10 Dec. 2021, 5 Oct. 2024
(10) p. 153 line 4 from above, the proof of Proposition 135
  (a) $\mathcal{M}^p_q(|x|^{t\beta},|x|^{q\beta})$
  (b) $\mathcal{M}^s{}_t(|x|^{t\beta},|x|^{t\beta})$
  (c) 10 Dec. 2021, 5 Oct. 2024
(11) p. 153 (15.6), the proof of Proposition 135
  (a) $I_\alpha f$
  (b) $\chi_Q I_\alpha f$
  (c) 30 Sept. 2024
(12) p. 154 line 3 from above, the proof of Proposition 135
  (a) $\lesssim$
  (b) $\le$

(c) 30 Sept. 2024

(13) p. 154 line 10 from below, the proof of Proposition 135
   (a) I
   (b) I$= w_{t\beta}(Q)^{\frac{1}{s}-\frac{1}{t}}\|(I_\alpha f)\chi_Q\|_{L^t(|x|^{t\beta})}$
   (c) 3 Jan. 2025

(14) p. 155 line 6 from above, the proof of Proposition 135
   (a) $w_{t\beta}(Q)$
   (b) $w_{t\beta}(2^jQ)$
   (c) 30 Sept. 2024

(15) p. 155 line 1 from below, the proof of Proposition 135(2)
   (a) $\mathcal{M}^p_q(|x|^{t\beta},|x|^{t\beta})$
   (b) $\mathcal{M}^s_t(|x|^{t\beta},|x|^{t\beta})$
   (c) 18 Dec. 2024

(16) p. 156 line 6 from below, Exercise 41
   (a) $\mathcal{M}^p_q(|x|^{t\beta},|x|^{t\beta})$
   (b) $\mathcal{M}^s_t(|x|^{t\beta},|x|^{t\beta})$
   (c) 18 Dec. 2024

(17) p. 156, Exercise 41
   (a) $\mathcal{M}^p_q(|x|^{t\beta},|x|^{t\beta})$
   (b) $\mathcal{M}^s_t(|x|^{t\beta},|x|^{t\beta})$
   (c) 30 Sept. 2024

(18) p. 156 line 3 from below
   (a) be a function, and let
   (b) and let
   (c) 30 Sept. 2024

(19) p. 157 line 12 from below
   (a) the Nakamura
   (b) Nakamura
   (c) 30 Sept. 2024

(20) p. 157 line 4 from below, Example 60
   (a) Here $\|\cdot\|^{\mathcal{D}}_{\mathcal{M}^p_q(1,w)}$. It is useful
   (b) It is useful
   (c) 3 Jan. 2025

(21) p. 157, line 2 from below, Exercise 60
   (a) type
   (b) -type
   (c) 30 Sept. 2024

(22) p. 158 Example 61 Replace the part starting with

   We claim that... (till the end of Example 61)

with

   We calculate $w(Q_0) = \displaystyle\int_{Q_0} |x|^\alpha dx \sim (R+|x_0|)^\alpha \cdot |Q_0|$ from Example **??** in the first book. Hence, $|\cdot|^\alpha \in \mathcal{B}_{p,q}$ holds if and only if $\alpha \ge -\frac{q}{p}n$.

3 Jan. 2025

(23) Add before Lemma 137 in page 159

   Let Q be a dyadic cube. Denote by Q^* the unique dyadic cube such that $Q \subset Q^*$ and $\ell(Q) = 2^{-1}\ell(Q^*)$.

(24) p. 159 line 13 from below, the proof of Lemma 137
   (a) $\Phi(Q^*)$.
   (b) $\Phi(Q^*)$ for all $Q \in \mathcal{D}$.
   (c) 30 Sept. 2024

(25) p. 159 line 10 from below, the proof of Lemma 137

   (a) Let τ be the doubling constant of w; $w(3Q) \le \tau w(Q)$
   (b) let τ be the tripling constant of w; $w(3Q) \le \tau w(Q)$
   (c) 30 Sept. 2024
(26) p. 159 line 8 from below, the proof of Lemma 137
   (a) $\le$
   (b) $\le 3^{\frac{n}{p}} \times$
   (c) 30 Sept. 2024
(27) p. 159 line 8 from below, the proof of Lemma 137
   (a) $=$
   (b) $= 3^{\frac{n}{p}} \times$
   (c) 30 Sept. 2024
(28) p. 159 line 7 from below, the proof of Lemma 137
   (a) $\max(2^{\frac{n}{q}}, 2^{-\frac{n}{q}}\tau)$
   (b) $\max(2^{\frac{n}{q}}, 3^{\frac{n}{p}} \times 2^{-\frac{n}{q}}\tau)$
   (c) 30 Sept. 2024
(29) p. 159 line 3 from below, Example 62
   (a) Theorem 288
   (b) Theorem 288 in the first book
   (c) 30 Sept. 2024

1.17. **Pages 160–169.**

(1) p. 161 line 3 from below, the proof of Lemma 139(1)
   (a) $|Q|^{-\frac{q}{p}+1}$
   (b) $|Q|^{\frac{q}{p}-1}$
   (c) 30 Sept. 2024
(2) p. 162 line 16 from below, the proof of Lemma 139
   (a) $\gamma_{p,q,\varepsilon}|Q|^{\beta-1+\frac{q}{p}}$
   (b) $|Q|^{\beta-1+\frac{q}{p}}$
   (c) 3 Jan. 2025
(3) p. 162 line 13 from below, the proof of Lemma 139
   (a) $b_{Q_0}^{(Q_0)}(x) \equiv |Q|^{\beta-1+\frac{q}{p}}|Q_0|^{\beta-1+\frac{q}{p}}|x|^{-n\beta}$
   (b) $b_{Q_0}^{(Q_0)}(x) \equiv \gamma_{p,q,\varepsilon}|Q|^{\beta-1+\frac{q}{p}}|Q_0|^{\beta-1+\frac{q}{p}}|x|^{-n\beta}$
(4) p. 162 line 8 from below (twice), the proof of Lemma 139
   (a) $=$
   (b) $\simeq$
   (c) 3 Jan. 2025
(5) p. 162 line 4 from below, the proof of Lemma 139
   (a) $\geq$
   (b) $\gtrsim$
   (c) 3 Jan. 2025
(6) p. 162 line 4 from below, the proof of Lemma 139
   (a) $b_{Q_0}(Q_0)(x)$
   (b) $b_{Q_0}^{(Q_0)}(x)$
   (c) 30 Sept. 2024
   p. 162 line 4 from below, the proof of Lemma 139
   (a) $=$
   (b) $\simeq$
   (c) 3 Jan. 2025
   p. 163 line 3 from above, the proof of Lemma 139
   (a) $=$
   (b) $\simeq$
   (c) 3 Jan. 2025
(7) p. 163 line 5 from above, the proof of Lemma 139
   (a) $\alpha < n\beta - n(q-1),.$
   (b) $\alpha < n\beta+n(q-1).$
   (c) 30 Sept. 2024
(8) p. 164 line 7 from above, Example 65
   (a) Example 58(3)(b)
   (b) Example 58(3)(a)
   (c) 30 Sept. 2024
(9) p. 165 line 3 from above, the proof of Theorem 140
   (a) $\leq$
   (b) $\lesssim$
   (c) 30 Sept. 2024
(10) p. 165 lines 13-15 from above, the proof of Theorem 140
   (a) $C_{p,q}$
   (b) C_p
   (c) 30 Sept. 2024
(11) p. 166 lines 6 and 8 from above, the proof of Theorem 141
   (a) a^j

(b) a
(c) 30 Sept. 2024

(12) p. 166 line 8 from above, the proof of Theorem 141
(a) Q_k^j
(b) Q_k^1
(c) 30 Sept. 2024

(13) p. 166 lines 14,16 from above, the proof of Theorem 141
(a) $\mathcal{U}$
(b) $\mathcal{U}^1$
(c) 30 Sept. 2024

(14) p. 166 line 8 from below, the proof of Theorem 141
(a) $\le$
(b) $\lesssim$
(c) 30 Sept. 2024

(15) p. 167 lines 1, 4 and 5 from above, the proof of Theorem 141
(a) $\{Q_j^k\}$
(b) $\{Q_k^j\}$
(c) 30 Sept. 2024

(16) p. 167 line 5 from above, the proof of Theorem 141(3)
(a) In view of $2^n \le \lambda'_\sigma a([\sigma]_{A_\infty})$,
(b) Since $a([\sigma]_{A_\infty}) \gg 2^n$,
(c) 30 Sept. 2024

(17) p. 167 line 6 from above, the proof of Theorem 141
(a) Lemma 296
(b) Lemma 296 in the first book
(c) 30 Sept. 2024

(18) p. 167 line 13 from below, the proof of Theorem 141
(a) $m_{Q^0}(\sigma(Q^0)$
(b) $m_{Q^0}(\sigma)$
(c) 3 Jan. 2025

(19) p. 167 line 12 from below, the proof of Theorem 141
(a) Q_0
(b) Q^0
(c) 3 Jan. 2025

(20) p. 167 line 10 from below (twice), the proof of Theorem 141
(a) Q_0
(b) Q^0
(c) 3 Jan. 2025

(21) p. 168 line 12 from below, Lemma 143
(a) of the right-hand
(b) on the right-hand
(c) 30 Sept. 2024

(22) p. 168 line 8 from below, Take out
Fix any Q containing x.
to two lines above before "Write"

(23) p. 169 line 12 from above, Lemma 145
(a) 2^l
(b) $2^l R$
(c) 30 Sept. 2024

1.18. **Pages 170–179.**

(1) p. 170 line 5 from below, the proof of Lemma 146
   (a) $\lesssim$
   (b) $\le$
   (c) 30 Sept. 2024
(2) p. 171 line 5 from above, Theorem 147
   (a) $Q \in \mathcal{Q}$ w
   (b) $Q \in \mathcal{Q}$, w
   (c) 30 Sept. 2024
(3) We should have used (15.13) for (15.14) since they are identical. See pages 170 and 171 for Lemma 146 and Theorem 147. We do not touch them.
(4) p. 171, line 7 from above (Theorem 147),
   (a) $\mathcal{M}_u^{\frac{q}{pu}}$
   (b) $\mathcal{M}_u^{\frac{pu}{q}}$
   (c) 30 Sept. 2024
(5) p. 171, lines 16, 7 and 5 from below (the proof of Theorem 147),
   (a) $\mathcal{M}_u^{\frac{q}{pu}}$
   (b) $\mathcal{M}_u^{\frac{pu}{q}}$
   (c) 30 Sept. 2024
(6) p. 171 lines 13 and 12 from below (the proof of Theorem 147),
   (a) $\|f\|_{\mathcal{M}^p_q(1,w)}^{1-\frac{q}{p}}$
   (b) $\|f\|_{\mathcal{M}^p_q(1,w)}{}^{1-\frac{q}{p}}$
   (c) 30 Sept. 2024
(7) p. 171 line 12 from below (the proof of Theorem 147),
   (a) $\|Mf\|_{\mathcal{M}^p_q(1,w)}^{\frac{q}{p}}$
   (b) $\|Mf\|_{\mathcal{M}^p_q(1,w)}{}^{\frac{q}{p}}$
   (c) 30 Sept. 2024
(8) p. 172, line 3 from above (Theorem 148),
   (a) $\mathcal{M}_u^{\frac{q}{pu}}$
   (b) $\mathcal{M}_u^{\frac{pu}{q}}$
   (c) 30 Sept. 2024
(9) p. 172, the right-hand side of (15.19)
   (a) $\ell(Q_0)$
   (b) $|Q_0|$
   (c) 30 Sept. 2024
(10) p. 172 line 10 from below, Slide "for any $Q_0 \in \mathcal{Q}$" two lines below.
(11) p. 172 line 1 from below, the proof of Theorem 149
   (a) (14.9)
   (b) (15.17)
   (c) 30 Sept. 2024
(12) p. 173 line 1 from above, the proof of Theorem 151
   (a) We may assume that the function
   (b) We may assume that
   (c) 3 Jan. 2025
(13) p. 173 lines 2 and 6 from above, the proof of Theorem 149
   (a) $M_\alpha f$
   (b) $(M_\alpha f)\chi_{Q_0}$
   (c) 30 Sept. 2024

(14) p. 173 line 2 from above, the proof of Theorem 149
  (a) Theorem 179
  (b) Theorem 179 in the first book
  (c) 30 Sept. 2024
(15) p. 173 line 4 from above, the proof of Theorem 149
  (a) $\chi_{E_Q}\ell(Q)^\alpha m_{3Q}(f)$
  (b) $\chi_{E_Q}(x)\ell(Q)^\alpha m_{3Q}(f)$
  (c) 30 Sept. 2024
(16) p. 173 line 12 from above, the proof of Theorem 149
  (a) $\lesssim$
  (b) $\le$
  (c) 30 Sept. 2024
(17) p. 173 line 14 from above, the proof of Theorem 149
  (a) (14.9)
  (b) (15.17)
  (c) 30 Sept. 2024
(18) p. 173 line 11 from below, the proof of Theorem 149
  (a) $u \equiv V$
  (b) $u \equiv w^t$
  (c) 30 Sept. 2024
(19) p. 173 line 11 from below, the proof of Theorem 149
  (a) $[wb^{\frac{1}{p}}]^{-p'}$
  (b) $[wb^{\frac{1}{q}}]^{-q'}$
  (c) 30 Sept. 2024
(20) p. 173 line 10 from below, the proof of Theorem 149
  (a) $= 3^{-nq}$
  (b) $=$
  (c) 30 Sept. 2024
(21) p. 173 line 6 from below, the proof of Theorem 151
  (a) Thus,
  (b) Since we can decduce that $M_{c,\sigma}$ is bounded on $L^q(\sigma)$ for any σ using the Besicovitch covering lemma,
  (c) 3 Jan. 2025
(22) p. 174 line 3 from above, the proof of Theorem 149
  (a) $\frac{u(E_Q)}{\sigma(E_Q)}$
  (b) $\dfrac{u(E_Q)}{\sigma(E_Q)}$
  (c) 30 Sept. 2024
(23) p. 174 line 6 from above, the proof of Theorem 149
  (a) $=$
  (b) $\simeq$
  (c) The constant 3^{-nt} lies behind
  (d) 30 Sept. 2024
(24) p. 174 line 7 from above, the proof of Theorem 149
  (a) $\lesssim$
  (b) $\le$
  (c) 30 Sept. 2024
(25) p. 174 line 9 from above, the proof of Theorem 149
  (a) (14.9)
  (b) (15.17)
  (c) 30 Sept. 2024

(26) p. 174 line 11 from above, the proof of Theorem 149
  (a) $=$
  (b) $\lesssim$
  (c) 30 Sept. 2024
(27) p. 175 line 5 from above, the proof of Theorem 149
  (a) $\ell(Q_0)$
  (b) $|Q_0|$
  (c) 30 Sept. 2024
(28) p. 175 line 7 from above, the proof of Theorem 149
  (a) $\lesssim (\|f\|_{\mathcal{M}^p_q(1,w^q)})^{q-p}$
  (b) $\lesssim |Q_0|^{1-\frac{q}{p}}(\|f\|_{\mathcal{M}^p_q(1,w^q)})^{t-q}$
  (c) 30 Sept. 2024
(29) p. 175 line 9 from above, the proof of Theorem 149
  (a) $\ell(Q_0)$
  (b) $|Q_0|$
  (c) 30 Sept. 2024
(30) p. 175 lines 9 from above, the proof of Theorem 149
  (a) $(\|f\|_{\mathcal{M}^p_q(1,w^q)})^{q-p}$
  (b) $(\|f\|_{\mathcal{M}^p_q(1,w^q)})^{t-q}$
  (c) 30 Sept. 2024
(31) p. 175 line 10 from above, the proof of Theorem 149
  (a) $(\|f\|_{\mathcal{M}^p_q(1,w^q)})^q$
  (b) $(\|f\|_{\mathcal{M}^p_q(1,w^q)})^t$
  (c) 30 Sept. 2024
(32) p. 175 line 7 from below, the proof of Theorem 149
  (a) $|\cdot|^{-n}\chi_{B(1)} \in \mathcal{M}^p_q(1,|x|^{t\beta})$
  (b) $|\cdot|^{-n}\chi_{B(1)} \notin \mathcal{M}^p_q(1,|x|^{q\beta})$
  (c) 30 Sept. 2024
(33) p. 175 line 3 from below, the proof of Theorem 150
  (a) $|Q|^{\frac{1}{s}-\frac{1}{t}}\|I_\alpha f\|_{L^t(Q)} \lesssim \|f\|_{\mathcal{M}^s_t(1,|x|^{t\beta})}$
  (b) $|Q|^{\frac{1}{s}-\frac{1}{t}}\|(M_\alpha f)\chi_Q\|_{L^t(|x|^{t\beta})} \lesssim \|f\|_{\mathcal{M}^p{}_q(1,|x|^{q\beta})}$
  (c) 30 Sept. 2024
(34) p. 176 line 2 from above, the proof of Theorem 149
  (a) We
  (b) For the first term, we
  (c) 3 Jan. 2025
(35) p. 176 line 5 from above, the proof of Theorem 150
  (a) $2^{-\frac{jn}{q'}-j\beta}$
  (b) $2^{-\frac{jn}{q'}+j\beta}$
  (c) 30 Sept. 2024
(36) p. 176 line 5 from above, the proof of Theorem 150
  (a) $|f(z)|$
  (b) $|f(z)|^q$
  (c) 30 Sept. 2024
(37) p. 176 line 11 from above, the proof of Theorem 150
  (a) type
  (b) -type
  (c) 30 Sept. 2024
(38) p. 176 line 13 from above (one line above the statement of Theorem 151),
  (a) $\mathcal{M}^{\frac{q}{pu}}_u$

   (b) $\mathcal{M}_u^{\frac{pu}{q}}$
   (c) 30 Sept. 2024
(39) p. 176 line 14 from above, Theorem 151
   (a) $0 \le \alpha < n$
   (b) $0 < \alpha < n$
   (c) 30 Sept. 2024
(40) p. 176, line 7 from below (Theorem 151),
   (a) $\mathcal{M}_u^{\frac{q}{pu}}$
   (b) $\mathcal{M}_u^{\frac{pu}{q}}$
   (c) 30 Sept. 2024
(41) p. 177 line 8 from above, Delete the definition of $\mathcal{E}_j$ in p. 177, the proof of Theorem 151. 30 Sept. 2024
(42) p. 177 lines 10 and 11 from above, the proof of Theorem 151
   (a) $\langle hw \rangle_Q$
   (b) $m_Q(hw)$
   (c) 30 Sept. 2024
(43) p. 177 lines 6 from below, the proof of Theorem 151
   (a) $\mathrm{d}x$
   (b) $\mathrm{d}x,$
   (c) 30 Sept. 2024
(44) p. 177 line 4 from below, the proof of Theorem 151
   (a) $\langle hw \rangle_Q$
   (b) $m_Q(hw)$
   (c) 30 Sept. 2024
(45) p. 177 lines 3 and 1 from below, the proof of Theorem 151
   (a) a^{k+1}
   (b) a^{j+1}
   (c) 30 Sept. 2024
(46) p. 177 line 9 from above, the proof of Theorem 151
   (a) Lemma 296
   (b) Lemma 296 in the first book
   (c) 30 Sept. 2024
(47) p. 178 line 5 from above, the proof of Theorem 151
   (a) By $\langle hw \rangle_{Q_k^j}$
   (b) By the estimate $m_{Q_k^j}(hw)$
   (c) 30 Sept. 2024
(48) p. 178 line 11 from above (the proof of Theorem 151)
   (a) $\|f\|_{\mathcal{M}_q^p(1,w)}^{1-\frac{q}{u}}$
   (b) $\|f\|_{\mathcal{M}_q^p(1,w)}{}^{1-\frac{q}{u}}$
   (c) 30 Sept. 2024
(49) p. 179 lines 5, 6, 7, 13 and 14 from above (the proof of Theorem 151)
   (a) $\|f\|_{\mathcal{M}_q^p(1,w)}^{1-\frac{q}{u}}$
   (b) $\|f\|_{\mathcal{M}_q^p(1,w)}{}^{1-\frac{q}{u}}$
   (c) 30 Sept. 2024
(50) p. 179 lines 6, and 14 from above (the proof of Theorem 151)
   (a) $\|Mf\|_{\mathcal{M}_q^p(1,w)}^{\frac{q}{u}}$
   (b) $\|Mf\|_{\mathcal{M}_q^p(1,w)}{}^{\frac{q}{u}}$
   (c) 30 Sept. 2024

1.19. **Pages 180–189.**

(1) p. 180 line 2 from above, the proof of Theorem 151
  (a) $m^{(q)}_{Q_0}$
  (b) $m^{(q)}_{Q_0^{(m)}}$
(2) p. 180, lines 9, 10 (twice) and 11 from above (the proof of Theorem 151)
  (a) $\mathcal{M}_u^{\frac{q}{pu}}$
  (b) $\mathcal{M}_u^{\frac{pu}{q}}$
  (c) 30 Sept. 2024
(3) p. 180 line 9 from below, the proof of Theorem 152
  (a) $\mathcal{M}^p_q(1, w^t)$
  (b) $\mathcal{M}^s_t(1, w^t)$
  (c) 30 Sept. 2024
(4) p. 180 line 7 from below, the proof of Theorem 152
  (a) By Hölder's inequality, $\|f_m\|_{\mathcal{M}^p_q(1,w^q)} \lesssim \|\chi_{Q_m}\|_{\mathcal{M}^p_q(1,w^q)}(\log m)^{\frac{\alpha}{n}}$.
  (b) We calculate, using the Hölder inequality,

$$\log m \times \|\chi_{Q_m}\|_{\mathcal{M}^s_t(1,w^t)} \lesssim \|f_m\|_{\mathcal{M}^p_q(1,w^q)} \lesssim \|\chi_{mQ_m}\|_{\mathcal{M}^s_t(1,w^t)}(\log m)^{\frac{\alpha}{n}}.$$

  (c) 30 Sept. 2024
(5) p. 180 line 4 from below, the proof of Theorem 152
  (a) $f \in L^\infty_{\rm c}(\mathbb{R}^n)$
  (b) $f \in L^\infty_{\rm c}(\mathbb{R}^n) \cap \mathbb{M}^+(\mathbb{R}^n)$
  (c) 3 Jan. 2025
(6) p. 181 line 12 from below, the proof of Theorem 152
  (a) due to
  (b) similarly to
  (c) 30 Sept. 2024
(7) p. 181 line 12 from below, the proof of Theorem 152
  (a) $-\dfrac{n}{t} \le \beta < \dfrac{n}{q'}$
  (b) $-\dfrac{n}{s} \le \beta < \dfrac{n}{p'}$
  (c) 30 Sept. 2024
(8) p. 182 line 12 from above, the proof of Theorem 154
  (a) $f_{j,1} \equiv f \cdot \chi_{2Q}$ and $f_{j,2} \equiv f \cdot \chi_{(2Q)^c}$.
  (b) $f_{j,1} \equiv f_j \cdot \chi_{2Q}$ and $f_{j,2} \equiv f_j \cdot \chi_{(2Q)^c}$.
  (c) 30 Sept. 2024
(9) p. 183 line 6 from above, the proof of Theorem 154
  (a) $\le$
  (b) $\lesssim$
  (c) 30 Sept. 2024
(10) p. 183 line 12 from below, the proof of Theorem 154
  (a) $\Phi(Q_m)$
  (b) $2\Phi(Q_m)$
  (c) 30 Sept. 2024
(11) p. 184 line 11 from above, Theorem 156
  (a) $1 \le$
  (b) $1 <$
  (c) 3 Jan. 2025
(12) p. 184 line 11 from below, Theorem 157

   (a) $1 \le q$
   (b) $1 < q$
   (c) 30 Sept. 2024
(13) p. 184 line 5 from below, the proof of Theorem 157
   (a) $f_{j,2} \equiv f\chi_{\mathbb{R}^n\setminus 2Q}$.
   (b) $f_{j,2} \equiv f_j\chi_{\mathbb{R}^n\setminus 2Q}$.
   (c) 30 Sept. 2024
(14) p. 185 line 7 from above, the proof of Theorem 157
   (a) $\sim$
   (b) $\lesssim$
   (c) 30 Sept. 2024
(15) p. 185 line 10 from above, Theorem 158
   (a) Let $1 < q \le u < \infty$
   (b) Let $0 \le \alpha < n$, $1 < q \le u, p < \infty$
   (c) 30 Sept. 2024
(16) p. 185 line 12 from above, Theorem 158
   (a) $\mathcal{M}_u^{\frac{q}{pu}}$
   (b) $\mathcal{M}_u^{\frac{pu}{q}}$
   (c) 30 Sept. 2024
(17) p. 185, Proposition 159
   (a) Let
   (b) Let $0 \le \alpha < n$
   (c) 30 Sept. 2024
(18) p. 185 line 12 from above, Theorem 158
   (a) $\mathcal{M}_u^{\frac{p}{qu}}$
   (b) $\mathcal{M}_u^{\frac{pu}{q}}$
   (c) 30 Sept. 2024
(19) p. 185 line 2 from below, the proof of Proposition 159
   (a) $\|M_{\alpha,1}f_j(x)\|_{\ell^r} \le (\|\{f_j\}_{\ell^r}\|_{\mathcal{M}^p_q(1,w)})^{1-\frac{q}{u}}\,\|Mf_j(x)\|_{\ell^r}^{\frac{q}{u}}$
   (b) $\left\|\{M_{\alpha,1}f_j(x)\}_{j=1}^\infty\right\|_{\ell^r} \le (\left\|\|\{f_j\}_{j=1}^\infty\|_{\ell^r}\right\|_{\mathcal{M}^p_q(1,w)})^{1-\frac{q}{u}}(\left\|\{Mf_j(x)\}_{j=1}^\infty\right\|_{\ell^r})^{\frac{q}{u}}$
   (c) 30 Sept. 2024
(20) p. 185 line 1 from below, the proof of Proposition 159
   (a) $\|M_{\alpha,2}f_j(x)\|_{\ell^r} \le (\|\{f_j\}_{\ell^r}\|_{\mathcal{M}^p_q(1,w)})^{1-\frac{q}{u}}\,\|Mf_j(x)\|_{\ell^r}^{\frac{q}{u}}$
   (b) $\left\|\{M_{\alpha,2}f_j(x)\}_{j=1}^\infty\right\|_{\ell^r} l(\left\|\|\{f_j\}_{j=1}^\infty\|_{\ell^r}\right\|_{\mathcal{M}^p_q(1,w)})^{1-\frac{q}{u}}(\left\|\{Mf_j(x)\}_{j=1}^\infty\right\|_{\ell^r})^{\frac{q}{u}}$
   (c) 30 Sept. 2024
(21) p. 186 line 10 from above and line 3 from below (15.27) the proof of Proposition 159
   (a) $\|M_{\alpha,2}f_j(x)\|_{\ell^r}$
   (b) $\left\|\{M_{\alpha,2}f_j(x)\}_{j=1}^\infty\right\|_{\ell^r}$
   (c) 30 Sept. 2024
(22) p. 186 lines 10 and 11 (15.27), the proof of Proposition 159
   (a) m_Q
   (b) m_{Q_j}
   (c) 30 Sept. 2024
(23) p. 186 line 6 from below, the proof of Proposition 159
   (a) If we combine this fact
   (b) Recall that $w \in A_q$. Hence w is a doubling weight. If we combine this fact
   (c) 30 Sept. 2024
(24) p. 187 line 1 from above, the proof of Proposition 159
   (a) Höder

   (b) Hölder
   (c) 3 Jan. 2025
(25) p. 187 line 3 from above, the proof of Proposition 159
   (a) $\|M_{\alpha,2}f_j(x)\|_{\ell^r}$
   (b) $\left\|\{M_{\alpha,2}f_j(x)\}_{j=1}^{\infty}\right\|_{\ell^r}$
   (c) 30 Sept. 2024
(26) p. 187 line 15 from below
   (a) Proposition 159
   (b) Theorem 154 and Proposition 159
   (c) 9 Oct. 2024
(27) p. 187 line 10 from below, the proof of Proposition 159
   (a) $\|\{f_j\}_{j=1}^{\infty}\|_{\mathcal{M}_q^p(\ell^r;1,w)}$
   (b) $\|\{f_j\}_{j=1}^{\infty}\|_{\mathcal{M}_q^p(\ell^r;1,w)}{}^{1-\frac{q}{u}}$
   (c) 30 Sept. 2024
(28) p. 187 Corollary 160
   (a) Let
   (b) Let $0 < \alpha < n$
(29) p. 187 line 7 should have been deleted, 9 Oct. 2024.
(30) p. 187, line 2 from below (Corollary 160)
   (a) $\mathcal{M}_u^{\frac{q}{pu}}$
   (b) $\mathcal{M}_u^{\frac{pu}{q}}$
   (c) 30 Sept. 2024
(31) p. 188, line 3 from above
   (a) type
   (b) -type
   (c) 30 Sept. 2024
(32) p. 188 line 6 from below, the proof of Theorem 161
   (a) Consequently, for any $f \in L^1(Q^0) \cap \mathbb{M}^+(\mathbb{R}^n) \setminus \{0\}$,
   (b) Consequently,
   (c) 3 Jan. 2025
(33) p. 188 line 2 from below, the proof of Theorem 161
   (a) $\|\chi_{Q^0\setminus Z}\|_{\mathcal{M}_q^p(1,M[w\chi_{Q^0}])}$,
   (b) $\|\chi_{Q^0\setminus Z}\|_{\mathcal{M}_q^p(1,w)}$,
   (c) 3 Jan. 2025

1.20. **Pages 190–199.**

(1) p. 190, line 5 from below
  (a) local weighted Morrey
  (b) weighted local Morrey
  (c) 30 Sept. 2024
(2) p. 190, line 4 from below
  (a) local weighted Morrey
  (b) weighted local Morrey
  (c) 30 Sept. 2024
(3) p. 192 line 8 from above, the proof of Lemma 163
  (a) $\leq$
  (b) $\lesssim$
  (c) 30 Sept. 2024
(4) p. 193 line 14 from above, the proof of Theorem 164
  (a) Let us prove (2) assuming (1).
  (b) Let us prove (1) assuming (2).
  (c) 30 Sept. 2024
(5) p. 193 line 10 from below, the proof of Theorem 164
  (a) Together with the A_q-property at Q
  (b) Together with the *(local)* A_q-property at Q
  (c) 30 Sept. 2024
(6) p. 194, line 8 from below
  (a) local weighted Morrey
  (b) weighted local Morrey
  (c) 30 Sept. 2024
(7) p. 195 line 1 from above, the proof of Proposition 167
  (a) Theorem 288
  (b) Theorem 288 in the first book
  (c) 30 Sept. 2024
(8) p. 195 line 7 from below, the proof of Proposition 167
  (a) as before. Note that
  (b) Note that
  (c) 30 Sept. 2024
(9) Delete the period in p. 196 line 2 from above.
(10) p. 196 lines 4 and 5 from above,
  (a) $|R^*|^{\frac{\beta}{q}-\frac{\beta}{p}}|c(R^*)|^{-\frac{\beta}{q}}|R^*|^{\frac{1}{q'}} \lesssim |R^*|^{1-\frac{\beta}{p}}$
  (b) $\ell(R^*)^{\frac{n+\beta}{q}-\frac{n+\beta}{p}}|c(R^*)|^{-\frac{\beta}{q}}|R^*|^{\frac{1}{q'}} \sim \ell(R^*)^{n-\frac{n+\beta}{p}}$
  (c) 30 Sept. 2024
(11) p. 196, line 11 from above
  (a) local weighted Morrey
  (b) weighted local Morrey
  (c) 30 Sept. 2024
(12) p. 196, line 12 from above
  (a) type
  (b) -type
  (c) 30 Sept. 2024
(13) p. 196 line 3 from below, the proof of Proposition 168
  (a) $R \in \mathcal{Q}^\sharp(R)$
  (b) $R \in \mathcal{Q}^\sharp(Q)$
  (c) 30 Sept. 2024
(14) p. 197 line 11 from above, the proof of Proposition 168

 (a) $\frac{q}{p}n + \beta > 0$
 (b) $\frac{q}{p}n + \beta \geq 0$
 (c) 30 Sept. 2024
(15) p. 197 line 4 from below, Theorem 169
 (a) $u \in \mathcal{G}$ and that
 (b) $u \in \mathcal{G}$, that
 (c) 30 Sept. 2024
(16) p. 197 line 3 from below, Theorem 169
 (a) *164*
 (b) 164
 (c) 30 Sept. 2024
(17) p. 197 line 2 from below, Theorem 169
 (a) for any $k \in \mathbb{N}$
 (b) for any $k \in \mathbb{N}_0$ and $Q_0 \in \mathrm{L}\mathcal{Q}$
 (c) 30 Sept. 2024
(18) p. 198 line 6 from above, the proof of Theorem 169
 (a) Write $\omega_j^k \equiv \omega_{2^{-n-2}}(Tf; Q_k^j)$,
 (b) Write $\omega_{j,k} \equiv \omega_{2^{-n-2}}(Tf; Q_k^j)$.
 (c) Replace the comma with the period
 (d) 6 Oct. 2024
(19) We should have removed the marginal typo in p. 198 line 7 from below
(20) p. 198 line 6 from below, the proof of Theorem 169
 (a) If we use
 (b) Recall that w is "locally" in A_q over $2Q^0$ thanks to the equivalent conditions in Theorem 164. If we use
 (c) 30 Sept. 2024
(21) p. 199 line 2 from above
 (a) Lemma 217
 (b) Lemma 217 in the first book
(22) p. 199 line 3 from above, the proof of Theorem 169
 (a) Q
 (b) Q^0
(23) p. 199 line 12 from above, Theorem 170
 (a) *164*
 (b) 164
(24) p. 199 line 11 from above, Theorem 170
 (a) for $j = 1, 2, \ldots, n$
 (b) all $j = 1, 2, \ldots, n$

1.21. **Pages 200–209.**

(1) p. 200 line 3 from above, the proof of Lemma 171
  (a) $\gtrsim$
  (b) $\lesssim$
  (c) 30 Sept. 2024
(2) p. 200 line 7 from above, the proof of Lemma 171
  (a) χ_{Q_0}
  (b) χ_{2Q_0}
  (c) 30 Sept. 2024
(3) p. 200 line 14 from above, the proof of Corollary 172
  (a) $S = S_1, S_2, S_3 = R$
  (b) $R = S_0, S_1, S_2, S_3 = S$
  (c) 18 Dec. 2024
(4) p. 200 line 14 from below, the proof of Corollary 172
  (a) $R = Q_0, Q_1, Q_2, Q_3$ such that R and $2Q_3$
  (b) Q_0, Q_1, Q_2, Q_3 such that $R = 2Q_0$ and Q_3
  (c) 30 Sept. 2024
(5) p. 200 line 10 from below, the proof of Corollary 172
  (a) $\|\chi_{Q_3}\|_{\mathrm{L}\mathcal{M}^{\mathrm{p}}_{\mathrm{q}}(\mathrm{u},\mathrm{w})} \sim \|\chi_Q\|_{\mathrm{L}\mathcal{M}^{\mathrm{p}}_{\mathrm{q}}(\mathrm{u},\mathrm{w})}$
  (b) $\|\chi_{Q_3}\|_{\mathrm{L}\mathcal{M}^{\mathrm{p}}_{\mathrm{q}}(\mathrm{u},\mathrm{w})} \sim \|\chi_R\|_{\mathrm{L}\mathcal{M}^{\mathrm{p}}_{\mathrm{q}}(\mathrm{u},\mathrm{w})} \gtrsim \|\chi_Q\|_{\mathrm{L}\mathcal{M}^{\mathrm{p}}_{\mathrm{q}}(\mathrm{u},\mathrm{w})}$
  (c) 30 Sept. 2024
(6) p. 201 lines 3–4 from above, the proof of Corollary 172
  (a) u and w satisfy the equivalent conditions in Theorem *164*
  (b) w satisfies the equivalent conditions in Proposition 166
  (c) 30 Sept. 2024
(7) p. 201 line 10 from above, Proposition 173
  (a) $w(Q)^{\frac{1}{p}} m_Q(|f|) \lesssim w(Q')^{\frac{1}{p}} m_Q(|f|) \lesssim \|f\|_{L^p(w)}$
  (b) $w(Q)^{\frac{1}{q}} m_Q(|f|) \lesssim w(Q')^{\frac{1}{q}} m_Q(|f|) \lesssim \|f\|_{L^q(w)}$
(8) p. 201 line 17 from above, the proof of Proposition 173
  (a) $1 < p < s < \infty$
  (b) $1 < p \le s < \infty$ and $0 < \alpha < n$
  (c) 30 Sept. 2024
(9) p. 201 line 6 from below, Theorem 174
  (a) for any
  (b) For any
  (c) 30 Sept. 2024
(10) p. 201 line 2 from below, Theorem 174
  (a) for
  (b) For
  (c) Also add a comma.
  (d) 30 Sept. 2024
(11) p. 202 line 10 from below, the proof of Proposition 175
  (a) Propositon 168
  (b) Theorem 169
  (c) 30 Sept. 2024
(12) p. 202 line 12 from above.
  (a) type
  (b) -type
  (c) 30 Sept. 2024
(13) p. 202 lines 3 and 1 from below, the proof of Proposition 175
  (a) $\ell(R)^\beta$

   (b) $\ell(R)^\alpha$
   (c) 30 Sept. 2024
(14) p. 203 line 1 from above, the proof of Proposition 175
   (a) if $R \in \mathcal{Q}(2Q)$
   (b) if $R \in \mathcal{Q}(2Q)$ and $Q \in \mathrm{L}\mathcal{Q}$
   (c) 3 Jan. 2025
(15) p. 203 line 2 from above (twice), the proof of Proposition 175
   (a) $\ell(R)^\beta$
   (b) $\ell(R)^\alpha$
   (c) 30 Sept. 2024
(16) p. 203 line 5 from above, the proof of Proposition 175
   (a) $\ell(R)^{n-\beta}$
   (b) $\ell(R)^{n-\alpha}$
   (c) 30 Sept. 2024
(17) p. 203 line 5 from above, the proof of Proposition 175
   (a) $\ell(R^*)^{n-\beta}$
   (b) $\ell(R^*)^{n-\alpha}$
   (c) 30 Sept. 2024
(18) p. 203 line 10 from above, the proof of Proposition 175
   (a) $\ell(R)^\beta$
   (b) $\ell(R)^\alpha$
   (c) 30 Sept. 2024
(19) p. 202 lines 3 and 1 from below, p. 203 lines 2(twice), 5 and 10 from above, the proof of Proposition 175
   (a) $\ell(R)^\beta$
   (b) $\ell(R)^\alpha$
(20) p. 203 line 15 from above, the proof of Proposition 175
   (a) $|\cdot|^{-\frac{n}{p}+\beta}$
   (b) $|\cdot|^{-\frac{n}{p}-\beta}$
   (c) 30 Sept. 2024
(21) p. 203, lines 3 and 1 from below, Proposition 176
   (a) $\mathrm{L}\mathcal{M}^p_q(|x|^{t\beta}, |x|^{t\beta})$
   (b) $\mathrm{L}\mathcal{M}^s_t(|x|^{t\beta}, |x|^{t\beta})$
   (c) 30 Sept. 2024
(22) p. 204 line 3 (the second one) from above, the proof of Proposition 176
   (a) $\mathrm{L}\mathcal{M}^p_q(|x|^{t\beta}, |x|^{t\beta})$
   (b) $\mathrm{L}\mathcal{M}^s_t(|x|^{t\beta}, |x|^{t\beta})$
   (c) 30 Sept. 2024
(23) p. 204 line 4 from above, the proof of Proposition 176
   (a) Then
   (b) then
   (c) 30 Sept. 2024
(24) p. 204 line 4 (the first one) from above, the proof of Proposition 176
   (a) $\mathrm{L}\mathcal{M}^p_q(|x|^{t\beta}, |x|^{t\beta})$
   (b) $\mathrm{L}\mathcal{M}^s_t(|x|^{t\beta}, |x|^{t\beta})$
   (c) 30 Sept. 2024
(25) p. 204 lines 11, 10, 8, 7, 4 from below
   (a) $\ell(R)^\beta$
   (b) $\ell(R)^\alpha$
   (c) 18 Dec. 2024
(26) p. 204 lines 10, 4 and 2 from below, the proof of Proposition 176
   (a) $\mathrm{L}\mathcal{M}^p_q(|x|^{t\beta}, |x|^{t\beta})$

   (b) $\mathrm{L}\mathcal{M}^s_t(|x|^{t\beta}, |x|^{t\beta})$
   (c) 30 Sept. 2024
(27) p. 204 lines 10, 4 and 2 from below, the proof of Proposition 176
   (a) $\|\chi_Q\|_{\mathrm{L}\mathcal{M}^p_q(|x|^{t\beta},|x|^{t\beta})}$
   (b) $\|\chi_Q\|_{\mathrm{L}\mathcal{M}^s_t(|x|^{t\beta},|x|^{t\beta})}$
   (c) 30 Sept. 2024
(28) p. 205 line 4 from above, the proof of Proposition 175
   (a) $|x|^{t\beta}(2^{-j}R)^{\frac{1}{p}-\frac{1}{q}}$
   (b) $\ell(2^{-j}R)^{(t\beta+n)\left(\frac{1}{p}-\frac{1}{q}\right)}$
   (c) 3 Jan. 2025
(29) p. 205 lines 5 and 6 from above, the proof of Proposition 176
   (a) $\|f\|_{\mathcal{M}^p_q(|x|^{t\beta},|x|^{q\beta})}$
   (b) $\|f\|_{\mathrm{L}\mathcal{M}^p_q(|x|^{t\beta},|x|^{q\beta})}$
   (c) 30 Sept. 2024
(30) p. 205 line 6 from above, the proof of Proposition 176
   (a) $\beta - n + \dfrac{n}{q'} - \lambda + \varepsilon$
   (b) $\alpha - n + \dfrac{n}{q'} - \lambda + \varepsilon$
   (c) 30 Sept. 2024
(31) p. 205 line 8 from above the proof of Proposition 176
   (a) $\ell(R)^\beta$
   (b) $\ell(R)^\alpha$
   (c) 18 Dec. 2024
(32) p. 205 line 8 from above, the proof of Proposition 176
   (a) $\|\chi_Q\|_{\mathrm{L}\mathcal{M}^p_q(|x|^{t\beta},|x|^{t\beta})}$
   (b) $\|\chi_Q\|_{\mathrm{L}\mathcal{M}^s_t(|x|^{t\beta},|x|^{t\beta})}$
   (c) 30 Sept. 2024
(33) p. 205 line 11 from above, the proof of Proposition 176
   (a) $\mathrm{L}\mathcal{M}^p_q(|x|^{t\beta}, |x|^{t\beta})$
   (b) $\mathrm{L}\mathcal{M}^s_t(|x|^{t\beta}, |x|^{t\beta})$
   (c) 30 Sept. 2024
(34) p. 205 line 11 from above, the proof of Proposition 176
   (a) M_β
   (b) M_α
   (c) 30 Sept. 2024
(35) p. 205 line 13 from above, the proof of Proposition 176
   (a) I_β
   (b) I_α
   (c) 30 Sept. 2024
(36) p. 205 line 13 from above, the proof of Proposition 176
   (a) $\mathrm{L}\mathcal{M}^p_q(|x|^{t\beta}, |x|^{t\beta})$
   (b) $\mathrm{L}\mathcal{M}^s_t(|x|^{t\beta}, |x|^{t\beta})$
   (c) 30 Sept. 2024
(37) p. 205 line 14 from above, the proof of Proposition 176
   (a) $M_\beta \lesssim I_\beta$
   (b) $M_\alpha \lesssim I_\alpha$
   (c) 30 Sept. 2024
(38) p. 205 line 15 from above, the proof of Proposition 176
   (a) < 0
   (b) $< n$
   (c) 30 Sept. 2024

(39) p. 205 line 1 from below, Exercise 48
  (a) $2^n(2^n-1)$
  (b) $2^n(2^n-1)$ for all $a>0$
  (c) 3 Jan. 2025
(40) p. 206, line 10 from above, twice
  (a) type
  (b) -type
  (c) 30 Sept. 2024
(41) p. 206 line 15 from below, §15.5.1
  (a) commtators
  (b) commutators
  (c) 30 Sept. 2024
(42) p. 206 line 13 from below, §15.5.1
  (a) Example 67
  (b) Example 67 below
(43) p. 206 line 8 from below, §15.1.2
  (a) Zhong, and Jia
  (b) Zhong and Jia
  (c) 30 Sept. 2024
(44) p. 206 line 6 from below, §15.1.2
  (a) Iida et. al.
  (b) Iida et al.
  (c) 30 Sept. 2024
(45) p. 207, line 14 from above, Section 15.1.3
  (a) weighted generalized Morrey space
  (b) generalized weighted Morrey space
  (c) 13 Dec. 2024
(46) p. 208 line 10 from above, §15.2
  (a) $\mathbf{G}(\mathbf{B}))$
  (b) $\mathbf{G}(\mathbf{B})$
  (c) 30 Sept. 2024
(47) p. 208 line 22 from above, take off §15.2.
(48) p. 208 line 9 from below, §15.2.2
  (a) $\mathcal{B}_\alpha$
  (b) $\mathfrak{B}_\alpha$
(49) p. 208, line 3 from below
  (a) type
  (b) -type
  (c) 30 Sept. 2024
(50) Take out the sentences stating with

    A fundamental result on the boundedness of fractional integral operators

  in page 209 (§15.2.4) to p. 208 line 21 from above. 30 Sept. 2024
(51) p. 209 line 1 from below,
  (a) $\mathcal{M}_u^{\frac{p}{qu}}$
  (b) $\mathcal{M}^{\frac{pu}{q}}{}_u$

1.22. **Pages 210–219.**

(1) p. 210, line 1 from above, §15.2.5
  (a) necesssary
  (b) necessary
  (c) 26 Sept. 2023
(2) p. 210, line 19 from above, §15.2.7
  (a) Shi
  (b) . Shi
  (c) 26 Sept. 2023
(3) p. 210, line 2 from below, §15.3.1
  (a) Nakmaura
  (b) Nakamura
  (c) 26 Sept. 2023
(4) p. 211, line 1 from above
  (a) Nakmaura
  (b) Nakamura
  (c) 26 Sept. 2023
(5) p. 211, line 6 from above
  (a) local weighted Morrey
  (b) weighted local Morrey
  (c) 30 Sept. 2024
(6) Take out

    See [356, Theorem 1.4] for Theorem 149.

  in p. 211, line 8 from above to the end of §15.2.5
(7) p. 211, line 8 from above, §15.3.2
  (a) local weighted Morrey
  (b) weighted local Morrey
  (c) 30 Sept. 2024
(8) p. 211, line 14 from below, one line above Definition 35
  (a) weighted generalized Morrey space
  (b) generalized weighted Morrey space
  (c) 13 Dec. 2024
(9) p. 211, Definition 35 (twice)
  (a) $\mathcal{M}^{\varphi}_q(w,w)$
  (b) $\mathrm{W}\mathcal{M}^{\varphi}_q(w,w)$
  (c) Also add: The weak weighted generalized Morrey space $\mathrm{W}\mathcal{M}^{\varphi}_q(w,w)$ is the set of all $f \in L^0(\mathbb{R}^n)$ for which $\|f\|_{\mathrm{W}\mathcal{M}^{\varphi}_q(w,w)}$ is finite.
  (d) 21 Aug. 2024
(10) Take out the passage from the first sentence of §15.3.3 till Example 67 and move them to General remarks in §15.1.
(11) Take off the passage from the paragraph starting with "As for" in p. 211 line 3 from below and ending with "their closed subspaces [20]" in p. 212 line 9 from above and move them to General remarks in §15.2. 7 Oct. 2024
(12) p. 214, line 10 from above
  (a) Ragusa
  (b) Hatano
  (c) 17 Oct. 2024
  (d) Take off [379]
(13) p. 214, line 15 from above
  (a) notaton
  (b) notation

   (c) 26 Sept. 2023
(14) p. 214, line 18 from above
   (a) $\|g\|_{\Phi_{\frac{n}{q}-\frac{n}{p},\theta}}$
   (b) $\|g\|_{\Phi_{\frac{n}{q}-\frac{n}{p},\theta}(0,\infty)}$
   (c) 20 Dec. 2024
(15) p. 215, line 15 from above
   (a) type
   (b) -type
   (c) 30 Sept. 2024
(16) p. 215, line 1 from below
   (a) Theorem 19
   (b) Theorem 19 in the first book
(17) p. 216, lines 9, 12, 13, 16 (twice) and 17 from above, the proof of Theorem 180
   (a) $\|\|\|Mf\|_{L^q(B(\cdot))}\|_{\Phi_{\frac{n}{q}-\frac{n}{p},\theta}}$
   (b) $\|\|\|Mf\|_{L^q(B(\cdot))}\|_{\Phi_{\frac{n}{q}-\frac{n}{p},\theta}(0,\infty)}$
   (c) 20 Dec. 2024
(18) p. 217, line 6 from above, the first line of Section 16.1.3
   (a) singular operator
   (b) singular integral operator
   (c) 7 Oct. 2024
(19) p. 217, line 24 from below, Theorem 183
   (a) $\lesssim_q$
   (b) $\lesssim_{n,p,q}$
   (c) 7 Oct. 2024
(20) p. 217, line 18 from below, Theorem 184
   (a) $\lesssim_q$
   (b) $\lesssim_{n,p,q}$
   (c) 7 Oct. 2024
(21) p. 217, line 14 from below, four lines above Theorem 185
   (a) singular operator
   (b) singular integral operator
   (c) 7 Oct. 2024
(22) p. 217, line 10 from below, Theorem 185
   (a) Let $1 < q < p < \infty$, $1 < \theta < \infty$, and let T be a singular integral operator.
   (b) Let $1 < q < p < \infty$ and $1 < \theta < \infty$
   (c) 7 Oct. 2024
(23) p. 217, line 5 from below, Theorem 186
   (a) Let $1 < q < p < \infty$, $1 < \theta < \infty$, and let T be a singular integral operator.
   (b) Let $1 < q < p < \infty$ and $1 < \theta < \infty$
   (c) 7 Oct. 2024
(24) p. 218, line 3 from above
   (a) type
   (b) -type
   (c) 30 Sept. 2024
(25) p. 218, line 9 from above, the proof of Theorem 187
   (a) $L^q(2B)$
   (b) $L^q(B(2r))$
   (c) 18 Dec. 2024
(26) p. 218, line 9 from above, the proof of Theorem 187
   (a) $r^{\frac{n}{p}-\frac{n}{q}+\frac{n}{t}}\int_r^\infty \|f\|_{L^1(B(s))}\frac{\mathrm{d}s}{s^{n+1-\alpha}}$

   (b) $r^{\frac{n}{p}-\frac{n}{q}+\frac{n}{t}}\int_r^\infty \|f\|_{L^1(B(\rho))}\frac{\mathrm{d}\rho}{\rho^{n+1-\alpha}}$
   (c) 18 Dec. 2024
(27) p. 218, line 10 from below,
   (a) Adams type
   (b) Adams-type
   (c) 17 Dec. 2024

1.23. Pages 220–229.

(1) p. 220 line 5 from above
  (a) generalized local Morrey-type spaces
  (b) local general Morrey-type spaces
  (c) 16 Dec. 2024
(2) p. 220, line 8 from below, Definition 39
  (a) After (16.1) add: In the above, a natural modification is made for $\theta = \infty$.
  (b) 19 Dec. 2024
(3) p. 220 lines 1 and 2 from below, two lines above Example 71,
  (a) Morrey spaces
  (b) Morrey-type spaces
  (c) 18 Dec. 2024
(4) p. 221 line 2 from below, Theorem 191
  (a) $w \in \mathbb{M}^+(0,\infty)\backslash\{0\}$
  (b) $\varphi \in \mathbb{M}^+(0,\infty)\backslash\{0\}$
  (c) 18 Dec. 2024
(5) p. 222 lines 7 and 11 from above, the proof of Theorem 191
  (a) $L^p(B(t_0))$
  (b) $L^q(B(t_0))$
  (c) 18 Dec. 2024
(6) p. 222 line 6 from below, the proof of Theorem 191
  (a) $L^p(B(x,r))$
  (b) $L^q(B(x,r))$
  (c) 18 Dec. 2024
(7) p. 222 line 5 from below, the proof of Theorem 191
  (a) $\varphi \notin \Omega_{q\theta}$
  (b) $\varphi \in \Omega_{q\theta}$
  (c) 18 Dec. 2024
(8) p. 224 line 11 from above, Corollary 195
  (a) Then if
  (b) If
  (c) 19 Dec. 2024
(9) p. 224 line 3 from below, the proof of Lemma 196
  (a) $r^{-1-\frac{n\theta_1}{r}}$
  (b) $r^{-1-\frac{n\theta_1}{q}}$
  (c) 18 Dec. 2024
(10) p. 225 line 5 from above,
  (a) Thus, $\mathcal{M}^{\varphi_1}_{q\theta_1}(\mathbb{R}^n) \hookrightarrow \mathcal{M}^{\varphi_2(2\cdot)}_{q\theta_2}(\mathbb{R}^n)$.
  (b) Thus, $\mathcal{M}^{\varphi_1}_{q\theta_1}(\mathbb{R}^n) \hookrightarrow \mathcal{M}^{\varphi_2(2\cdot)}_{q\theta_2}(\mathbb{R}^n)$ is a consequence of Lemma 193.
  (c) 19 Dec. 2024
(11) p. 225 line 4 from below,
  (a) $\sqrt[n]{\tau^{-1}}$
  (b) $\tau^{-\frac{1}{n}}$
  (c) 19 Dec. 2024
(12) p. 225 line 3 from below,
  (a) $\lesssim$
  (b) $=$
  (c) 18 Dec. 2024
(13) p. 226 line 2 from above, Remark 7
  (a) $\sim$

(b) $\simeq$
(c) 19 Dec. 2024

(14) p. 226 line 6 from above, Theorem 198
(a) $\varphi_k(R^{-\frac{1}{n}})^{\theta_k} R^{\frac{\theta_k}{q_k}-1}$
(b) $\varphi_k(R^{-\frac{1}{n}})^{\theta_k} R^{\frac{\theta_k}{q}-1}$
(c) 19 Dec. 2024

(15) p. 226 line 8 from above, Theorem 198
(a) $L^{q_1}(w_1)$ to $L^{q_2}(w_2)$
(b) $L^{\frac{\theta_1}{q}}(w_1)$ to $L^{\frac{\theta_2}{q}}(w_2)$
(c) 18 Dec. 2024

(16) p. 226 line 13 from above,
(a) globalize Theroem 198 as we did in Section 16.2.2.
(b) globalize Theroem 198.
(c) 19 Dec. 2024

(17) p. 226 line 7 from below, Theorem 200
(a) $L^{\frac{\theta_1}{q}}(\varphi_1(\cdot^{-1})\cdot^{\frac{\theta_1}{q}-1})$ to $L^{\frac{\theta_2}{q}}(\varphi_2(\cdot^{-1})\cdot^{\frac{\theta_2}{q}-1})$
(b) $L^{\frac{\theta_1}{q}}(\varphi_1(\cdot^{-1})^{\theta_1}\cdot^{\frac{\theta_1}{q}-1})$ to $L^{\frac{\theta_2}{q}}(\varphi_2(\cdot^{-1})^{\theta_2}\cdot^{\frac{\theta_2}{q}-1})$
(c) 16 Dec. 2024

(18) p. 227 line 4 from above, the proof of Theorem 200
(a) $= \|f\|_{\mathcal{M}_{q\theta_2}^{\varphi_2(2\cdot)}}$
(b) $\simeq \|f\|_{\mathcal{M}_{q\theta_2}^{\varphi_2(\cdot/2)}}$
(c) 16 Dec. 2024

(19) p. 227 line 10, 12, 15 from above, the proof of Theorem 200
(a) II
(b) II^{θ_2}
(c) 16 Dec. 2024

(20) p. 227, line 2 from below,
(a) $\lesssim$
(b) $=$
(c) 16 Dec. 2024

(21) p. 228 line 7 from above, Corollary 208
(a) $L^{\frac{\theta_1}{q}}(\varphi_1(\cdot^{-1})\cdot^{\frac{\theta_1}{q}-1})$ to $L^{\frac{\theta_2}{q}}(\varphi_2(\cdot^{-1})\cdot^{\frac{\theta_2}{q}-1})$
(b) $L^{\frac{\theta_1}{q}}(\varphi_1(\cdot^{-1})^{\theta_1}\cdot^{\frac{\theta_1}{q}-1})$ to $L^{\frac{\theta_2}{q}}(\varphi_2(\cdot^{-1})^{\theta_2}\cdot^{\frac{\theta_2}{q}-1})$
(c) 16 Dec. 2024

(22) p. 228 line 19 from above
(a) general local Morrey-type spaces
(b) local general Morrey-type spaces
(c) 16 Dec. 2024

(23) p. 228 line 22 from above, Theorem 202
(a) $1 < q_1 \le \theta \le q_2 < \infty$
(b) $1 < q_2 \le \theta \le q_1 < \infty$
(c) 16 Dec. 2024

(24) p. 228 line 25 from above, Theorem 202
(a) $\|M(\chi_B {W_1}^{-\frac{1}{q_1'}})\|_{L^{q_2}({W_2}^{q_2})} \lesssim \left(\int_B W_1(x)^{-\frac{q_1}{q_1'}}\mathrm{d}x\right)^{\frac{1}{q_1}}$
(b) $\|M(\chi_B {W_1}^{-q_1'})\|_{L^{q_2}({W_2}^{q_2})} \lesssim \left(\int_B W_1(x)^{-q_1'}\mathrm{d}x\right)^{\frac{1}{q_1}}$
(c) 16 Dec. 2024

(25) p. 229 line 3 from above, Theorem 202

(a) $\|M(\chi_B {W_1}^{-\frac{1}{q'}})\|_{L^q({W_2}^q)} \lesssim \left(\int_B W_1(x)^{-\frac{q}{q'}} \mathrm{d}x\right)^{\frac{1}{q}}$
(b) $\|M(\chi_B {W_1}^{-q'})\|_{L^q({W_2}^q)} \lesssim \left(\int_B W_1(x)^{-q'} \mathrm{d}x\right)^{\frac{1}{q}}$
(c) 16 Dec. 2024

(26) p. 228 line 9 from below, (16.11)
(a) $(x \in \mathbb{R}^n)$
(b) $(x \in \mathbb{R}^n)$.
(c) 19 Dec. 2024

1.24. **Pages 230–239.**

(1) p. 230 line 2 from above, Theorem 203(3)
   (a) $L^\infty_{\rm c}(\mathbb{R}^n)$
   (b) $L^\infty_{\rm c}(\mathbb{R}^n)$.
   (c) 19 Dec. 2024
(2) p. 230 line 8 from below, Lemma 205
   (a) $0 \le \delta$
   (b) $0 < \delta$
   (c) 19 Dec. 2024
   (d) If $\delta = 0$, we can not use Corollary 220 in the first book.
(3) p. 230 line 8 from below,
   (a) $\varphi \in \Omega_{q\theta}$
   (b) $\varphi \in {\rm L}\Omega_{\rm q\theta}$
   (c) 16 Dec. 2024
(4) p. 230 line 5 from below,
   (a) $B(t^{n-\delta q})$
   (b) $B(t^{\frac{1}{\delta q-n}})$
   (c) 16 Dec. 2024
(5) p. 231 line 3 from above, the proof of Lemma 205
   (a) $r^{\frac{\delta}{\delta q-n}}$
   (b) $r^{-\frac{\delta}{\delta q-n}}$
   (c) 16 Dec. 2024
(6) p. 231 line 5 from above
   (a) variant to
   (b) variant of
   (c) 16 Dec. 2024
(7) p. 231, line 8 from above, the proof of Lemma 205
   (a) $\varphi(r^{-\frac{q}{n}})r^{-\frac{q}{n}}$
   (b) $\varphi(r^{-\frac{q}{n}})r$
   (c) 16 Dec. 2024
(8) p. 231 line 11 from below, the proof of Lemma 206
   (a) $\sim$
   (b) $\simeq$
   (c) 16 Dec. 2024
(9) p. 231 line 10 from below, the proof of Lemma 206
   (a) $\sim$
   (b) $=$
   (c) 16 Dec. 2024
(10) p. 231, line 10 from below, the proof of Lemma 206
   (a) $\varphi(r)r$
   (b) $\varphi(r)r^{-\frac{n}{q}}$
(11) p. 231, line 9 from below,
   (a) $\sim$
   (b) $\simeq$
   (c) 16 Dec. 2024
(12) p. 231, line 9 from below,
   (a) $\varphi(r^{-\frac{q}{n}})r^{-\frac{q}{n}}$
   (b) $\varphi(r^{-\frac{q}{n}})r$
   (c) 16 Dec. 2024
(13) p. 231, line 6 from below, Theorem 207

   (a) $\varphi_1 \in \Omega_{q\theta_1}$ and $\varphi_2 \in \Omega_{q\theta_2}$
   (b) $\varphi_1 \in {\rm L}\Omega_{{\rm q}\theta_1}$ and $\varphi_2 \in {\rm L}\Omega_{{\rm q}\theta_2}$
   (c) 16 Dec. 2024
(14) p. 231, line 5 from below, Theorem 207
   (a) $\mathbb{M}^{\uparrow}(0,\infty)$
   (b) $\mathbb{M}^{\downarrow}(0,\infty)$
   (c) 16 Dec. 2024
(15) p. 231, line 4 from below, Theorem 207
   (a) $\varphi_2(r^{-\frac{q}{n}})r^{-\frac{q}{n}}$
   (b) $\varphi_2(r^{-\frac{q}{n}})r$
   (c) 16 Dec. 2024
(16) p. 231, line 3 from below, Theorem 207
   (a) $\varphi_1(r^{-\frac{q}{n}})r^{-\frac{q}{n}}$
   (b) $\varphi_1(r^{-\frac{q}{n}})r$
   (c) 16 Dec. 2024
(17) p. 232, line 1 from above, Theorem 207
   (a) Lemma 205
   (b) Lemma 206
   (c) 16 Dec. 2024
(18) p. 232, line 1 from above, the proof of Theorem 207
   (a) $\hat{g}_\delta$
   (b) $\|f\|_{L^q(B(r^{-\frac{q}{n}}))}$
   (c) 16 Dec. 2024
(19) p. 232, line 2 from above, the proof of Theorem 207
   (a) $\|Tf\|_{{\rm L}\mathcal{M}^{\varphi}_{{\rm q}\theta}}$
   (b) $\|Tf\|_{{\rm L}\mathcal{M}^{\varphi_2}_{{\rm q}\theta_2}}$
   (c) 16 Dec. 2024
(20) p. 232, line 2 from above, the proof of Theorem 207
   (a) $\varphi_1(r^{-\frac{q}{n}})r^{-\frac{q}{n}}$
   (b) $\varphi_1(r^{-\frac{q}{n}})r$
   (c) 17 Dec. 2024
(21) p. 232, line 3 from above, the proof of Theorem 207
   (a) $\sim$
   (b) $\simeq$
   (c) 17 Dec. 2024
(22) p. 232, line 7 from above, Corollary 208
   (a) $\mathbb{M}^{\uparrow}(0,\infty)$
   (b) $\mathbb{M}^{\downarrow}(0,\infty)$
   (c) 17 Dec. 2024
(23) p. 233, line 8 from above,
   (a) $M_\alpha, 0 \le \alpha < n$
   (b) M_α for $0 \le \alpha < n$.
   (c) 17 Dec. 2024
(24) p. 233, line 11 from above, Lemma 210
   (a) Then if
   (b) If
   (c) 17 Dec. 2024
(25) p. 233, line 20 from above, Lemma 211
   (a) Then if
   (b) If
   (c) 19 Dec. 2024

(26) p. 233, line 20 from above, Lemma 211
  (a) $L^{\theta_1}(r^{-1-\frac{\theta_1}{q_1}})$
  (b) $L^{\theta_1}(r^{-1-\frac{n\theta_1}{q_1}})$
  (c) 17 Dec. 2024
(27) p. 233, line 20 from above, Lemma 211
  (a) $n\left(\frac{1}{q_1}-\frac{1}{q_2}\right)_+$
  (b) $n\left(\frac{1}{q_1}-\frac{1}{q_2}\right)_+ = \max\left(0, n\left(\frac{1}{q_1}-\frac{1}{q_2}\right)\right)$
  (c) 16 Dec. 2024
(28) p. 233, line 1 from below, the proof of Lemma 211
  (a) $\left(\int_0^\infty \cdots \frac{\mathrm{d}r}{r}\right)^{\frac{1}{\theta_2}}$
  (b) $\left\{\int_0^\infty \cdots \frac{\mathrm{d}r}{r}\right\}^{\frac{1}{\theta_2}}$
  (c) 17 Dec. 2024
(29) p. 234, line 4 from above,
  (a) $\varphi_2(r)t^\alpha$
  (b) $\varphi_2(r)$
  (c) 17 Dec. 2024
(30) p. 234, line 8 from below, Lemma 212
  (a) $g(v)dv$
  (b) $g(v)\mathrm{d}v$
  (c) 17 Dec. 2024
(31) p. 234, line 6 from below,
  (a) $\sim \left\{\int_0^\infty \left(\varphi_1(t^{\frac{1}{\alpha q_1 - n}})^{q_1}\int_0^t g(v)dv\right)^{\frac{\theta}{q_1}}\frac{\mathrm{d}t}{t}\right\}^{\frac{1}{\theta}}$
  (b) $\simeq \left\{\int_0^\infty \left(\varphi_1(t^{\frac{1}{\alpha q_1 - n}})t^{-\frac{n}{q_1(\alpha q_1 - n)}}g(t)\right)^{\frac{\theta}{q_1}}\frac{\mathrm{d}t}{t}\right\}^{\frac{1}{\theta}}$
  (c) 17 Dec. 2024
(32) p. 234, line 5 from below, the proof of Lemma 212
  (a) $g(v)dv$
  (b) $g(v)\mathrm{d}v$
  (c) 17 Dec. 2024
(33) p. 235, lines 3 from above, the proof of Lemma 212
  (a) $\sim$
  (b) $\simeq$
  (c) 19 Dec. 2024
(34) p. 235, lines 5 from above, the proof of Lemma 212
  (a) $\sim$
  (b) $\simeq$
  (c) 19 Dec. 2024
(35) p. 235, line 5 from above, the proof of Lemma 212
  (a) $\left\{\int_0^\infty (\cdots)^\theta \frac{dt}{t}\right\}^{\frac{1}{\theta}}$
  (b) $\left\{\int_0^\infty \left(\cdots\right)^\theta \frac{dt}{t}\right\}^{\frac{1}{\theta}}$
  (c) 17 Dec. 2024

(36) p. 235, line 6 from above,
   (a) $\sim$
   (b) $=$
   (c) 17 Dec. 2024
(37) p. 235, line 12 from below,
   (a) $\left\{\int_0^\infty \left(\varphi_2(t^{\frac{1}{\alpha q_2 - n}})^{q_2}\mathcal{H}h(t)\right)^{\frac{\theta}{q_1}} \frac{\mathrm{d}t}{t}\right\}^{\frac{1}{\theta}} \lesssim \left\{\int_0^\infty \left(\varphi_1(t^{\frac{1}{\alpha q_1 - n}})^{q_1}h(t)\right)^{\frac{\theta}{q_1}} \frac{\mathrm{d}t}{t}\right\}^{\frac{1}{\theta}}$
   (b) $\int_0^\infty \left(\varphi_2(t^{\frac{1}{\alpha q_1 - n}})^{q_1} t\mathcal{H}h(t)\right)^{\frac{\theta}{q_1}} \frac{\mathrm{d}t}{t} \lesssim \int_0^\infty \left(\varphi_1(t^{\frac{1}{\alpha q_1 - n}})^{q_1} t^{-\frac{n}{\alpha q_1 - n}} h(t)\right)^{\frac{\theta}{q_1}} \frac{\mathrm{d}t}{t}$
   (c) 17 Dec. 2024
(38) p. 235, line 11 from below,
   (a) $\mathcal{M}^{\varphi_1}_{q_1\theta}(\mathbb{R}^n)$ to $\mathcal{M}^{\varphi_2}_{q_2\theta}(\mathbb{R}^n)$
   (b) $\mathrm{L}\mathcal{M}^{\varphi_1}_{\mathrm{q}_1\theta}(\mathbb{R}^{\mathrm{n}})$ to $\mathrm{L}\mathcal{M}^{\varphi_2}_{\mathrm{q}_2\theta}(\mathbb{R}^{\mathrm{n}})$
(39) p. 236, line 9 from above, §16.1.1
   (a) type
   (b) -type
   (c) 30 Sept. 2024
(40) p. 236, line 14 from below, §16.1.4
   (a) considerd
   (b) considered
   (c) 26 Sept. 2023
(41) p. 236, line 10 from below §16.1.4
   (a) type
   (b) -type
   (c) 30 Sept. 2024
(42) p. 236, line 8 from below, §16.1.4
   (a) type
   (b) -type
   (c) 30 Sept. 2024
(43) p. 236, line 4 from below, §16.2
   (a) type
   (b) -type
   (c) 30 Sept. 2024
(44) p. 236, line 2 from below, §16.2
   (a) Definition 1.1
   (b) Definition 1
   (c) 17 Dec. 2024
(45) p. 236 line 2 from below, §16.2
   (a) generalized Morrey-type spaces
   (b) general Morrey-type spaces
   (c) 19 Dec. 2024
(46) p. 237, line 2 from above, §16.2
   (a) Morrey
   (b) Morrey-type
   (c) 30 Sept. 2024
(47) p. 237 line 4 from above, §16.2
   (a) Morrey spaces
   (b) Morrey-type spaces
   (c) 19 Dec. 2024
(48) p. 237 line 11 from below, §16.2.3
   (a) generalied
   (b) general

(c) 17 Dec. 2024

(49) p. 237 line 7 from below, §16.2.4
  (a) et. al.
  (b) et al.
  (c) 17 Dec. 2024

1.25. **Pages 240–249.**

(1) p. 240 line 3 from above, Definition 40
  (a) Lozanovsky
  (b) Lozanovski
  (c) 17 Dec. 2024
(2) p. 240 line 6 from above, Definition 40
  (a) Remove
  (b) λ, for some $\lambda > 0$
  (c) 21 Aug. 2024
(3) p. 240 line 2 and 3 from below, the proof of Lemma 214
  (a) $|\lambda f + \rho g|$
  (b) $|f + g|$
  (c) 17 Dec. 2024
(4) p. 241 line 4 from above, the proof of Lemma 215
  (a) $\|f\|_{E_0(\mu)^{1-\theta}E_1(\mu)^\theta}$
  (b) $\|f^j\|_{E_0(\mu)^{1-\theta}E_1(\mu)^\theta}$
  (c) 21 Aug. 2024
(5) p. 241 line 11 from below, the proof of Lemma 214
  (a) $\displaystyle\sum_{j=J}^{\infty}$
  (b) $\displaystyle\sum_{j=J+1}^{\infty}$
  (c) 17 Dec. 2024
(6) p. 241 line 5 from below, Lemma 215
  (a) . Then
  (b) for all $j \in \mathbb{N}$. Then
  (c) 9 Oct. 2024
(7) p. 242 line 2 from above, the proof of Lemma 215
  (a) add before "Since"
  (b) satisfying $\|f_{j,0}\|_{X_0}, \|f_{j,1}\|_X < 1$
  (c) 21 Aug. 2024
(8) p. 242 line 17 from below, the proof of Theorem 216
  (a) Exericse
  (b) Exercise
  (c) 17 Dec. 2024
(9) p. 243 line 22 from below
  (a) Calderón-–Lozanovski product
  (b) Calderón product
  (c) 17 Dec. 2024
(10) p. 243 line 13 from below
  (a) in the right-hand
  (b) on the right-hand
  (c) 30 Sept. 2024
(11) p. 243 line 7 from below, the proof of Theorem 219 (twice)
  (a) $E_1(\mu)$
  (b) $E_0(\mu)$
  (c) 17 Dec. 2024
(12) p. 244 line 11 from above,
  (a) inequalities (17.2)–(17.3)
  (b) inequalities (17.2) and (17.3)

(c) 17 Dec. 2024

(13) p. 244 line 7 from below, Proposition 221
- (a) $u \le v$
- (b) $v \le u$
- (c) 17 Dec. 2024

(14) p. 244 line 5 from below, the proof of Proposition 221
- (a) Theorem 19
- (b) Theorem 19 in the first book
- (c) 17 Dec. 2024

(15) p. 244 line 3 from below, the proof of Proposition 221
- (a) $q \le v$
- (b) $v \le q$
- (c) 17 Dec. 2024

(16) p. 244 line 1 from below, the proof of Proposition 221
- (a) as in
- (b) as in Example 16 in
- (c) 21 Aug. 2024

(17) p. 245 line 5 from above, Example 74
- (a) $\frac{q_1}{p_1}$
- (b) $\frac{q_1}{p_1}$.
- (c) 17 Dec. 2024

(18) p. 245, line 11 from above, the proof of Proposition 221
- (a) where $j \in \{0, 1\}$
- (b) where $j = 0, 1$
- (c) 17 Dec. 2024

(19) p. 245 line 12 from above, Example 74
- (a) $\|f\|_{\mathcal{M}^p_p}^{\frac{p}{q_j}}$
- (b) $\|f\|_{\mathcal{M}^p_q}{}^{\frac{q}{q_j}}$
- (c) 17 Dec. 2024

(20) p. 245 line 13 from above, Example 74
- (a) $|f|^{\frac{q\theta}{p_1}}$
- (b) $|f|^{\frac{q\theta}{q_1}}$
- (c) 17 Dec. 2024

(21) p. 245, line 16 from above, Example 74
- (a) we have,
- (b) we have
- (c) 17 Dec. 2024

(22) p. 245 line 15 from above, Example 74
- (a) $\frac{p(1-\theta)}{q_0} + \frac{p\theta}{q_1}$
- (b) $\frac{p(1-\theta)}{p_0} + \frac{p\theta}{p_1}$
- (c) 21 Aug. 2024

(23) p. 245 line 5 from below, Proposition 222
- (a) $\neq$
- (b) $=$
- (c) 18 Dec. 2024

(24) p. 245 line 2 from below, the proof of Proposition 222
- (a) defined
- (b) defined by
- (c) 21 Aug. 2024

(25) p. 246 line 3 from above, the proof of Proposition 222

(a) $\{F_j\}_{j=1}^{\infty}$
(b) Then $\{F_j\}_{j=1}^{\infty}$
(c) 26 Sept. 2023

(26) p. 246 line 4 from above,
(a) $\chi_F \in (\mathcal{M}^p_q(\mathbb{R}^n) \cap \mathcal{M}^{p_1}_{q_1}(\mathbb{R}^n)) \setminus \mathcal{M}^{p_0}_{q_0}(\mathbb{R}^n)$.
(b) $\chi_F \in \mathcal{M}^p_q(\mathbb{R}^n)$.

(27) p. 246 line 11 from above, the proof of Proposition 222
(a) $\|g\|_{\mathcal{M}^{p_1}_{q_1}} = \max \mathfrak{A}g$,
(b) $\|\chi_F\|_{\mathcal{M}^{p_1}_{q_1}} \sim 1$ and $\|g\|_{\mathcal{M}^{p_1}_{q_1}} = \max \mathfrak{A}g$
(c) 22 Dec. 2024

(28) p. 246 line 12 from above, the proof of Proposition 222
(a) $f \in \mathcal{M}^{p_0}_{q_0}(\mathbb{R}^n)$.
(b) $f \in \mathcal{M}^{p_0}_{q_0}(\mathbb{R}^n)$, since $\chi_F \notin \mathcal{M}^{p_0}_{q_0}(\mathbb{R}^n)$.
(c) 26 Sept. 2023

(29) p. 246 line 14 from below, the proof of Proposition 222
(a) genearlized
(b) generalized
(c) 26 Sept. 2023

(30) p. 246 line 9 from below, Proposition 224
(a) add as an assumption
(b) ${\varphi_0}^{q_0} = {\varphi_1}^{q_1}$.
(c) 26 Sept. 2023

(31) p. 247 line 3 from above, the proof of Proposition 224
(a) inequalities (17.5)–(17.6)
(b) inequalities (17.5) and (17.6)
(c) 19 Dec. 2024

(32) p. 247, line 9 from above, the proof of Proposition 224
(a) where $j \in \{0, 1\}$
(b) where $j = 0, 1$
(c) 17 Dec. 2024

(33) p. 247, line 10 from above, the proof of Proposition 224
(a) for $j = 0, 1$
(b) for $j = 0, 1$ in view of (17.4).
(c) 26 Sept. 2023

(34) p. 247, line 6 from below, Exercise 57
(a) $q\theta$
(b) q
(c) 26 Sept. 2023

(35) p. 248, Definition 41
(a) $L^0(\mathbb{R}^n)$
(b) $L^0(\mu)$
(c) 16 Jan. 2024

(36) p. 248, Definition 41
(a) $M > 0$ such that
(b) $M > 0$ such that for all $f \in E(\mu)$
(c) 16 Jan. 2024

(37) p. 249, line 4 from above, Example 76
(a) Hence
(b) Hence, if $(X, \mathcal{B}, \mu)$ is $(\mathbb{R}^n, \mathcal{L}, \mathrm{d}x)$
(c) 16 Jan. 2024

(38) p. 249, line 14 from above, Example 77
(a) $\beta = \{\beta_k\}_{k=0}^{\infty}$

(b) $\beta = \{\beta_k\}_{k=-\infty}^{\infty}$
(c) 16 Jan. 2024

(39) p. 249, line 14 from above, Example 77
(a) $\{2^{\alpha_0 k}\beta_k\}_{k=0}^{\infty}$
(b) $\{2^{\alpha_0 k}\beta_k\}_{k=-\infty}^{\infty}$
(c) 16 Jan. 2024

(40) p. 249, line 14 from above, Example 77
(a) $\ell^{q_0}(\mathbb{N}_0)$
(b) $\ell^{q_0}(\mathbb{Z})$
(c) 16 Jan. 2024

(41) p. 249, line 15 from above, Example 77
(a) $F = \{f_k\}_{k=0}^{\infty}$
(b) $F = \{f_k\}_{k=-\infty}^{\infty}$
(c) 16 Jan. 2024

(42) p. 249, line 17 from above, Example 77 (twice)
(a) $k = 0$
(b) $k = -\infty$
(c) 16 Jan. 2024

(43) p. 249, line 15 from above, Example 77
(a) $k = 0$
(b) $k = -\infty$
(c) 16 Jan. 2024

(44) p. 249, line 17 from above, Example 77
(a) $\left\{\sum_{k=0}^{\infty}\left(\left\|\sum_{k=0}^{\infty} 2^{k\alpha_1}\beta_k g f_k\right\|_{L^{p_1}}\right)^{q_1}\right\}^{\frac{1}{q_1}}$
(b) $\left\{\sum_{l=-\infty}^{\infty}\left(\left\|2^{l\alpha_1}\beta_l g f_l\right\|_{L^{p_1}}\right)^{q_1}\right\}^{\frac{1}{q_1}}$
$= \left\{\sum_{l=-\infty}^{\infty}\left(\left\|\sum_{k=-\infty}^{\infty} 2^{k\alpha_1}\beta_k g f_k \chi_{B(2^l)\setminus B(2^{l-1})}\right\|_{L^{p_1}}\right)^{q_1}\right\}^{\frac{1}{q_1}}$
(c) 16 Jan. 2024

(45) p. 249, line 17 from above, Example 77
(a) $\left\{\sum_{k=0}^{\infty}(2^{k\alpha_0}\|\beta_k f_k\|_{L^{p_0}})^{q_0}\right\}^{\frac{1}{q_0}}$
(b) $\left\{\sum_{k=-\infty}^{\infty}(2^{k\alpha_0}\|\beta_k f_k\|_{L^{p_0}})^{q_0}\right\}^{\frac{1}{q_0}}$
(c) 16 Jan. 2024

(46) p. 249, line 19 from above, Example 77
(a) $\left\{\sum_{k=0}^{\infty}\left(\left\|\sum_{k=0}^{\infty} 2^{k\alpha_1}\beta_k g \chi_{B(2^k)\setminus B(2^{k-1})}\right\|_{L^{p_2}}\right)^{q_1}\right\}^{\frac{1}{q_1}}$
(b) $\left\{\sum_{k=-\infty}^{\infty}\left(\left\|2^{k\alpha_1}\beta_k g \chi_{B(2^k)\setminus B(2^{k-1})}\right\|_{L^{p_2}}\right)^{q_1}\right\}^{\frac{1}{q_1}}$
(c) 16 Jan. 2024

(47) p. 249, line 19 from above, Example 77
(a) $\left\{\sum_{k=0}^{\infty}(2^{k\alpha_0}\|\beta_k\|_{L^{p_0}})^{q_0}\right\}^{\frac{1}{q_0}}$

(b) $\left\{\sum_{k=-\infty}^{\infty}(2^{k\alpha_0}\|\beta_k\|_{L^{p_0}})^{q_0}\right\}^{\frac{1}{q_0}}$

(c) 16 Jan. 2024

(48) p. 249, line 21 from above, Example 77

(a) $\left\{\sum_{k=0}^{\infty}\left(\left\|\sum_{k=0}^{\infty}2^{k\alpha_2}g\chi_{B(2^k)\setminus B(2^{k-1})}\right\|_{L^{p_2}}\right)^{q_2}\right\}^{\frac{1}{q_2}}$

(b) $\left\{\sum_{k=-\infty}^{\infty}\left(\left\|2^{k\alpha_2}g\chi_{B(2^k)\setminus B(2^{k-1})}\right\|_{L^{p_2}}\right)^{q_2}\right\}^{\frac{1}{q_2}}$

(c) 16 Jan. 2024

1.26. **Pages 250–259.**

(1) p. 251 line 12 from above, the proof of Proposition 227
  (a) We thus may
  (b) Thus, we may
  (c) 7 Oct. 2024
(2) p. 251 line 15 from above, the proof of Proposition 227
  (a) delete
  (b) and some Z of measure zero
  (c) 26 Sept. 2023
(3) p. 251 line 17 from above, Theorem 18
  (a) Theorem 18
  (b) Theorem 18 in the first book
  (c) 19 Dec. 2024
(4) p. 251 line 2 from below,
  (a) inquality
  (b) inequality
  (c) 17 Dec. 2024
(5) p. 252 line 10 from above, Proposition 231
  (a) $q_2 \le p_2$
  (b) $q_2 = q_1 \le p_2$
  (c) 5 Jan. 2025
(6) p. 252 line 13 from above, the proof of Proposition 231
  (a) $f\chi_Q \in \mathcal{M}^{p_0}_{q_0}(\mathbb{R}^n)$
  (b) $f\chi_Q \in \mathcal{M}^{p_1}_{q_1}(\mathbb{R}^n)$
  (c) 19 Dec. 2024
(7) p. 253 line 7 from above, the proof of Proposition 232
  (a) Letting
  (b) by the aforementioned equality. Letting
  (c) 26 Sept. 2023
(8) p. 253 line 4 from below, Example 79
  (a) Replace the whole statement with the following:

Let $0 < q < p < \infty$, $\gamma > 0$ and let $R = 2^{\frac{p}{p-q}} - 1$. We write

$$F_N \equiv \left\{ y + \sum_{k=1}^{N} R(1+R)^{k-1} a_k \,:\, \{a_k\}_{k=1}^{N} \subset \{0,1\}, y \in [0,1] \right\}$$

and let $E^N \equiv \{x + N! \,:\, x \in F_{[1+\log_2 N]}\}$ for each $N \in \mathbb{N}$. Let

eq:140812-11 (1.1)
$$\Phi_\gamma \equiv \sum_{N > N_0} N^{-\gamma} \chi_{E^N},$$

where N_0 is large enough. Let $0 < \theta < 1$, $0 < r_0, r_1, r < \infty$, $1 < q_0, q_1, q < p$, and

$$\frac{1}{q} = \frac{1-\theta}{q_0} + \frac{\theta}{q_1}, \quad \frac{1}{r} = \frac{1-\theta}{r_0} + \frac{\theta}{r_1}.$$

Let T be the multiplication operator generated by Φ_γ. With some suitable choice of the parameters, we aim to disprove that T is bounded from $\mathcal{M}^p_q(\mathbb{R}^n)$ to $L^r(\mathbb{R}^n)$ although T is bounded from $\mathcal{M}^p_{q_0}(\mathbb{R}^n)$ to $L^{r_0}(\mathbb{R}^n)$ and T is bounded from $\mathcal{M}^p_{q_1}(\mathbb{R}^n)$ to $L^{r_1}(\mathbb{R}^n)$.

  (b) 6 Dec. 2024
(9) p. 254 line 7 from above, Proposition 235

(a) Replace the whole statement with the following:
If $\gamma > 2$, then $\Phi_\gamma \in \mathrm{PWM}(\mathcal{M}^p_1(\mathbb{R}), L^1(\mathbb{R}))$.
(b) 6 Dec. 2024

(10) p. 254, line 8 from above, the proof of Proposition 235 (twice)
(a) E_N
(b) $[y, y+1]$
(c) 6 Dec. 2024

(11) p. 254 line 8 from above, the proof of Proposition 235
(a) for any N
(b) and $|E^N| = 2^{[1+\log_2 N]}$ for any $N \in \mathbb{N}$ and $y \in \mathbb{R}$ because

$$\|\Phi_\gamma f\|_{L^1} \le \sum_{N>N_0} N^{-\gamma} 2^{[1+\log_2 N]} \|f\|_{\mathcal{M}^p_1}.$$

(c) 6 Dec. 2024

(12) p. 254, line 9 from above, Proposition 236
(a) Replace the whole statement with the following:
Let $p > r$. Then $\Phi_\gamma \in \mathrm{PWM}(L^p(\mathbb{R}), L^r(\mathbb{R}))$ if and only if

$$\gamma > \frac{2}{r} - \frac{2}{p}.$$

(b) 6 Dec. 2024

(13) p. 254, the proof of Proposition 236
(a) Replace the whole statement with the following:
Define $u > 0$ by

$$\frac{1}{r} = \frac{1}{u} + \frac{1}{p}.$$

We have only to look for the condition for Φ_γ to belong to $L^u(\mathbb{R})$. Since

$$\Phi_\gamma^u = \sum_{N>N_0} N^{-\gamma u} \chi_{E^N},$$

the condition we are looking for is $\gamma u > 2$.
(b) 6 Dec. 2024

(14) p. 254, line 8 from below, Proposition 237
(a) Replace the whole statement with the following:
If the parameters γ and r satisfy $0 < r < \infty$ and $\gamma r \le 2$, then $\Phi_\gamma \notin \mathrm{PWM}(\mathcal{M}^p_q(\mathbb{R}), L^r(\mathbb{R}))$.
(b) 6 Dec. 2024

(15) p. 254, the proof of Proposition 237
(a) Replace the whole statement with the following:
Let $f = \chi_{\bigcup_{N=N_0}^\infty E^N} \in \mathcal{M}^p_q(\mathbb{R})$. We use (1.1) to obtain

$$\int_{\mathbb{R}} |Tf(x)|^r \mathrm{d}x = \sum_{N>N_0} N^{-\gamma r} \int_{E^N} |f(x)|^r \mathrm{d}x = \sum_{N>N_0} N^{-\gamma r} 2^{[1+\log_2 N]} = \infty,$$

as desired.
(b) 6 Dec. 2024

(16) p. 254, lines 4 and 3 from below
(a) Replace these two lines with the following:
Let $\gamma = 2.01$, $p = q_1 = 8$, $q = 4$, $r = \frac{28}{31}$, $q_0 = r_0 = 1$, $r_1 = \frac{8}{9}$, $\theta = \frac{6}{7}$.
(b) 26 Sept. 2023, 6 Dec. 2024

(17) p. 255 line 9 from above, Lemma 238
(a) $g \in F_j(\mu)$

   (b) $g_j \in F_j(\mu)$
   (c) 17 Dec. 2024
(18) p. 256 line 6 from above, Proposition 239
   (a) $\alpha = \frac{n}{p_0} - \frac{n}{p_1} - \frac{n}{q_0} + \frac{n}{q_1}$
   (b) $\alpha = -\frac{n}{p_0} + \frac{n}{p_1} + \frac{n}{q_0} - \frac{n}{q_1}$
   (c) 17 Dec. 2024
(19) p. 256 line 8 from above, the proof of Proposition 239
   (a) $j = 0, 1, 2.$
   (b) $j = 0, 1.$
   (c) 17 Dec. 2024
(20) p. 256 line 9 from above, the proof of Proposition 239
   (a) $\dot{K}^{\alpha_j}_{p_j\infty}$
   (b) $\dot{K}^{\alpha_j}_{q_j\infty}$
   (c) 17 Dec. 2024
(21) p. 256 line 10 from above, the proof of Proposition 239
   (a) $\dot{K}^{\alpha}_{p\infty}$
   (b) $\dot{K}^{\alpha}_{q\infty}$
   (c) 17 Dec. 2024
(22) p. 256 line 12 from above, the proof of Proposition 239
   (a) "only if part"
   (b) "only if" part
   (c) 17 Dec. 2024
(23) p. 256 line 12 from above, the proof of Proposition 239
   (a) If $q_1 < q_2$, then $\mathrm{PWM}(L^{q_1}(O), L^{q_2}(O))$
   (b) If $q_0 < q_1$, then $\mathrm{PWM}(L^{q_0}(O), L^{q_1}(O))$
   (c) 17 Dec. 2024
(24) p. 256 line 14 from above, the proof of Proposition 239
   (a) is trivial thanks to Lemma 375 in the first book.
   (b) is trivial.
   (c) 19 Dec. 2024
(25) p. 256 line 19 from above,
   (a) $\alpha = \frac{n}{p_0} - \frac{n}{q_0}$
   (b) $\alpha = -\frac{n}{p_0} + \frac{n}{q_0}$ also swap the order of the definition of q_2 and α.
   (c) 17 Dec. 2024
(26) p 256 line 4 from below,
   (a) Morrey spaces
   (b) Morrey-type spaces
   (c) 17 Dec. 2024
(27) p. 257, line 1 from above, the proof of Proposition 241
   (a) squence
   (b) sequence
   (c) 26 Sept. 2023
(28) p. 257, line 8 from above
   (a) Propsition
   (b) Proposition
   (c) 26 Sept. 2023
(29) p. 257, line 20 from above is not needed, the proof of Proposition 242, 26 Sept. 2023
(30) p. 257, line 21 from above, the proof of Proposition 241
   (a) $a + [0, 1]$

(b) $a+[0,1]^n$
(c) 26 Sept. 2023

(31) p. 257 line 22 from above, second inequality, the proof of Proposition 242
(a) $\lesssim$
(b) $\sim$
(c) 19 Dec. 2024

(32) p. 257, line 22 from above, the proof of Proposition 242
(a) Add
(b) from (12.8)
(c) 26 Sept. 2023

(33) p. 257, Example 80
(a) add
(b) from Theorem 27
(c) 26 Sept. 2023

(34) p. 258, line 3 from below, Example 81
(a) $\lesssim 1$.
(b) $\lesssim 1$ by Lemma 25.
(c) 26 Sept. 2023

(35) p. 258, line 10 from below, Example 81
(a) (17.8),
(b) (17.8), (17.9)
(c) 26 Sept. 2023

(36) p. 258, line 3 from below, Example 81
(a) Hence
(b) A geometric observation shows
(c) 26 Sept. 2023

(37) p. 258, line 3 from below, Example 81
(a) $\|_{\mathcal{M}^{\varphi_2}_{q_2}} \lesssim 1$. Thus
(b) $\|_{\mathcal{M}^{\varphi_0}_{q_1}} \lesssim \|f\|_{\mathcal{M}^{\varphi_1}_{q_1}}$. Thus
(c) 26 Sept. 2023

(38) p. 259 line 6 from above, Theorem 243 contains the class $\mathcal{G}_\infty$, which should have been defined as the set of all constant functions.

(39) p. 259 line 6 from below (17.13), twice
(a) α
(b) $\alpha^{\frac{q_2}{q_0 q_1}}$
(c) 19 Dec. 2024

(40) p. 259, (17.11), the proof of Theorem 243
(a) $\geq$
(b) $\gtrsim$
(c) 26 Sept. 2023

(41) p. 259, one line above (17.12), the proof of Theorem 243
(a) Let $\varepsilon > 0$ be fixed. Let us
(b) Let us
(c) 26 Sept. 2023

(42) p. 259 line 3 from below, Theorem 243, (17.12)
(a) $\|g\|_{\mathcal{M}^{\varphi_2}_{q_2}}{}^{\frac{q_2}{q_0}} \lesssim \left((1+\varepsilon)\varphi_2(r) m^{(q_2)}_{B(a,r)}(g)\right)^{\frac{q_2}{q_0}} \leq \varphi_0(r)\|f\|_{L^{q_0}(B(a,r))}$
(b) $\|g\|_{\mathcal{M}^{\varphi_2}_{q_2}}{}^{\frac{q_2}{q_0}} \leq \left(2\varphi_2(r) m^{(q_2)}_{B(a,r)}(g)\right)^{\frac{q_2}{q_0}} \lesssim \varphi_0(r) r^{-\frac{n}{q_0}}\|f\|_{L^{q_0}(B(a,r))}$
(c) 17 Dec. 2024

(43) p. 259 line 1 from below, second inequality, the proof of Proposition 242
(a) $\lesssim$

(b) $\leq$
(c) 19 Dec. 2024

1.27. **Pages 260–269.**

(1) p. 260 lines 1–3, We should have taken these three lines out of the proof. 27 Dec. 2024
(2) p. 260 line 8 from above, Example 82
  (a) (s_k, r_k)
  (b) (r_k, s_k)
  (c) 17 Dec. 2024
(3) p. 260 line 10 from above, Example 82
  (a) $q_1 \le q_0$
  (b) $q_1 < q_0$
  (c) 17 Dec. 2024
(4) p. 260, line 13 from above, Example 82
  (a) $\geq$
  (b) $=$
  (c) 26 Sept. 2023
(5) p. 260, line 15 from above, follows from (17.8). 12 Sept. 2024
(6) p. 260 line 15 from below, Example 82
  (a) $\gtrsim 2^{\frac{k}{q_0}}$
  (b) $\geq 2^{\frac{k}{q_0}}$
  (c) 17 Dec. 2024
(7) p. 260, line 6 from below Theorem 244 (A)
  (a) ${\varphi_0}^{-q_0}{\varphi_1}^{q_1}$
  (b) ${\varphi_0}^{q_0}{\varphi_1}^{-q_1}$
  (c) 19 Dec. 2024
(8) p. 260, line 4 from below Theorem 244 (C)
  (a) in the sense of coincidence of norms
  (b) in the sense of sets
  (c) 12 Sept. 2024
(9) p. 261, line 4 from above
  (a) It sometimes
  (b) The space $\mathrm{BMO}_\varphi(\mathbb{R}^n)$ is the set of all $f \in L^1(\mathbb{R}^n)$ (modulo constant functions) for which $\|f\|_{\mathrm{BMO}_\varphi} < \infty$.
  (c) It sometimes
  (d) 4 Jan. 2025
(10) p. 261, line 4 from above
  (a) we defined
  (b) we define
  (c) 4 Jan. 2025
(11) p. 261, line 19 from above, Example 83
  (a) is not
  (b) does not belong to
  (c) 4 Jan. 2025
(12) p. 261 line 6 from below, (twice) Lemma 245
  (a) $\dot{\mathrm{BMO}}_\varphi$
  (b) BMO_φ
  (c) Remove $\cdot$.
  (d) 19 Dec. 2024
(13) p. 262, line 16 from above, the proof of Lemma 245
  (a) $\geq$
  (b) $\gtrsim$
  (c) 4 Jan. 2025
(14) p. 262 line 14 from below, the proof of Lemma 245

   (a) $\geq$
   (b) $\gtrsim$
   (c) 12 Sept. 2024
(15) p. 262 line 7 from below, Lemma 246
   (a) Delete for $0 < t' < t < \infty$
   (b) 17 Dec. 2024
(16) p. 262 line 5 from below, Lemma 246
   (a) add
   (b) Here the implicit constant depends on g
   (c) 12 Sept. 2024
(17) p. 263 line 1 from above, the proof of Lemma 246
   (a) $m_Q(g) = 1$
   (b) $g \in L^\infty(\mathbb{R}^n)$
   (c) 12 Sept. 2024
(18) p. 263 line 3 from above, the proof of Lemma 246
   (a) $\leq$
   (b) $=$
   (c) 12 Sept. 2024
(19) p. 263 line 2 from below, the proof of Theorem 247
   (a) $\mathrm{BMO}^+\varphi$
   (b) BMO_φ
   (c) Remove $\cdot$.
   (d) 19 Dec. 2024
(20) p. 264 line 7 from above, Example 84
   (a) $\|f\|_{\mathrm{Lip}^\alpha}$
   (b) $\|f\|_{\mathrm{Lip}^\alpha} < \infty$
   (c) 12 Sept. 2024
(21) p. 264 line 10 from below, the proof of Theorem 248
   (a) $\|f\|_{\mathrm{BMO}^+_\varphi}$
   (b) $(\|f\|_{\mathrm{BMO}_\varphi} + \|f\|_{L^p})$
   (c) 4 Jan. 2025
(22) p. 264 lines 6 and 5 from below, the proof of Theorem 248
   (a) $\Phi^*(r) - \Phi^*(1) \gtrsim \Phi^*(r)$
   (b) $\Phi_*(r) - \Phi_*(1) \gtrsim \Phi_*(r)$
   (c) 17 Dec. 2024
(23) p. 264 line 1 from below, the proof of Theorem 248
   (a) $g \in \mathrm{BMO}^+_\psi$
   (b) $g \in \mathrm{BMO}_\psi$
   (c) 17 Dec. 2024
(24) p. 265 line 2 and 5 from above, the proof of Theorem 248
   (a) $\|g\|_{\mathrm{BMO}^+_\psi}$
   (b) $\|g\|_{\mathrm{BMO}_\psi}$
   (c) 19 Dec. 2024
(25) p. 265 lines 4 and 5 from above, the proof of Theorem 248
   (a) $\|f\|_{\mathrm{BMO}^+_\varphi}$
   (b) $\|f\|_{\mathrm{BMO}_\varphi}$
   (c) 19 Dec. 2024
(26) p. 265 line 9 from above, Example 85
   (a) $2 + |\log \min(1, r)|$
   (b) $\log 2 + |\log \min(1, r)|$
   (c) 19 Dec. 2024

(27) p. 265 line 9 from below, the proof of Theorem 249
  (a) $\mathrm{BMO}_\varphi^+ = \mathrm{BMO}_\varphi$
  (b) $\mathrm{BMO}_\varphi^+ \subset \mathrm{BMO}_\varphi$
  (c) 19 Dec. 2024

(28) p. 266 line 3 from above, Proposition 250
  (a) $\mathcal{G}_n$ and $\varphi(+0) = \lim_{\varepsilon \downarrow 0} \varphi(\varepsilon) = \infty$.
  (b) $\mathcal{G}_n$.
  (c) 22 Dec. 2024

(29) p. 266 line 12 from above, the proof of Proposition 250 (1)
  (a) Let $r < 10^{-n}$
  (b) Let $r \le 1$ and $|a| \ge 2nr$.
  (c) 18 Dec. 2024

(30) Take

We write

$$d \equiv \sup_{y \in Q(a,r), z \in Q(a,3r) \setminus Q(a,r)} |y - z|, \quad D \equiv \inf_{y \in Q(a,r)} |y|.$$

in page 266 line 13 from above to one line above:(1) should start with this sentence.

(31) p. 266 line 17 from above, the proof of Proposition 250 (1)
  (a) Since φ
  (b) Since φ, Φ_*
  (c) 18 Dec. 2024

(32) p. 266 line 5 from below, the proof of Proposition 250 (2)
  (a) 10^{-n}
  (b) 1
  (c) 18 Dec. 2024

(33) p. 266 line 3 from below, the proof of Proposition 250
  (a) $x_0 \in Q(a, 3r) \setminus Q(a, r)$
  (b) $x_0 \in \partial Q(a, r)$
  (c) 12 Sept. 2024

(34) p. 266 line 2 from below, the proof of Proposition 250
  (a) $|g(x) - g(x_0)| \lesssim \frac{1}{\Phi^*(D)} \lesssim \frac{1}{\varphi(r)\Phi^*(|a|)} \sim \frac{1}{w_\varphi(a,r)}$.
  (b) $m_{Q(a,r)}(|g(x) - m_{Q(a,r)}(g)|) \lesssim \frac{1}{\Phi^*(D)} \lesssim \frac{1}{\varphi(r)\Phi^*(|a|)} \sim \frac{1}{w_\varphi(a,r)}$ from our assumption (17.18).
  (c) 22 Dec. 2024

(35) p. 267, line 1 from above, the proof of Proposition 250(3)
  (a) $\min(|a|/2n, 10^{-n})$
  (b) $\max(|a|/2n, 1)$
  (c) 7 Oct. 2024

(36) Add as (4) in the proof of Proposition 250 in page 267: (4) Assume that $1 > r \ge |a|/2n$. In this case $\omega_\varphi(a,r) \sim \varphi(r)\Phi_*(r)$. Then for $x \in Q(x,r)$ and $x_0 \in Q(a,3r) \setminus Q(a,r)$, $d \le 4nr$ and hence

$$m_{Q(a,r)}(|g - g(x_0)|) \lesssim \frac{1}{\varphi(4nr)\Phi_*(4nr)} \sim \frac{1}{\omega_\varphi(a,r)}.$$

So, in this case our claim is true. 19 Dec. 2024

(37) p. 267 line 12 from above, Proposition 251
  (a) $g \in \mathrm{BMO}_\varphi$
  (b) $g \in \mathrm{BMO}_\psi$
  (c) 19 Dec. 2024

(38) p. 267 line 13 from above, the proof of Proposition 251
  (a) $g \in \mathrm{BMO}_\varphi$ according to Theorem 249

   (b) $g \in \mathrm{BMO}_\psi$ according to Theorem 248
   (c) 19 Dec. 2024
(39) p. 267 line 7 from below, Corollary 252
   (a) Suppose
   (b) Let $\varphi \in \mathbb{M}^{\downarrow}(0,\infty)$ satisfy $\frac{1}{\varphi} \in \mathcal{G}_n$. Suppose
   (c) 22 Dec. 2024
(40) p. 268, Example 87, (2)
   (a) $\Phi^*(|x|) \gtrsim 1+|x|$
   (b) $\Phi^*(|x|) \lesssim 1+|x|$
   (c) 17 Sept. 2024
(41) p. 268, line 17 from above, Proposition 253
   (a) $|g(x+y)-g(x)| \lesssim \frac{\varphi(y)}{\Phi_+(|y|)}$
   (b) $|g(x+y)-g(x)| \lesssim \frac{1}{\varphi(|y|)\Phi_+(|y|)}$
   (c) 17 Sept. 2024
(42) p. 268, line 17 from above, Proposition 253,
   (a) $\mathrm{BMO}_\varphi^+ \cap L^p$
   (b) $\mathrm{BMO}_\varphi \cap L^p$
   (c) 19 Dec. 2024
(43) p. 268 lines 15 and 14 from below, the proof of Proposition 253
   (a) The proof is akin to Proposition 250.
   (b) Simply reexamine Proposition 250.
(44) p. 268, Exercise 59
   (a) Show that $f \in L^p(\mu)$.
   (b) Show that $f \in L^p(\mu)$.
   (c) 17 Sept. 2024 Compare Exercise 59 with Example 75.
   (d) Example 75 and Exercise 59 are related
(45) p. 269, line 1 from above, Exercise 61
   (a) $\in E_2(\mu)$
   (b) $\in E_3(\mu)$
   (c) 4 Jan. 2025
(46) p. 269, line 2 from above (twice), Exercise 61
   (a) $E_2(\mu)$
   (b) $E_3(\mu)$
   (c) 4 Jan. 2025
(47) p. 269, line 3 from above, Exercise 61
   (a) .
   (b) . Furthermore $E_3(\mu)$ enjoys the Fatou property.
   (c) 4 Jan. 2025
(48) p. 269, Exercise 61(2)
   (a) $L^\infty(\mu) \approx \mathrm{PMW}(E_2(\mu), E_2(\mu))$
   (b) $L^\infty(\mu) = \mathrm{PMW}(E_2(\mu), E_2(\mu))$
   (c) 17 Sept. 2024
(49) p. 269, line 8 from above, Exercise 62
   (a) $\frac{1}{p} = \frac{1}{p_1} + \frac{1}{p_2}, \frac{1}{q} = \frac{1}{q_1} + \frac{1}{q_2}$
   (b) $\frac{1}{p} = \frac{1}{p_1} + \frac{1}{p_2}, \quad \frac{1}{q} = \frac{1}{q_1} + \frac{1}{q_2}$
   (c) 7 Oct. 2024
(50) p. 269, Theorems 18 and 19 in Exercise 63 meant the ones in the first book, 17 Sept. 2024.
(51) p. 269, Exercise 64
   (a) $\|g\|_{L^\infty}$.
   (b) $\|g\|_{L^\infty}$. Compare Exercise 64 with Exercise 61.

   (c) 17 Sept. 2024.
   (d) Exercise 64 falls within the scope of Exercise 61.
(52) p. 269, line 11 from above, Exercise 66
   (a) $\varphi_3 = \varphi_1\varphi_2$.
   (b) $\varphi_3 = \varphi_1\varphi_2$, $\varphi_1, \varphi_2, \varphi_3 > 0$.
   (c) 4 Jan. 2025
(53) p. 269, Exercise 68
   (a) Suppose
   (b) Let $\varphi \in \mathbb{M}^{\downarrow}(0,\infty)$, $\frac{1}{\varphi} \in \mathcal{G}_n$. Suppose
   (c) 17 Sept. 2024

1.28. **Pages 270–279.**

(1) p. 270, line 18 from above:
   (a) the parameter q
   (b) the parameter q in the Morrey space $\mathcal{M}^p_q(\mathbb{R}^n)$
   (c) 4 Jan. 2025
(2) p. 270, line 8 from below, Theorem 254
   (a) $\frac{1}{r_0} = \frac{1}{q_0} + \frac{1}{p_0} - \frac{\alpha}{n}$.
   (b) $\frac{r}{r_0} = \frac{p}{p_0}, \quad \frac{1}{r_0} = \frac{1}{q_0} + \frac{1}{p_0} - \frac{\alpha}{n}$.
   (c) 22 Dec. 2024
(3) p. 270, lines 5-6 from below (Theorem 254). This sentence should have been added to Theorem 255, 17 Sept. 2024.
(4) p. 270 line 2 from below, the proof of Theorem 254
   (a) For each
   (b) in the first book. For each
   (c) 19 Dec. 2024
(5) p. 270 line 1 from below, the proof of Theorem 254
   (a) kernel
   (b) nutshell
   (c) 17 Dec. 2024
(6) p. 270, line 1 from below (the proof of Theorem 254).
   (a) as usual.
   (b) as usual. Write $\mathcal{D}^\flat(Q^0) = \mathcal{D}(Q^0)$.
   (c) 17 Sept. 2024.
(7) p. 271 line 1 from above, the proof of Theorem 254
   (a) . As for
   (b) for $* \in \{\sharp, \flat\}$. As for
   (c) 17 Sept. 2024
(8) p. 271 line 4 from above, the proof of Theorem 254
   (a) $Q \in \mathcal{D}^\sharp$
   (b) $Q \in \mathcal{D}^\flat$
   (c) 17 Sept. 2024
(9) p. 272 (17.4) and three lines below (17.4), Remark 9
   (a) I_α
   (b) M_α
   (c) 17 Sept. 2024
(10) p. 272 line 1 from below, the proof of Theorem 255
   (a) $\frac{1}{q_0}$
   (b) $\frac{1 \pm \alpha^{-1}\varepsilon}{q_0}$
   (c) 22 Dec. 2024
(11) p. 273 line 1 from above, the proof of Theorem 255
   (a) $\|g\|_{\mathcal{M}^{q_0}_q}$
   (b) $\|g\|_{\mathcal{M}^{q_0\left(1\pm\frac{\varepsilon}{\alpha}\right)^{-1}}_{q\left(1\pm\frac{\varepsilon}{\alpha}\right)^{-1}}}$
   (c) 22 Dec. 2024
(12) p. 273, line 4 from above, one line above Theorem 256
   (a) variant for
   (b) variant of
   (c) 7 Oct. 2024
(13) p. 273, line 12 from above, the proof of Theorem 256
   (a) $r_0^{(-\epsilon)}$

(b) $r_0(-\epsilon)$
(c) 21 Aug. 2024

(14) p. 273 lines 2 and 1 from below, the proof of Proposition 257
(a) g
(b) χ_{F_j}
(c) 23 Sept. 2024

(15) p. 273 line 1 from below
(a) $f \in \mathcal{M}^{r_0}_r(\mathbb{R}^n)$ and $g \in \mathcal{M}^{\frac{n}{\alpha}}_r(\mathbb{R}^n)$
(b) $f \in \mathcal{M}^{r_0}_r(\mathbb{R}^n)$
(c) 27 Dec. 2024

(16) p. 274 line 1 from above, the proof of Proposition 257
(a) $\|I_\alpha[h\chi_{F_j}]\|_{\mathcal{H}^{r_0}_r} \lesssim \|h\|_{\mathcal{H}^{r_0}_r}$.
(b) $\|I_\alpha[g \cdot h] \cdot f\|_{L^1} \lesssim \|f\|_{\mathcal{M}^{r_0}_r} \|g\|_{\mathcal{M}^{n/\alpha}_r} \|h\|_{\mathcal{H}^{r_0'}_{r'}}$ for all $f \in \mathcal{M}^{r_0}_r(\mathbb{R}^n)$, $g \in \mathcal{M}^{n/\alpha}_r(\mathbb{R}^n)$ and $h \in \mathcal{M}^{r_0'}_{r'}(\mathbb{R}^n)$.
(c) 30 Dec. 2024

(17) p. 274 line 3 from above, the proof of Proposition 257
(a) χ_F.
(b) χ_{F_j}.
(c) 17 Oct. 2024

p. 274 lines 6 and 7 from above, the proof of Proposition 257
(a) $|[0,(1+R)^j]^n|^{\frac{1}{r}-\frac{1}{r_0}}\|\chi_{F_j}\|_{L^{r'}} = 2^{jn}(1+R)^{j\alpha-\frac{jn}{r_0}}$, implying that $\|\chi_{F_j}\|_{\mathcal{H}^{r_0'}_{r'}} \le 2^{jn}(1+R)^{j\alpha-\frac{jn}{r_0}}$.
(b) if we let $g = h = \chi_{F_j}$, $\|g\|_{\mathcal{M}^{\frac{n}{\alpha}}_r} \sim 1$ and $\|h\|_{\mathcal{H}^{r_0'}_{r'}} \lesssim 2^{jn}$
(c) 30 Dec. 2024

(18) p. 274 line 8 from above, the proof of Proposition 257
(a) $\sim j^{\frac{1}{r}}(1+R)^{\frac{ln}{r_0}-l\alpha}$,
(b) $\lesssim j^{\frac{1}{r}}$,
(c) 17 Oct. 2024

(19) p. 275, line 18 from above:
(a) the parameter q
(b) the parameter q in the Morrey space $\mathcal{M}^p_q(\mathbb{R}^n)$
(c) 4 Jan. 2025

(20) p. 276 line 2 from above, Remark 10(2)
(a) $\dot{B}^s_{p1}(\mathbb{R}^n)$
(b) $B^s_{p1}(\mathbb{R}^n)$
(c) 23 Sept. 2024

(21) p. 276 lines 7 and 10 from above, Remark 10(2)
(a) $\displaystyle\sum_{j=-\infty}^{\infty}$
(b) $\displaystyle\sum_{j=0}^{\infty}$
(c) 19 Dec. 2024

(22) p. 276 line 6 from below, Remark 11
(a) $j \in \mathbb{Z}.$,
(b) $j \in \mathbb{Z}$,
(c) 19 Dec. 2024

(23) p. 276 line 12 from below, Example 89
(a) $m \in \mathbb{Z}^n$ and $j \in \mathbb{Z}$.

   (b) fixed $m \in \mathbb{Z}^n$
   (c) 22 Dec. 2024
(24) p. 276 line 3 from below, Remark 11
   (a) Since $q > p$,
   (b) Since $q > p$, adding this inequality over $j \in \mathbb{Z}$, we obtain
   (c) 23 Sept. 2024
(25) p. 277 line 6 from above, Definition 44
   (a) Banach function lattice
   (b) Banach lattice
   (c) 23 Sept. 2024
(26) p. 277 line 13 from below, Definition 44
   (a) $[\alpha_2 + 1]$
   (b) $[\alpha_2 + 1]$, $j \in \mathbb{Z}$ and $m \in \mathbb{Z}^m$
   (c) 22 Dec. 2024
(27) p. 277 line 9 from below
   (a) We did not and will not define $\mathcal{S}'(\mathbb{R}^n)$.
   (b) 23 Sept. 2024
(28) p. 277 line 8 from below
   (a) a_{ij}
   (b) a_{jm}
   (c) 23 Sept. 2024
(29) p. 279 line 14 from above, one line above Lemma 260
   (a) well defined
   (b) well defined for a Banach function space X
   (c) 23 Sept. 2024
(30) p. 279 line 6 from below,
   (a) $\lesssim$
   (b) $\le$
   (c) 22 Dec. 2024

1.29. **Pages 280–289.**

(1) p. 280 line 1 from above, the proof of Lemma 260
  (a) $|\partial^\alpha a_{jm}|$
  (b) $|a_{jm}|$
  (c) 19 Dec. 2024
(2) p. 280 lines 12 from above, the proof of Lemma 260
  (a) Then
  (b) Define $\kappa_{jm}(x) = 2^{jn}\kappa(2^j x - m)$. Then
  (c) 19 Dec. 2024
(3) p. 280 lines 12 from above, the proof of Lemma 260
  (a) we have
  (b) for each $j \in \mathbb{Z}$ and $m \in \mathbb{Z}^n$, we have
  (c) 19 Dec. 2024
(4) p. 280, line 13 from above, the proof of Lemma 260
  (a) so that $\chi_{Q_{00}} \le \kappa \le \chi_{\mathbb{R}^n \setminus 3Q_{00}}$ for each $j \in \mathbb{Z}$ and $m \in \mathbb{Z}^n$.
  (b) . Set $\kappa_{jm}(x) = 2^{jn}\kappa(2^j x - m)$, $x \in \mathbb{R}^n$ for $j \in \mathbb{Z}$ and $m \in \mathbb{Z}^n$.
  (c) 23 Sept. 2024
(5) p. 280, line 17 from above, the proof of Lemma 260
  (a) $\mathcal{M}^p_E(\mathbb{R}^n)$
  (b) $\mathfrak{M}^p_E(\mathbb{R}^n)$
  (c) 23 Sept. 2024
(6) p. 280 line 7 from below, the proof of Lemma 260
  (a) (t^{q_0}, t^{q_1})
  (b) $(t^{\frac{1}{q_0}}, t^{\frac{1}{q_1}})$
  (c) 17 Dec. 2024
(7) p. 280 line 4 from below, the proof of Lemma 260
  (a) (t^{u_0}, t^{u_1})
  (b) $(t^{\frac{1}{u_0}}, t^{\frac{1}{u_1}})$
  (c) 17 Dec. 2024
(8) p. 281 line 4 from above
  (a) the one used
  (b) the one used by Ragusa
  (c) 17 Oct. 2024
(9) p. 281 line 5 from above
  (a) $\mathfrak{M}^p_{L^{q,r}}(\mathbb{R}^n)$. However, this leads to confusion since we defined the global Morrey-type space $\mathcal{M}^p_{q\theta}(\mathbb{R}^n)$ in Chapter 16.
  (b) $\mathcal{R}^{p,q,\lambda}(\mathbb{R}^n)$.
  (c) 17 Oct. 2024
(10) p. 281, line 9 from above, the proof of Theorem 261
  (a) $\mathcal{M}^p_{L^{q,r}}(\mathbb{R}^n)$
  (b) $\mathfrak{M}^p_{L^{q,r}}(\mathbb{R}^n)$
  (c) 23 Sept. 2024
(11) p. 281 line 8 from below,
  (a) $\|f\chi_Q\|_{\mathbf{q}}$
  (b) $\|f\chi_Q\|_{L^{\mathbf{q}}}$
  (c) 17 Dec. 2024
(12) p. 281, line 7 from below,
  (a) let p satisfy $\frac{n}{p} = \sum_{j=1}^n \frac{1}{q_j}$, so that $\mathcal{M}^p_{\mathbf{q}}(\mathbb{R}^n) = L^{\mathbf{q}}(\mathbb{R}^n)$.
  (b) if p satisfies $\frac{n}{p} = \sum_{j=1}^n \frac{1}{q_j}$, then $\mathcal{M}^p_{\mathbf{q}}(\mathbb{R}^n) = L^{\mathbf{q}}(\mathbb{R}^n)$.
  (c) 23 Sept. 2024

(13) p. 282 line 9 from above, Exercise 73
   (a) $\mathcal{M}^p_{\mathcal{M}^p_{\mathfrak{q}}}$
   (b) $\mathfrak{M}^p_{\mathcal{M}^p_{\mathfrak{q}}}$
   (c) 26 Sept. 2023
(14) p. 283, line 8 from above, §17.2.2
   (a) Riuesset
   (b) Rieusset
   (c) 26 Sept. 2023
(15) p. 283 line 9 from above, §17.2.2
   (a) See
   (b) . See
   (c) 19 Dec. 2024
(16) p. 283, line 10 from above, §17.2.2
   (a) Riuesset
   (b) Rieusset
   (c) 26 Sept. 2023
(17) p. 283 line 10 from above, §17.2.2.
   (a) 228, 227
   (b) 227, 228
   (c) 19 Dec. 2024
(18) p. 283, line 3 from below, §17.3
   (a) discuessed
   (b) discussed
   (c) 26 Sept. 2023
(19) p. 284 lines 18, 17 from below, §17.3.2
   (a) 417, 415, 416
   (b) 415, 416, 417
   (c) 19 Dec. 2024
(20) Take off the last sentence in §17.4.2 in page 285.
(21) p. 285 line 12 from below,
   (a) Mizuta, Ohno, Rafeiro, Shimomura, Samko and Sawano
   (b) Mizuta, Nakai, Ohno, Rafeiro, Samko, Sawano, Shimomura and Sobukawa
   (c) 17 Dec. 2024
(22) p. 285 line 2 from below,
   (a) inverstigated
   (b) investigated
   (c) 17 Dec. 2024
(23) p. 288 line 3 from above,
   (a) $(\mathcal{X}_0 + \mathcal{X}_1) \times [0, \infty)$
   (b) $[0, \infty) \times (\mathcal{X}_0 + \mathcal{X}_1)$
   (c) 17 Dec. 2024
(24) p. 288 line 5 from above,
   (a) $K(x, t)$
   (b) $K(t, x)$
   (c) 17 Dec. 2024
(25) We should have written lines 2 and 1 from below in pag 288 right after the proof of Lemma 264. This concerns the abbreviation of inf
(26) p. 289 line 7 from below
   (a) $\|K(\cdot, x)\|_{\Phi_{\theta,q}(0,\infty)}$
   (b) $\|K(\cdot, x)\|_{\Phi_{\theta,q}(0,\infty)}$
   (c) 20 Dec. 2024
(27) p. 289, Example 91

(a) norms
(b) quasi-norms for all $0 < \theta < 1$ and $0 < q \leq \infty$
(c) 23 Sept. 2024

(28) p. 289, line 2 from below
(a) we define
(b) consider its *a-convexification given by*
(c) 23 Sept. 2024

1.30. **Pages 290–299.**

(1) p. 290, line 1 from above, Theorem 266
  (a) Let $E(\mu)$
  (b) Let $0<\theta<1$ and $E(\mu)$
(2) p. 290, line 2 from above, Theorem 266
  (a) If $f \in [E(\mu), L^\infty(\mu)]_{\theta,\infty} =$,
  (b) If $f \in (E(\mu), L^\infty(\mu))_{\theta,\infty} \approx$,
  (c) 23 Sept. 2024, 21 Dec. 2024
(3) p. 290, line 3 from above, the proof of Theorem 266
  (a) Then
  (b) Fix $t>0$. Then
  (c) 19 Dec. 2024
(4) p. 290, line 6 from above, the proof of Theorem 266
  (a) $\lesssim$
  (b) $\le$
  (c) 19 Dec. 2024
(5) p. 290, line 8 from above, the proof of Theorem 266
  (a) $\le (2^{-l}\|f\|_{\mathrm{W}E^{\frac{1}{1-\theta}}(\mu)})^{\frac{1}{1-\theta}}$.
  (b) $= (\|\chi_{(2^l,\infty]}(|f|)\|_{E^{\frac{1}{1-\theta}}(\mu)})^{1-\theta} \le (2^{-l}\|f\|_{\mathrm{W}E^{\frac{1}{1-\theta}}(\mu)})^{1-\theta}$
  (c) 19 Dec. 2024
(6) p. 290, line 10 from above, the proof of Theorem 266
  (a) If we insert this relation, then we obtain
$$t^{-\theta}K(t,f;E(\mu),L^\infty(\mu)) \lesssim \|f\|_{\mathrm{W}E^{\frac{1}{1-\theta}}(\mu)}$$
  for any $t>0$.
  (b) If we insert this relation into the above inequality, then we obtain
$$\begin{aligned} K(t,f;E(\mu),L^\infty(\mu)) &\lesssim \sum_{l=-\infty}^{\infty} \min(2^{l\theta}(\|f\|_{\mathrm{W}E^{\frac{1}{1-\theta}}(\mu)})^{1-\theta}, 2^l t) \\ &\lesssim \int_0^\infty \min(u^\theta(\|f\|_{\mathrm{W}E^{\frac{1}{1-\theta}}(\mu)})^{1-\theta}, tu)\frac{\mathrm{d}u}{u} \\ &\simeq \|f\|_{\mathrm{W}E^{\frac{1}{1-\theta}}(\mu)}. \end{aligned}$$
(7) p. 290, line 16 from below, the proof of Theorem 266
  (a) If $f \in (E(\mu), L^\infty(\mu))_{\theta,\infty} \cap \mathbb{M}^+(\mathbb{R}^n)$,
  (b) If $f \in (E(\mu), L^\infty(\mu))_{\theta,\infty}$,
  (c) 23 Sept. 2024
(8) p. 290 line 14 from below,
  (a) $K(\|\chi_{(y,\infty]}(|f|)\|_{E(\mu)}, f; E(\mu), L^\infty(\mu))$
  (b) $K(\|\chi_{(y,\infty]}(|f|)\|_{E(\mu)}, f; E(\mu), L^\infty(\mu))$.
(9) p. 291, line 1 from above, the proof of Theorem 266
  (a) any decomposition
  (b) a suitable decomposition
  (c) 23 Sept. 2024
(10) p. 291, lines 5, 8 from above, the proof of Theorem 266
  (a) $\le K$
  (b) $\le 2K$
  (c) 23 Sept. 2024
(11) p. 291, line 9 from above, the proof of Theorem 266
  (a) $\le$

   (b) $\lesssim$
   (c) 23 Sept. 2024
(12) p. 291, line 13 from above, Remark 12
   (a) This definition
   (b) As it stands, the definition of $\overline{\mathcal{X}}_{\theta,q}$
   (c) 23 Sept. 2024
(13) p. 291 line 14 from above, Remark 12
   (a) $\theta = 0$ of $\theta \geq 1$
   (b) $\theta = 0$ or $\theta \geq 1$
   (c) 17 Dec. 2024
(14) p. 291, line 15 from above, Remark 12
   (a) if $K(t, x) > 0$ for $t > 0$, $\|x\|_{\overline{\mathcal{X}}_{\theta,q}} = \infty$ for any $x \in \mathcal{X}_0 + \mathcal{X}_1$
   (b) if $x \in (\mathcal{X}_0 + \mathcal{X}_1) \setminus \{0\}$ satisfies $K(t, x) > 0$ for $t > 0$, $\|x\|_{\overline{\mathcal{X}}_{\theta,q}} = \infty$
   (c) 23 Sept. 2024
(15) p. 291, line 17 from above, Remark 12
   (a) for $\theta \geq 1$
   (b) for $\theta \geq 1$ and $q < \infty$
   (c) 23 Sept. 2024
(16) p. 291, line 3 from below, Lemma 267
   (a) $(\mathcal{X}_0, \mathcal{X}_1)$
   (b) $\overline{\mathcal{X}} = (\mathcal{X}_0, \mathcal{X}_1)$
   (c) 23 Sept. 2024
(17) p. 292, line 2 from above twice, the proof of Lemma 267
   (a) Banach
   (b) quasi-Banach
   (c) 19 Dec. 2024
(18) p. 292, line 5 from above, Lemma 268
   (a) $(\mathcal{X}_0, \mathcal{X}_1)$
   (b) $\overline{\mathcal{X}} = (\mathcal{X}_0, \mathcal{X}_1)$
   (c) 23 Sept. 2024
(19) p. 292, line 13 from above, Theorem 269
   (a) $(\mathcal{X}_0, \mathcal{X}_1)$
   (b) $\overline{\mathcal{X}} = (\mathcal{X}_0, \mathcal{X}_1)$
   (c) 23 Sept. 2024
(20) p. 292, line 13 from above, Theorem 269
   (a) $(\mathcal{Y}_0, \mathcal{Y}_1)$
   (b) $\overline{\mathcal{Y}} = (\mathcal{Y}_0, \mathcal{Y}_1)$
   (c) 23 Sept. 2024
(21) p. 292, line 10 from below, the proof of Theorem 312
   (a) $\dfrac{\|T\|_{\mathcal{X}_0 \to \mathcal{Y}_0}}{\|T\|_{\mathcal{X}_1 \to \mathcal{Y}_1}}$
   (b) $\dfrac{\|T\|_{\mathcal{X}_1 \to \mathcal{Y}_1}}{\|T\|_{\mathcal{X}_0 \to \mathcal{Y}_0}}$
   (c) 17 Dec. 2024
(22) p. 292, lines 3 and 2 from below, the proof of Theorem 269
   (a) $x_0 \in \mathcal{X}_1$
   (b) $x_0 \in \mathcal{X}_0$
   (c) 21 Dec. 2024
(23) p. 293, line 1 from above, Definition 48
   (a) Let
   (b) Let $\overline{\mathcal{X}} = (\mathcal{X}_0, \mathcal{X}_1)$ be a compatible couple of quasi-Banach spaces. Let
   (c) 23 Sept. 2024

(24) p. 293, lines 9 and 10, Definition 49
 (a) a pair of Banach spaces and $\theta \in (0,1)$.
 (b) a couple of Banach spaces.
 (c) 21 Dec. 2024
(25) p. 293, lines 13 and 16 from above, Definition 49
 (a) X
 (b) $\mathcal{X}$
 (c) 23 Sept. 2024
(26) p. 293 line 12 from below, Example 93
 (a) $f \in \mathrm{BC}(\mathbb{R}^n)$
 (b) $f \in \mathrm{BC}^1(\mathbb{R}^n)$
 (c) 21 Dec. 2024
(27) p. 295 (18.5), line 4 from above, the proof of Lemma 271
 (a)).
 (b)),
 (c) 22 Dec. 2024
(28) p. 295, line 8 from above, the proof of Lemma 271
 (a) . Then
 (b) and
 (c) 23 Sept. 2024
(29) p. 295, line 11 from below, the proof of Lemma 271
 (a) $\|x_{1\kappa}\|_{E_1(\mu)}$
 (b) $t\|x_{1\kappa}\|_{E_1(\mu)}$
 (c) 23 Sept. 2024
(30) p. 295, line 7 from below, the proof of Lemma 271 (twice)
 (a) 0
 (b) 1
 (c) 23 Sept. 2024
(31) p. 295 lines 2 from below,
 (a) $(E_0(\mu)^{\rho_0}_{\rm power}, E_1(\mu)^{\rho_1}_{\rm power})_{\theta,\infty}$
 (b) $(E_0(\mu)^{\rho_0}_{\rm power}, E_1(\mu)^{\rho_1}_{\rm power})_{\eta,\infty}$
 (c) 17 Dec. 2024
(32) p. 296 lines 2,5,10 from above,
 (a) $(E_0(\mu)^{\rho_0}_{\rm power}, E_1(\mu)^{\rho_1}_{\rm power})_{\theta,\infty}$
 (b) $(E_0(\mu)^{\rho_0}_{\rm power}, E_1(\mu)^{\rho_1}_{\rm power})_{\eta,\infty}$
 (c) 17 Dec. 2024
(33) p. 296 line 10 from above (18.6), the proof of Lemma 271
 (a) $\geq (\|x\|_{(E_0(\mu),E_1(\mu))_{\theta,\infty}})^\rho$
 (b) $\geq (2^{-1}\|x\|_{(E_0(\mu),E_1(\mu))_{\theta,\infty}})^\rho$.
 (c) 22 Dec. 2024
(34) p. 296 line 6 from below, the proof of Lemma 271
 (a) Lemma 271
 (b) (18.6)
 (c) 21 Dec. 2024
(35) p. 296, line 3 from below, the proof of Theorem 270
 (a) $2^{\rho_1} \leq \dfrac{2^{(k+1)\rho_1} J(2^{k+1}, x; \overline{E}(\mu))^{\rho_0-\rho_1}}{2^{k\rho_1} J(2^k, x; \overline{E}(\mu))^{\rho_0-\rho_1}} \leq 2^{\rho_0}$, we have
 (b) $2^{\rho_1} \leq \dfrac{2^{(l+1)\rho_1} J(2^{l+1}, x; \overline{E}(\mu))^{\rho_0-\rho_1}}{2^{l\rho_1} J(2^l, x; \overline{E}(\mu))^{\rho_0-\rho_1}} \leq 2^{\rho_0}$, we have
 (c) 23 Sept. 2024
(36) p. 297 line 6 from above, the proof of Theorem 270

   (a) $2^{\theta q r}$
   (b) $2^{k\theta q}$
   (c) 23 Sept. 2024
(37) p. 297 line 9 from above, the proof of Theorem 270
   (a) $s^{-\theta r}$
   (b) $s^{-\theta q}$
   (c) 23 Sept. 2024
(38) p. 297 line 9 from above, the proof of Theorem 270
   (a) $)^r$
   (b) $)^q$
   (c) 23 Sept. 2024
(39) p. 297 line 9 from above, the proof of Lemma 271(2)
   (a) $E_0(\mu)^{\rho_0}_{\rm power}, E_1(\mu)^{\rho_1}_{\rm power}$
   (b) $\overline{E}(\mu)$
(40) p. 297 line 10 from above,
   (a) $\lesssim$
   (b) $\le$
(41) p. 297 Exercise 75
   (a) Verify Remark 12 by using
   (b) Compare Remark 12 with
   (c) 23 Sept. 2024
(42) p. 297 line 1 from below,
   (a) $\lambda \ge 0$
   (b) $\lambda \in \mathbb{R}$
(43) p. 298, line 2 from above
   (a) $\Phi^{\uparrow}_{\lambda,q}(0,\infty) \equiv \Phi^{\uparrow}_{\lambda,q}(0,\infty) \cap \mathbb{M}^{\uparrow}(0,\infty)$
   (b) $\Phi^{\uparrow}_{\lambda,q}(0,\infty) \equiv \Phi_{\lambda,q}(0,\infty) \cap \mathbb{M}^{\uparrow}(0,\infty)$
   (c) 23 Sept. 2024
(44) p. 298, line 5 from above, (18.2)
   (a) $+\varphi(s)$
   (b) $+\varphi(s)\chi_{(s,\infty)}(t)$
   (c) 21 Dec. 2024
(45) p. 298, line 12 from above, Definition 50
   (a) all increasing
   (b) all non-decreasing
   (c) 23 Sept. 2024
(46) p. 298 line 19 from above
   (a) $\|K(\cdot,f;\Phi^{\uparrow}_{\lambda_0,q_0}(0,\infty),\Phi^{\uparrow}_{\lambda_1,q_1}(0,\infty))\|_{\Phi_{\theta,q}}$
   (b) $\|K(\cdot,f;\Phi^{\uparrow}_{\lambda_0,q_0}(0,\infty),\Phi^{\uparrow}_{\lambda_1,q_1}(0,\infty))\|_{\Phi_{\theta,q}(0,\infty)}$
   (c) 20 Dec. 2024
(47) p. 298, line 9 from below, Lemma 272
   (a) $0 < q_0, q_1, q \le \infty$
   (b) $0 < q_0, q_1 \le \infty$
   (c) 22 Dec. 2024
(48) p. 298, line 9 from below, Lemma 272
   (a) Delete and $\lambda_0 \ne \lambda_1$.
   (b) 22 Dec. 2024
(49) p. 299 line 8 from above, the proof of Lemma 272
   (a) $\inf_{\varphi_0 \in \Phi_{\lambda_0,q_0}(0,\infty), \varphi-\varphi_0 \in \Phi_{\lambda_1,q_1}(0,\infty)}$
   (b) $\inf_{\varphi_0 \in \Phi^{\uparrow}{}_{\lambda_0,q_0}(0,\infty), \varphi-\varphi_0 \in \Phi^{\uparrow}{}_{\lambda_1,q_1}(0,\infty)}$

(c) 21 Dec. 2024

(50) p. 299, line 12 from above, Lemma 273
(a) Let $0 < q_0, q_1, q \le \infty$, $0 < \theta < 1$, $\lambda_0, \lambda_1 > 0$, $\lambda_0 \ne \lambda_1$ and define $\lambda \equiv (1-\theta)\lambda_0 + \theta\lambda_1$.
(b) Let $0 < q_0, q_1, \le \infty$, $\lambda_0, \lambda_1 > 0$.
(c) 22 Dec. 2024

(51) p. 299, line 16 from above, the proof of Lemma 273
(a) Let
(b) We assume that $q_0 < \infty$; otherwise we modify the proof slightly. Let
(c) 22 Dec. 2024

(52) p. 299 line 3 from below, the proof of Lemma 273
(a) $\sim$
(b) $\simeq$
(c) 21 Dec. 2024

1.31. **Pages 300–309.**

(1) p. 300 line 2 from above, Take off $\lambda_0 \neq \lambda_1$ in the statement of Lemma 274 since this follows from $\lambda_0 < \lambda_1$.
(2) p. 300 line 5 from above
  (a) **Proof.**
  (b) **Proof.** We assume that $q_0 < \infty$; otherwise we modify the proof slightly.
  (c) 21 Dec. 2024
(3) p. 300, line 8 from above, the proof of Lemma 274
  (a) $\Phi_{\theta(\lambda_1-\lambda_0)-1,q}(0,\infty)$
  (b) $\Phi_{\theta(\lambda_1-\lambda_0)q_0-1,\frac{q}{q_0}}(0,\infty)$
  (c) 21 Dec. 2024
(4) p. 300, line 10 from above, the proof of Lemma 274
  (a) $\|\varphi\|_{\Phi_{\lambda_0,q}(0,\infty)}{}^q$
  (b) $\|\varphi\|_{\Phi_{\lambda_0,q}(0,\infty)}{}^{q_0}$
  (c) 21 Dec. 2024
(5) p. 300, lines 5 and 9 from below, Theorem 275
  (a) $\Phi_{\theta,q}$
  (b) $\Phi_{\lambda,q}$
  (c) 21 Dec. 2024
(6) p. 301, line 2 from below, the proof of Theorem 275
  (a) $K(t,f;\cdots$
  (b) $K(t,\varphi;\cdots$
(7) p. 301, line 1 from below, the proof of Theorem 275
  (a) $\inf\limits_{\substack{\varphi_0\in\Phi^{\uparrow}_{\lambda_0,q_0}(0,\infty)\\ \varphi-\varphi_0\in\Phi_{\lambda_1,q_1}(0,\infty)}}$
  (b) $\inf\limits_{\substack{\varphi_0\in\Phi^{\uparrow}_{\lambda_0,q_0}(0,\infty)\\ \varphi-\varphi_0\in\Phi^{\uparrow}_{\lambda_1,q_1}(0,\infty)}}$
  (c) 23 Dec. 2024
(8) p. 302, line 9 from above, the proof of Theorem 275
  (a) $\Phi_{\theta,q}(0,\infty)$
  (b) $\Phi_{1-\theta,q}(0,\infty)$
  (c) 23 Sept. 2024
(9) p. 302, line 2 from below, the proof of Theorem 275(3)
  (a) $\sim$
  (b) $\simeq$
(10) p. 302, line 2 from below, the proof of Theorem 275(3)
  (a) $\Phi_{\theta(\lambda_0-\lambda_1),q}$
  (b) $\Phi_{\theta(\lambda_1-\lambda_0),q}(0,\infty)$
  (c) 21 Dec. 2024
(11) p. 302, line 2 from below, the proof of Lemma 274
  (a) $\lambda > 0$
  (b) $\lambda\in\mathbb{R}$
  (c) 22 Dec. 2024
(12) p. 302, line 1 from below, the proof of Theorem 275(3)
  (a) $\lesssim$
  (b) $\leq$
(13) p. 302, line 1 from below, the proof of Theorem 275(3)
  (a) $\Phi_{\theta(\lambda_0-\lambda_1),q}$
  (b) $\Phi_{\theta(\lambda_1-\lambda_0),q}(0,\infty)$

(c) 21 Dec. 2024

(14) p. 303, line 2 from below, the proof of Theorem 275(3)
  (a) $\Phi_{\theta(\lambda_0-\lambda_1),q}$
  (b) $\Phi_{\theta(\lambda_1-\lambda_0),q}(0,\infty)$
  (c) 21 Dec. 2024

(15) p. 303, line 3 from below, the proof of Theorem 275(3)
  (a) $\Phi_{\theta(\lambda_0-\lambda_1),q}$
  (b) $\Phi_{\theta(\lambda_1-\lambda_0),q}(0,\infty)$
  (c) 21 Dec. 2024

(16) p. 303 line 5 from above, the proof of Theorem 275(3)
  (a) $\mathrm{I}_2 \le \|J_2\|_{\Phi_{(1-\theta)(\lambda_1-\lambda_0),q}(0,\infty)}$
  (b) $\mathrm{I}_2 = \|J_2\|_{\Phi_{(1-\theta)(\lambda_0-\lambda_1),q}(0,\infty)}$

(17) p. 303 line 6 from below, Definition 51
  (a) a linear subspace $\mathcal{X}$
  (b) a linear space $\mathcal{X}$
  (c) 19 Dec. 2024

(18) p. 303 line 1 from below,
  (a) $\lambda \ge 0$
  (b) $\lambda \in \mathbb{R}$

(19) p. 304, Example 94
  (a) add
  (b) Let $\sigma > 0$, $\lambda > 0$ and $0 < q \le \infty$. Let F be as in Definition 51. Then $\|f\|_{\Phi_{\lambda,q}(F^\sigma)} = (\|f\|_{\Phi_{\sigma^{-1}\lambda,\sigma q}(F)})^\sigma$ for all $f \in \mathcal{Z}$.
  (c) 23 Sept. 2024

(20) p. 304 line 12 from below,
  (a) Olsen
  (b) Olsen's
  (c) 17 Dec. 2024

(21) p. 304, lines 10 and 9 from below, Lemma 276
  (a) Define σ by
$$\frac{1}{\sigma} = \frac{1-\theta}{\sigma_0} + \frac{\theta}{\sigma_1}.$$
  (b) Define σ and η by
$$\frac{1}{\sigma} = \frac{1-\theta}{\sigma_0} + \frac{\theta}{\sigma_1}, \quad \eta \equiv \frac{\theta\sigma}{\sigma_1}.$$
  (c) 22 Dec. 2024

(22) p. 304, line 6 from below, Lemma 276
  (a) $(F^\sigma, F^{\sigma_0}, F^{\sigma_1})$
  (b) $(F^\sigma, F_0^{\sigma_0}, F_1^{\sigma_1})$
  (c) 22 Dec. 2024

(23) p. 304, line 1 from below
  (a) decompsotion
  (b) decomposition
  (c) 26 Sept. 2023

(24) p. 305 line 2 from above, the proof of Lemma 276
  (a) quasi-additivity
  (b) the weak quasi-additivity

(25) p. 305 line 3 from above, the proof of Lemma 276
  (a) $t^{\lambda_0\sigma_0} \sup_{s>0} s^{-\lambda_0\sigma} F_0{}^{\sigma_0} f_0(s) + t^{\lambda_1\sigma_1} \sup_{s>0} s^{-\lambda_1\sigma} F_1{}^{\sigma_1} f_1(s)$
  (b) $t^{\lambda_0\sigma_0} \sup_{s>0} s^{-\lambda_0\sigma_0} F_0{}^{\sigma_0} f_0(s) + t^{\lambda_1\sigma_1} \sup_{s>0} s^{-\lambda_1\sigma_1} F_1{}^{\sigma_1} f_1(s)$

(26) p. 305 line 4 from above, the proof of Lemma 276
   (a) $t^{\lambda_0\sigma_0}\|{F_0}^{\sigma_0}f_0\|_{\Phi_{\lambda_0\sigma_0,\infty}}+t^{\lambda_1\sigma_1}\|{F_1}^{\sigma_1}f_1\|_{\Phi_{\lambda_1\sigma_1,\infty}}$
   (b) $t^{\lambda_0\sigma_0}\|{F_0}^{\sigma_0}f_0\|_{\Phi_{\lambda_0\sigma_0,\infty}(0,\infty)}+t^{\lambda_1\sigma_1}\|{F_1}^{\sigma_1}f_1\|_{\Phi_{\lambda_1\sigma_1,\infty}(0,\infty)}$
   (c) 20 Dec. 2024
(27) p. 305 line 5 from above, the proof of Lemma 276
   (a) $t^{\tau_0}\|{F_0}^{\sigma_0}f_0\|_{\Phi\lambda_0\sigma_0,\frac{q_0}{\sigma_0}}+t^{\tau_1}\|{F_1}^{\sigma_1}f_1\|_{\Phi\lambda_1\sigma_1,\frac{q_1}{\sigma_1}}$
   (b) $t^{\tau_0}\|{F_0}^{\sigma_0}f_0\|_{\Phi_{\lambda_0\sigma_0,\frac{q_0}{\sigma_0}}(0,\infty)}+t^{\tau_1}\|{F_1}^{\sigma_1}f_1\|_{\Phi_{\lambda_1\sigma_1,\frac{q_1}{\sigma_1}}(0,\infty)}$
   (c) 20 Dec. 2024
(28) p. 305 line 11 from above, the proof of Lemma 276
   (a) quasi-additivity
   (b) the weak quasi-additivity
(29) p. 305 lines 11 and 6 from below, the proof of Theorem 277
   (a) $(\|f\|_{\Phi_{\lambda\sigma,\frac{q}{\sigma}}(F^\sigma)})^q$
   (b) $(\|f\|_{\Phi_{\lambda\sigma,\frac{q}{\sigma}}(F^\sigma)})^{\frac{q}{\sigma}}$
(30) p. 305, line 11 from below, the proof of Theorem 277
   (a) Lemma 276
   (b) Lemma 276 and Example 94
   (c) 23 Sept. 2024
(31) p. 305, line 15 from below, Theorem 277
   (a) If $(F^\sigma, F^{\sigma_0}, F^{\sigma_1})$
   (b) If $\lambda_0\sigma_0 \neq \lambda_1\sigma_1$ and $(F^\sigma, F_0^{\sigma_0}, F_1^{\sigma_1})$
   (c) 22 Dec. 2024
(32) Add:

   We will assume $q_0, q_1, q < \infty$; otherwise naturally modify the proof.

   in the beginning of the proof of Theorem 277 in pages 305. 23 Sept. 2024
(33) p. 305, line 9 from below, the proof of Theorem 277
   (a) Lemma 276
   (b) Lemma 276 and Example 94
   (c) 22 Dec. 2024
(34) p. 305, line 11 from below, the proof of Theorem 277
   (a) $)^q$
   (b) $)^{\frac{q}{\sigma}}$
   (c) 22 Dec. 2024
(35) p. 305, line 7 from below, the proof of Theorem 277
   (a) variables
   (b) variables and an analogue of Theorem 270
   (c) 23 Sept. 2024
(36) p. 305, line 6 from below, the proof of Theorem 277
   (a) $)^q$
   (b) $)^{\frac{q}{\sigma}}$
   (c) 22 Dec. 2024
(37) p. 306, line 12 from above, Theorem 278
   (a) weak A-B
   (b) weakly A-B
   (c) 22 Dec. 2024
(38) Add:

   We will assume $q_0, q_1, q < \infty$; otherwise naturally modify the proof.

   in the beginning of the proof of Theorem 277 in pages 305. 23 Sept. 2024
(39) p. 306 line 11 from below, the proof of Theorem 278,
   (a) $(\Phi_{\lambda_0,q_0}(F_0), \Phi_{\lambda_1,q_1}(F_1))$

   (b) $(\Phi_{\lambda_0,q_0}(F_0),\Phi_{\lambda_1,q_1}(F_1))_{\theta,q}$
   (c) 19 Dec. 2024
(40) p. 306 line 1 from below, the proof of Theorem 278
   (a) $\overset{\frac{\sigma_1}{\sigma}\text{ power}}{\Phi_{\lambda_1,q_1}(F_1)}$
   (b) $\Phi_{\lambda_1,q_1}(F_1)^{\frac{\sigma_1}{\sigma}\text{ power}}$
   (c) 22 Dec. 2024
(41) p. 307 line 3 from above, the proof of Theorem 278
   (a) $\left(\int_0^\infty F_1 f_{1,s}(\rho)^{q_1}\frac{\mathrm{d}\rho}{\rho^{1+\lambda_1 q_1}}\right)^{\frac{\sigma_0}{q_1\sigma}}$
   (b) $\left(\int_0^\infty F_1 f_{1,s}(\rho)^{q_1}\frac{\mathrm{d}\rho}{\rho^{1+\lambda_1 q_1}}\right)^{\frac{\sigma_1}{q_1\sigma}}$
   (c) 22 Dec. 2024
(42) p. 307, line 6 from above, the proof of Theorem 278
   (a) $\le$
   (b) $\lesssim$
   (c) 20 Dec. 2024
(43) p. 307, line 7 from above, Theorem 278
   (a) $t\left(\int_0^\infty [\rho^{-\lambda_1\sigma_1}(B_sFf)(\alpha\rho)^\sigma)^{\frac{q_1}{\sigma_1}}\frac{\mathrm{d}\rho}{\rho}\right]$
   (b) $t\left(\int_0^\infty [\rho^{-\lambda_1\sigma_1}(B_sFf)(\alpha\rho)^\sigma]^{\frac{q_1}{\sigma_1}}\frac{\mathrm{d}\rho}{\rho}\right)$
   (c) 23 Sept. 2024
   (d) Note that the positions of) and] were changed.
(44) p. 307, line 8 from above, the proof of Theorem 278
   (a) $\Phi_{\frac{\lambda_0\sigma_0}{\sigma},\frac{q_0\sigma}{\sigma_0}}$
   (b) $\Phi_{\frac{\lambda_0\sigma_0}{\sigma},\frac{q_0\sigma}{\sigma_0}}(0,\infty)$
   (c) 20 Dec. 2024
(45) p. 307, line 8 from above, the proof of Theorem 278
   (a) $\Phi_{\frac{\lambda_1\sigma_1}{\sigma},\frac{q_1\sigma}{\sigma_1}}$
   (b) $\Phi_{\frac{\lambda_1\sigma_1}{\sigma},\frac{q_1\sigma}{\sigma_1}}(0,\infty)$
   (c) 20 Dec. 2024
(46) p. 307, line 9 from above, the proof of Theorem 278
   (a) and a trivial estimate
   (b) :
   (c) 20 Dec. 2024
(47) p. 307, line 10 from above, the proof of Theorem 278
   (a) $\Phi_{\frac{\theta\sigma}{\sigma_1},q}$
   (b) $\Phi_{\frac{\theta\sigma}{\sigma_1},q}(0,\infty)$
   (c) 20 Dec. 2024
(48) p. 307, line 11 from above, the proof of Theorem 278
   (a) $\le\left\|\|A.Ff\|_{\Phi_{\frac{\lambda_0\sigma_0}{\sigma},\frac{q_0\sigma}{\sigma_0}}}+\cdot\|B.Ff\|_{\Phi_{\frac{\lambda_1\sigma_1}{\sigma},\frac{q_1\sigma}{\sigma_1}}}\right\|_{\Phi_{\frac{\theta\sigma}{\sigma_1},q}}$
   (b) $\sim\|Ff\|_{\Phi_{\lambda,q}(0,\infty)}=\|f\|_{\Phi_{\lambda,q}(F)}$
   (c) 22 Dec. 2024
(49) p. 307, line 12 from below, Theorem 279
   (a) weak A-B
   (b) weakly A-B
   (c) 22 Dec. 2024
(50) p. 307, line 11 from below, Theorem 279

   (a) $(F^\sigma, F^{\sigma_0}, F^{\sigma_1})$
   (b) $(F^\sigma, F_0^{\sigma_0}, F_1^{\sigma_1})$
   (c) 22 Dec. 2024
(51) p. 307 line 8 from below, Theorem 279
   (a) Φ_{λ_0,p_0}
   (b) Φ_{λ_0,q_0}
   (c) 22 Dec. 2024
(52) p. 307 line 8 from below, Theorem 279
   (a) Φ_{λ_1,p_1}
   (b) Φ_{λ_1,q_1}
   (c) 22 Dec. 2024
(53) p. 307, line 6 from below
   (a) for many examples
   (b) for many examples in the rest of this chapter
   (c) 23 Sept. 2024
(54) In Exercise 76 in p. 307,
   (a) $\mathcal{X}$
   (b) $\mathcal{X}$ and $F : \mathcal{Z} \to \mathbb{M}^\uparrow(0,\infty)$
   (c) 23 Sept. 2024
(55) In Exercise 76 in p. 307,
   (a) $\left(\Phi_{\lambda\sigma,\frac{q}{\sigma}}(F^\sigma)\right)^{\frac{1}{\sigma}}$
   (b) $\left(\Phi_{\lambda\sigma,\frac{q}{\sigma}}(F^\sigma)\right)^{\frac{1}{\sigma}} \equiv \{H^\sigma \,:\, H \in \Phi_{\lambda\sigma,\frac{q}{\sigma}}(F^\sigma)\}$.
   (c) 23 Sept. 2024
(56) p. 307 line 1 from below, Exercise 76
   (a) $\Phi_{\lambda,\theta}$
   (b) $\Phi_{\lambda,q}$
   (c) 22 Dec. 2024
(57) p. 308, Exercise 77 (3)
   (a) the Hardy operator
   (b) the dual Hardy operator
   (c) 23 Sept. 2024
(58) p. 309 line 9 from above, the proof of Lemma 280
   (a) As a result,
   (b) As a result, since $f^*(t)$ is a constant,
   (c) 22 Dec. 2024
(59) p. 309 line 12 from above,
   (a) L^∞
   (b) $L^\infty(\mu)$
   (c) 22 Dec. 2024

1.32. **Pages 310–319.**

(1) p. 310, line 3 from above, Theorem 282
  (a) $(L^{p_0}(\mu), L^\infty(\mu))_{\theta,p} = L^p(\mu)$
  (b) $(L^{p_0}(\mu), L^\infty(\mu))_{\theta,p} \approx L^p(\mu)$
  (c) 23 Sept. 2024
(2) p. 310, line 7 from above, the proof of Theorem 282
  (a) $(L^1(\mu), L^\infty(\mu))_{\theta,q} = L^{\frac{1}{1-\theta},q}(\mu)$
  (b) $(L^1(\mu), L^\infty(\mu))_{\theta,q} \approx L^{\frac{1}{1-\theta},q}(\mu)$
  (c) 23 Sept. 2024
(3) p. 310 line 12 from above, the proof of Theorem 282
  (a) $\|\mathcal{H}[f^*]\|_{\Phi_{\theta-1,q}}$
  (b) $\|\mathcal{H}[f^*]\|_{\Phi_{1-\theta,q}(0,\infty)}$
  (c) 22 Dec. 2024, 23 Sept. 2024
(4) p. 310 line 13 from above, the proof of Theorem 282
  (a) $\|f\|_{L^{\frac{1}{1-\theta},q}}$
  (b) $\|f\|_{L^{\frac{1}{1-\theta},q}(\mu)}$
  (c) 20 Dec. 2024
(5) p. 310 line 11 from below, the proof of Theorem 282 (twice)
  (a) $\|f\|_{L^{\frac{1}{1-\theta},q}}$
  (b) $\|f\|_{L^{\frac{1}{1-\theta},q}(\mu)}$
  (c) 20 Dec. 2024
(6) p. 310 line 9 from below, the proof of Theorem 282
  (a) $\|f\|_{L^{\frac{1}{1-\theta},q}}$
  (b) $\|f\|_{L^{\frac{1}{1-\theta},q}(\mu)}$
  (c) 20 Dec. 2024
(7) p. 310 line 8 from below,
  (a) Lemma 271
  (b) Lemma 281
  (c) 30 Dec. 2024
  p. 310 line 3 from below
  (a) a mapping
  (b) a general mapping
  (c) 30 Dec. 2024
(8) p. 311, line 4 from above
  (a) $\lambda_f\left(\frac{1}{t}\right)$
  (b) $\lambda_f\left(\frac{1}{t}\right) = \lambda_{f,\mu}\left(\frac{1}{t}\right)$
  (c) 23 Sept. 2024
(9) p. 311 line 8 from above
  (a) $\|H^{\frac{1}{p_0}} f\|_{\Phi_{1,p_0}}$
  (b) $\|H^{\frac{1}{p_0}} f\|_{\Phi_{1,p_0}(0,\infty)}$
  (c) 20 Dec. 2024
(10) p. 311 line 17 from below, Lemma 283
  (a) Then
  (b) Assume $p, p_0, p_1 < \infty$. Then
  (c) 23 Sept. 2024
(11) p. 311, line 14 from below, the proof of Lemma 283
  (a) $F^p(f_0 + f_1) = H(f_0 + f_1) = \lambda_{f_0+f_1}\left(\frac{1}{t}\right)$

   (b) $F^p(f_0+f_1)(t) = H(f_0+f_1)(t) = \lambda_{f_0+f_1}\left(\frac{1}{t}\right)$
   (c) 23 Sept. 2024
(12) p. 311, line 12 from below, the proof of Lemma 283
   (a) $F^p(f_0+f_1)$
   (b) $F^p(f_0+f_1)(t)$
   (c) 23 Sept. 2024
(13) p. 311 line 8 from below, Theorem 284
   (a) $f_{1,s} \equiv f \cdot \chi_{X\setminus G_s(f)}$
   (b) $f_{1,s} \equiv f \cdot \chi_{X\setminus G_s(f)}.$
   (c) 17 Dec. 2024
(14) p. 311, line 6 from below, the proof of Lemma 283
   (a) $F^p(f_{0,s}) = H(f_{0,s}) = |G_{\min(t,s)}(f)|$
   (b) $F^p(f_{0,s})(t) = H(f_{0,s})(t) = \mu(G_{\min(t,s)}(f))$
   (c) 23 Sept. 2024
(15) p. 311 line 3 from below, 3 lines above Theorem 284
   (a) $G_s(f)$
   (b) $X \setminus G_s(f)$
   (c) 18 Dec. 2024
(16) p. 311, line 2 from below, the proof of Lemma 283
   (a) ${F_0}^{p_0}(f_{0,s}) \le B_s[F^p f](t)$
   (b) ${F_1}^{p_1}(f_{1,s})(t) = \mu\left\{x \in X \,:\, \frac{1}{t} < |f(x)| \le \frac{1}{s}\right\} \le B_s[F^p f](t)$
   (c) 23 Sept. 2024
(17) p. 312 line 1 from above, Theorem 284
   (a) Let $0 < p_0, p_1 < \infty$
   (b) Let $0 < q \le \infty$, $0 < p_0, p_1 < \infty$
   (c) 23 Sept. 2024
(18) p. 312 line 4 from above, the proof of Theorem 284
   (a) $(L^{p_0}(\mu), L^{p_1}(\mu))_{\theta,q} = (\Phi_{1,p_0}(F_0), \Phi_{1,p_1}(F_1))_{\theta,q} = \Phi_{1,q}(F) = \Phi_{1,q}(H^{\frac{1}{p}})$
   (b) $(L^{p_0}(\mu), L^{p_1}(\mu))_{\theta,q} \approx (\Phi_{1,p_0}(F_0), \Phi_{1,p_1}(F_1))_{\theta,q} \approx \Phi_{1,q}(F) = \Phi_{1,q}(H^{\frac{1}{p}})$
(19) p. 332 line 5 from above, the proof of Theorem 284
   (a) Lemma 283
   (b) Lemma 283 and Theorem 279
(20) p. 312 line 6 from above, the proof of Theorem 284
   (a) $v = f^*(t)$
   (b) $t \mapsto t^{-1}$
   (c) 17 Dec. 2024
(21) p. 312 line 7 from above, the proof of Theorem 284
   (a) $\|(Hf)^{\frac{1}{p}}\|_{\Phi_{1,q}}$
   (b) $\|(Hf)^{\frac{1}{p}}\|_{\Phi_{1,q}(0,\infty)}$
   (c) 20 Dec. 2024
(22) p. 312, line 8 from above, the proof of Theorem 284
   (a) $\left\{\int_0^\infty \left(\frac{\lambda_{f,\mu}(t^{-1})}{t^p}\right)^{\frac{q}{p}} \frac{\mathrm{d}t}{t}\right\}^{\frac{1}{q}}$
   (b) $\left\{\int_0^\infty \left(\frac{\lambda_{f,\mu}(t^{-1})}{t^p}\right)^{\frac{q}{p}} \frac{\mathrm{d}t}{t}\right\}^{\frac{1}{q}} = \left\{\int_0^\infty (t^p\lambda_{f,\mu}(t))^{\frac{q}{p}} \frac{\mathrm{d}t}{t}\right\}^{\frac{1}{q}}$
   (c) 23 Sept. 2024
(23) p. 312, line 13 from above, Exercise 79
   (a) Theorem 191
   (b) Theorem 282

(c) 23 Sept. 2024

(24) p. 312 line 13 from below, Exercise 79
  (a) $L^{\frac{1}{1-\theta},1}$
  (b) $L^{\frac{1}{1-\theta},1}(\mu)$
  (c) 17 Dec. 2024

(25) p. 312, line 11 from below
  (a) the Morrey norm
  (b) (an equivalent form of) the Morrey norm
  (c) 9 Oct. 2024

(26) p. 313, line 1 from above
  (a) We disprove
  (b) We can disprove
  (c) 23 Sept. 2024

(27) p. 313, line 3 from above, Theorem 313
  (a) Replace the whole statement with the following:
    There exists a bounded linear operator $T : \mathcal{M}^8_1(\mathbb{R}) \to L^1(\mathbb{R})$ such that T, restricted to $L^8(\mathbb{R})$, maps $L^8(\mathbb{R})$ to $L^{\frac{8}{9}}(\mathbb{R})$ but that T does not map $\mathcal{M}^8_4(\mathbb{R})$ to $L^{\frac{28}{31}}(\mathbb{R})$.
  (b) 6 Dec. 2024

(28) p. 313 line 6 from above, the proof of Theorem 285
  (a) $L^p(B(x,r))$
  (b) $L^q(B(x,t))$
  (c) 18 Dec. 2024

(29) p. 313, line 7 from above, the proof of Theorem 285
  (a) $\Phi_{\lambda,\infty}(F) = \mathcal{M}^p_q(\mathbb{R}^n)$.
  (b) If we let $\lambda = \frac{n}{q} - \frac{n}{p}$, $\Phi_{\lambda,\infty}(F) = \mathcal{M}^p_q(\mathbb{R}^n)$.
  (c) 23 Sept. 2024

(30) p. 313 lines 9 and 10 (twice) from above, the proof of Theorem 285
  (a) $L^p(B(x,t))$
  (b) $L^q(B(x,t))$
  (c) 18 Dec. 2024

(31) p. 313 line 10 from above, the proof of Theorem 285
  (a) $+$
  (b) $+\sup\limits_{x\in\mathbb{R}^n}$

(32) p. 313, line 14 from below, the proof of Theorem 286
  (a) in the first book.
  (b) in the first book. Note that $\chi_F \in \mathcal{M}^p_q(\mathbb{R}^n) \cap \mathcal{M}^{p_1}_q(\mathbb{R}^n)$.
  (c) 23 Sept. 2024

(33) p. 313, line 4 from below
  (a) $\|f\|_{\Phi_{\frac{n}{q}-\frac{n}{p},u}(0,\infty)}$
  (b) $\|Ff\|_{\Phi_{\frac{n}{q}-\frac{n}{p},u}(0,\infty)}$
  (c) 28 Sept. 2024

(34) p. 313, line 3 from below
  (a) $\|f\|_{\Phi_{\frac{n}{q}-\frac{n}{p},\infty}}$
  (b) $\|Ff\|_{\Phi_{\frac{n}{q}-\frac{n}{p},\infty}(0,\infty)}$
  (c) 28 Sept. 2024

(35) p. 313, line 2 from below
  (a) this statement
  (b) Theorem 286
  (c) 23 Sept. 2024

(36) p. 314 line 1 from above, Theorem 287
  (a) $1 \le q$
  (b) $0 < q$
  (c) 19 Dec. 2024
(37) p. 314 line 1 from above, Theorem 287
  (a) $u_0, u_1, u > 0$
  (b) $0 < u_0, u_1, u \le \infty$
  (c) 19 Dec. 2024
(38) p. 314 line 3 from above, Theorem 287
  (a) $=$
  (b) $\approx$
  (c) 19 Dec. 2024
(39) p. 314, line 5 from above, the proof of Theorem 287
  (a) $\lambda_0 = \frac{1}{q} - \frac{1}{p_0}$, $\lambda_1 = \frac{1}{q} - \frac{1}{p_1}$, $\lambda = \frac{1}{q} - \frac{1}{p}$
  (b) $\lambda_0 = \frac{n}{q} - \frac{n}{p_0}$, $\lambda_1 = \frac{n}{q} - \frac{n}{p_1}$, $\lambda = \frac{n}{q} - \frac{n}{p}$
  (c) 7 Oct. 2024
(40) p. 314, line 5 from above, the proof of Theorem 287
  (a) $\sigma_0 = \sigma_1 = \sigma = q$
  (b) $\sigma_0 = \sigma_1 = \sigma = 1$
  (c) 7 Oct. 2024
(41) p. 314 lines 10 and 15 from above, the proof of Theorem 287
  (a) (F^q, F^q, F^q)
  (b) (F, F, F)
  (c) 19 Dec. 2024
(42) p. 314 line 11 from below, the proof of Theorem 287
  (a) $=$
  (b) $\approx$
  (c) 19 Dec. 2024
(43) p. 314 line 7 from below, the headder of §18.4.2
  (a) Morrey spaces
  (b) Morrey-type spaces
  (c) 19 Dec. 2024
(44) p. 315, line 12 from above, Theorem 288
  (a) $=$
  (b) $\approx$
  (c) 19 Dec. 2024
(45) p. 315, line 14 from above
  (a) The local Morrey-type
  (b) The generalized local Morrey-type
  (c) 23 Sept. 2024
(46) p. 315, line 10 from below, Theorem 289
  (a) $\lim_{t\downarrow\infty}$
  (b) $\lim_{t\uparrow\infty}$
  (c) 7 Oct. 2024
(47) p. 315 line 4 from below, Theorem 289
  (a) $=$
  (b) $\approx$
  (c) 19 Dec. 2024
(48) p. 316, line 2 from above, the proof of Theorem 289
  (a) $\Phi_{\lambda,q}$

   (b) $\Phi_{\lambda,u}$
   (c) 7 Oct. 2024
(49) p. 316 line 9 from below,
   (a) $(\cdots{}^p)\dfrac{dt}{t}$
   (b) $(\cdots)^p\dfrac{dt}{t}$
   (c) 19 Dec. 2024
(50) p. 316 line 6 from below, Lemma 290
   (a) $=$
   (b) $\approx$
   (c) 19 Dec. 2024
(51) p. 316, line 6 from below, Lemma 290
   (a) Remove the sentence starting with "In particular, $\cdots$".
   (b) 23 Sept. 2024
(52) p. 316 line 2 from below, Lemma 290
   (a) $1 \le p$
   (b) $0 < p$
   (c) 19 Dec. 2024
(53) p. 316, line 2 from below, the proof of Lemma 290
   (a) $\left(t^{-\tau}\int_{G_t}|f(x)|^p \mathrm{d}\mu(x)\right)^p$
   (b) $t^{-\tau p}\int_{G_t}|f(x)|^p \mathrm{d}\mu(x)$
   (c) 7 Oct. 2024
(54) p. 317 line 5 from above, Lemma 291
   (a) $=$
   (b) $\approx$
   (c) 19 Dec. 2024
(55) p. 317 line 7 from above, Lemma 291
   (a) $=$
   (b) $\approx$
   (c) 19 Dec. 2024
(56) p. 317, line 10 from above, Remark 13
   (a) $(L^p(X, w^{\tau_0}\mathrm{d}\mu), L^p(X, w^{\tau_1}\mathrm{d}\mu))_{\theta,q} = L^p(\mu)$, while
   (b) $(L^p(\mathrm{d}\mu), L^p(\mathrm{d}\mu))_{\theta,q} = L^p(\mu)$. In fact,
   (c) 23 Sept. 2024
(57) p. 317, line 13 from above, Remark 13
   (a) $L^p(\mu)$ with equivalence of norms
   (b) $L^p(\mu)$
   (c) 23 Sept. 2024
(58) p. 317, line 15 from above
   (a) $0 < \tau_0, \tau_1 < \infty$
   (b) $0 < \tau_0, \tau_1 < \frac{1}{p}$. and $0 < \tau_0, \tau_1 < \frac{1}{p}$. Assume $\tau_0 \ne \tau_1$. Let $0 < \theta < 1$ and $\tau = (1-\theta)\tau_0 + \theta\tau_1$.
   (c) 23 Sept. 2024
(59) p. 317, line 9 from below
   (a) $= L^p(X, w_k\mathrm{d}\mu)$
   (b) $\approx L^p(X, w_k\mathrm{d}\mu)$
   (c) 23 Sept. 2024
(60) p. 318 line 7 from above, Theorem 292
   (a) $= \mathrm{L}\mathcal{M}_{pq}^{\frac{p}{1-p\tau}}(G_{\tau_0,\tau_1}, \nu_{\tau_0,\tau_1})$
   (b) $\approx \mathrm{L}\mathcal{M}_{pq}^{\frac{p}{1-p\tau}}(G_{\tau_0,\tau_1}, \nu_{\tau_0,\tau_1})$

   (c) 19 Dec. 2024
(61) p. 318 line 8 from above, Theorem 292
   (a) $= L^p(X, {w_0}^{1-\theta}{w_1}^{\theta}\mathrm{d}\mu)$
   (b) $\approx L^p(X, {w_0}^{1-\theta}{w_1}^{\theta}\mathrm{d}\mu)$
   (c) 19 Dec. 2024
(62) p. 318, line 9 from above, the proof of Theorem 292
   (a) Theorem 287
   (b) Theorem 291
   (c) 23 Sept. 2024
(63) p. 318 line 10 from below, Lemma 293
   (a) $\le \infty$
   (b) $< \infty$
   (c) 19 Dec. 2024
(64) p. 318, line 9 from below, Lemma 293
   (a) ≤ 0
   (b) < 0
   (c) 23 Sept. 2024
(65) p. 319 line 16 from below, Theorem 294
   (a) $=$
   (b) $\approx$
   (c) 19 Dec. 2024
(66) p. 319 line 14 from below, Theorem 294
   (a) $=$
   (b) $\approx$
   (c) 19 Dec. 2024
(67) p. 319 line 8 from below, Exercise 80
   (a) $=$
   (b) $\approx$
   (c) 19 Dec. 2024
(68) p. 319 line 7 from below, Exercise 80
   (a) $=$
   (b) $\approx$
   (c) 19 Dec. 2024
(69) p. 319, line 3 from below
   (a) exhausitively
   (b) exhaustively
   (c) 26 Sept. 2023

1.33. **Pages 320–329.**

(1) p. 320 line 9 from below, §18.3.1
  (a) 5.2.1. Theorem
  (b) Theorem 5.2.1
  (c) 17 Dec. 2024
(2) p. 320 line 6 from below, §18.3.2
  (a) 5. 3. 1. Theorem
  (b) Theorem 5.3.1
  (c) 17 Dec. 2024
(3) p. 321 line 6 from above, §18.4.1
  (a) See the works by Burenkov and Nursultanov **, Nakai and Sobukawa ** and by Mastyło and Sawano **
  (b) See Burenkov and Nursultanov **, Nakai and Sobukawa ** and Mastyło and Sawano ** for real interpolation of local Morrey spaces.
  (c) 17 Dec. 2024
(4) p. 321 line 4 from below,
  (a) [57, Theorem 7]
  (b) [57, Theorem 6]
  (c) 19 Dec. 2024
(5) p. 325, line 6 from above, Lemma 296
  (a) Let
  (b) Let $0 < \theta < 1$ and $w \in \mathbb{C}$ be a complex number satisfying $|w| \ll 1$. Let
  (c) 12 Dec. 2024
(6) p. 325, line 7 from above, Lemma 296
  (a) $q : \overline{S} \to \mathbb{C}$
  (b) $q : \overline{S} \equiv \{z \in \mathbb{C} \,:\, 0 \le \mathrm{Re}(z) \le 1\} \to \mathbb{C}$
  (c) 12 Dec. 2024
(7) p. 326, lines 8, 11 and 16 from above, the proof of Lemma 296
  (a) $[-Qw \log |f|]$
  (b) $(-Qw \log |f|)$
  (c) 12 Dec. 2024
(8) p. 327, line 1 from above, the headder of Section 19.1.2, p. 327, line 2 from above, one line below the headder of Section 19.1.2,
  (a) Doestch
  (b) Doetsch
  (c) 12 Dec. 2024
(9) p. 328, line 8 from below, the proof of Lemma 300
  (a) add
  (b) Define $H(z) \equiv \log |F(z)|$ for $z \in \bar{S}$.
  (c) 23 Sept. 2024
(10) p. 329 line 5 from above, the proof of Lemma 300
  (a) $\mathrm{d}t$.
  (b) $\mathrm{d}t$
  (c) 24 Dec. 2024
(11) p. 329 lines 19, 17 (twice), 13 (twice) and 9 (twice) from below, the proof of Lemma 300
  (a) $\sin \pi\theta$
  (b) $\sin(\pi\theta)$
  (c) 17 Dec. 2024
(12) p. 329, line 9 from below, Lemma 300

(a) $\displaystyle\int_{\infty}^{-\infty} \frac{-H(iy)\pi \operatorname{sech}(\pi y)\sin \pi\theta}{1-\operatorname{sech}(\pi y)\cos(\pi\theta)} \mathrm{d}y$

(b) $\displaystyle\pi \int_{\infty}^{-\infty} \frac{-H(iy)\pi \operatorname{sech}(\pi y)\sin(\pi\theta)}{1-\operatorname{sech}(\pi y)\cos(\pi\theta)} \mathrm{d}y$

(c) 12 Dec. 2024

1.34. **Pages 330–339.**

(1) p. 330, line 11 from below
  (a) three-line theorem
  (b) three-line lemma
  (c) 21 Aug. 2024
(2) p. 330 lines 7 and 3 from below, the proof of Lemma 302(twice),
  (a) Doestch
  (b) Doetsch
  (c) 12 Dec. 2024
(3) p. 331, line 3 from above
  (a) Throughtout
  (b) Throughout
  (c) 12 Dec. 2024
(4) p. 331, line 4 from above
  (a) Remove the sentence starting with "Complex...".
  (b) 23 Sept. 2024
(5) p. 331, line 12 from below, Example 97
  (a) $t \in \mathbb{R} \mapsto f(t;\cdot) \in \mathcal{M}^p_q$
  (b) $t \in \mathbb{R} \mapsto f(t;\cdot) \in \mathcal{M}^p_q(\mathbb{R})$
  (c) 12 Dec. 2024
(6) p. 331, line 5 from below, Example 97
  (a) $2\sin\frac{1}{2} \cdot |B(R)|^{\frac{1}{p}-\frac{1}{q}}$
  (b) $2^{1+\frac{1}{p}-\frac{1}{q}} \sin\frac{1}{2} \cdot |B(2R)|^{\frac{1}{p}-\frac{1}{q}}$
  (c) 12 Dec. 2024
(7) We should have mentioned that Definition 55 is a duplication of what I have defined. See p. 332. 23 Sept. 2024
(8) p. 332 line 6 from above, one line below Definition 55
  (a) Doestch
  (b) Doetsch
  (c) 12 Dec. 2024
(9) p. 332, line 13 from above, Theorem 303
  (a) $z \in 1 + \mathbb{R}$
  (b) $z \in 1 + i\mathbb{R}$
  (c) 12 Dec. 2024
(10) p. 332 line 13 from above, Theorem 303
  (a) . holds.
  (b) holds, where μ_0 and μ_1 are defined by (19.13).
  (c) 23 Sept. 2024
  (d) I also have remarked that Theorem 303 is almost a duplication of Theorem 301.
(11) p. 332 line 7 from below, Definition 56
  (a) (1)
  (b) (2)
  (c) 23 Sept. 2024
(12) p. 333 line 8 from above, Example 99
  (a) $\lim\limits_{k\to\infty}$
  (b) $\lim\limits_{j\to\infty}$
  (c) 17 Dec. 2024
(13) p. 333 lines 17, 16, 15 (twice) from below, Example 100
  (a) E
  (b) $\mathcal{X}$

   (c) 12 Dec. 2024
(14) p. 333, line 17 from below, Example 100 one line above (1)
   (a) The norm is given by
   (b) unlike the previous setting. The norm is given by
   (c) 18 Dec. 2024
(15) p. 333, line 10 from below, Example 100
   (a) $\mathcal{F}(E(\mathbb{R}^n), L^\infty(\mathbb{R}^n))$
   (b) $\mathcal{F}(\mathcal{X}(\mathbb{R}^n), L^\infty(\mathbb{R}^n))$
   (c) 12 Dec. 2024
(16) p. 333 line 14 from below, Example 100
   (a) E^θ
   (b) $\mathcal{X}^{1-\theta}$
   (c) 18 Dec. 2024
(17) Page 333 line 13 from below, Example 100
   (a) Lemma 300
   (b) Theorem 301
   (c) 19 Dec. 2024
(18) p. 333, line 9 from below, Example 100
   (a) $\|F\|_{\mathcal{F}(E(\mathbb{R}^n), L^\infty(\mathbb{R}^n))}$
   (b) $\|F\|_{\mathcal{F}(E(\mathcal{X}, L^\infty)}$
   (c) 12 Dec. 2024
(19) p. 333, line 8 from below, Example 100
   (a) $\|F\|_{\mathcal{F}(E(\mathbb{R}^n), L^\infty(\mathbb{R}^n))}$
   (b) $\|F\|_{\mathcal{F}(E(\mathcal{X}, L^\infty)}$
   (c) 12 Dec. 2024
(20) p. 333, line 6 from below, Example 100
   (a) $\|F\|_{\mathcal{F}(E(\mathbb{R}^n), L^\infty(\mathbb{R}^n))}$
   (b) $\|F\|_{\mathcal{F}(E(\mathcal{X}, L^\infty)}$
   (c) 12 Dec. 2024
(21) p. 333, line 4 from below, Example 100
   (a) $\dfrac{1}{\theta}\int_{\mathbb{R}} |F(1+it)|\mu_1(t)\mathrm{d}t \in E(\mathbb{R}^n)$
   (b) $\dfrac{1}{1-\theta}\int_{\mathbb{R}} |F(it)|\mu_0(t)\mathrm{d}t \in \mathcal{X}(\mathbb{R}^n)$
   (c) 18 Dec. 2024
(22) p. 334, line 6 from above, Example 100
   (a) $p_1 = \infty > p_1$
   (b) $p_0 = \infty > p_1$
   (c) 23 Sept. 2024
(23) p. 334 lines 12 and 11 from below
   (a) Lip($\mathbb{R}$;
   (b) Lip($\mathbb{R}$,
   (c) 4 Jan. 2025
(24) p. 336 line 1 from below, the proof of Lemma 306
   (a) Define
   (b) Let $t > 0$. Define
   (c) 24 Dec. 2024
(25) p. 338, line 1 from above, Theorem 308
   (a) Banach spaces
   (b) complex Banach spaces
   (c) 12 Dec. 2024
(26) p. 338, line 4 from above, the proof of Theorem 308
   (a) $[\mathcal{X}_0, \mathcal{X}_1]_\theta{}^* \hookleftarrow [\mathcal{X}_0{}^*, \mathcal{X}_1{}^*]^\theta$

(b) $[\mathcal{X}_0, \mathcal{X}_1]_\theta{}^* \overset{1}{\hookleftarrow} [\mathcal{X}_0{}^*, \mathcal{X}_1{}^*]^\theta$
(c) 23 Sept. 2024

(27) p. 338, line 6 from above (19.23), the proof of Theorem 308
(a) $(\mathcal{X}_0{}^*, \mathcal{X}_1{}^*)_\theta$
(b) $[\mathcal{X}_0{}^*, \mathcal{X}_1{}^*]_\theta$
(c) 12 Dec. 2024

(28) p. 338, line 7 from above, the proof of Theorem 308:
(a) Therefore,
(b) Assume that g assumes its value in $\mathcal{X}_0 \cap \mathcal{X}_1$ and that g, restricted to S, is an $\mathcal{X}_0 \cap \mathcal{X}_1$-valued holomorphic function. Since $x^* = f'(\theta)$ and $x = g(\theta)$,
(c) 4 Jan. 2025

(29) p. 338, line 14 from above:
(a) $(\mathcal{X}_0 \cap \mathcal{X}_1)^*$
(b) $\mathcal{X}_0 \cap \mathcal{X}_1$
(c) 4 Jan. 2025

(30) p. 338, line 12 from below, the proof of Theorem 308
(a) $f^*(z)(g(z))$
(b) $f^{*\prime}(z)(g(z))$
(c) 12 Dec. 2024

(31) p. 338, line 10 from below (19.24), the proof of Theorem 308
(a) $(\mathcal{X}_0{}^*, \mathcal{X}_1{}^*)_\theta$
(b) $[\mathcal{X}_0{}^*, \mathcal{X}_1{}^*]^\theta$
(c) 12 Dec. 2024

(32) p. 338, line 9 from below, the proof of Theorem 308
(a) $x \in [\mathcal{X}_0{}^*, \mathcal{X}_1{}^*]^\theta$
(b) $x \in [\mathcal{X}_0{}^*, \mathcal{X}_1{}^*]^\theta$ and $\|x\|_{[\mathcal{X}_0,\mathcal{X}_1]_\theta{}^*} \le \|x\|_{[\mathcal{X}_0{}^*,\mathcal{X}_1{}^*]^\theta}$
(c) 23 Sept. 2024

(33) p. 338, line 8 from below, the proof of Theorem 308
(a) mapping
(b) the mapping
(c) 23 Sept. 2024

(34) p. 338, line 7 from below, the proof of Theorem 308
(a) $[\mathcal{X}_0, \mathcal{X}_1]_\theta{}^* \hookrightarrow [\mathcal{X}_0{}^*, \mathcal{X}_1{}^*]^\theta$
(b) $[\mathcal{X}_0, \mathcal{X}_1]_\theta{}^* \overset{1}{\hookrightarrow} [\mathcal{X}_0{}^*, \mathcal{X}_1{}^*]^\theta$
(c) 23 Sept. 2024

(35) p. 338, line 2 from below, the proof of Theorem 308
(a) add
(b) The space $L^1(\mathcal{X}_0) \oplus L^1(\mathcal{X}_1)$ is equipped with the norm

$$\|(x_0, x_1)\|_{L^1(\mathcal{X}_0)\oplus L^1(\mathcal{X}_1)} = \|x_0\|_{L^1(\mathcal{X}_0)} + \|x_1\|_{L^1(\mathcal{X}_1)} \quad ((x_0, x_1) \in L^1(\mathcal{X}_0) \oplus L^1(\mathcal{X}_1)).$$

(c) 23 Sept. 2024

(36) p. 339, line 1 from above, the proof of Theorem 308
(a) $L^1(\mu)^* = \mathrm{Lip}(\mathbb{R}; \mathcal{X}^*)$
(b) $L^1(\mathcal{X}_k)^* = \mathrm{Lip}(\mathbb{R},(\mathcal{X}_k)^*)$ for $k = 0, 1$
(c) 23 Sept. 2024

(37) p. 339, line 2 from above, the proof of Theorem 308
(a) $g_0 \in \mathrm{Lip}(\mathbb{R}; \mathcal{X}_0{}^*)$ and $g_1 \in \mathrm{Lip}(\mathbb{R}; \mathcal{X}_0{}^*)$
(b) $g_0 \in \mathrm{Lip}(\mathbb{R}, (\mathcal{X}_0)^*)$ and $g_1 \in \mathrm{Lip}(\mathbb{R}, (\mathcal{X}_1)^*)$

(38) p. 339 line 4(twice) from above, the proof of Theorem 308
(a) Lip($\mathbb{R}$;
(b) Lip($\mathbb{R}$,
(c) 4 Jan. 2025

(39) p. 339, line 11 from below, the proof of Theorem 308
  (a) $k_a(\tau)$
  (b) $k_a(\mu(j+i\tau))$
  (c) 23 Sept. 2024
(40) p. 339, line 10 from below, the proof of Theorem 308
  (a) $\tau \in \partial\Delta(1)$
  (b) $\tau \in \mathbb{R}$ and $j = 0, 1$
  (c) 23 Sept. 2024
(41) p. 339 lines 10 and 9 from below (both twice), the proof of Theorem 308
  (a) $\mathrm{Lip}(\mathbb{R};$
  (b) $\mathrm{Lip}(\mathbb{R},$
  (c) 4 Jan. 2025
(42) p. 339, line 8 from below, the proof of Theorem 308
  (a) We define
  (b) By using the density assumption we define
  (c) 23 Sept. 2024
(43) p. 339, line 8 from below, the proof of Theorem 308
  (a) by
  (b) so that
  (c) 23 Sept. 2024
(44) p. 339 line 5 from below, the proof of Theorem 308
  (a) $g(z) \equiv \displaystyle\int_{\frac{1}{2}}^{\mu^{-1}(z)} k(\zeta)\mathrm{d}\zeta$
  (b) $g(z) \equiv \displaystyle\int_{\mu^{-1}(\frac{1}{2})}^{z} k(\mu(\zeta))\mathrm{d}\zeta$
  (c) 18 Dec. 2024

1.35. **Pages 340–349.**

(1) p. 340 line 1 from above, the proof of Theorem 308
  (a) Lip($\mathbb{R}$;
  (b) Lip($\mathbb{R}$,
  (c) 4 Jan. 2025
(2) p. 340, line 4 from above, the proof of Theorem 308
  (a) $-g(\theta)$
  (b) $-ig(\theta)$
  (c) 23 Sept. 2024
(3) p. 340 line 6 from above, the proof of Theorem 308
  (a) $l = g'(\theta) \in [\mathcal{X}_0{}^*, \mathcal{X}_1{}^*]^\theta = [\mathcal{X}_0{}^*, \mathcal{X}_1{}^*]_\theta$
  (b) $x^* = g'(\theta) \in [\mathcal{X}_0{}^*, \mathcal{X}_1{}^*]^\theta$
  (c) 17 Dec. 2024
(4) p. 340, line 10 from above, Theorem 309
  (a) Banach spaces
  (b) complex Banach spaces
  (c) 12 Dec. 2024
(5) p. 340, line 16 from above, the proof of Theorem 309
  (a) $\le \|x\|_{[\mathcal{Y}_0,\mathcal{Y}_1]^\theta}$. Assume
  (b) ≤ 1 by assuming
  (c) 22 Dec. 2024
(6) p. 340, line 16 from above, the proof of Theorem 309
  (a) $\|x\|_{[\mathcal{Y}_0,\mathcal{Y}_1]^\theta} = 1$.
  (b) $\|x\|_{[\mathcal{X}_0,\mathcal{X}_1]^\theta} < 1 - \varepsilon$ for some $\varepsilon > 0$.
  (c) 23 Sept. 2024
(7) p. 340, line 17 from above, the proof of Theorem 309
  (a) $G \in \mathcal{G}(\mathcal{Y}_0, \mathcal{Y}_1)$
  (b) $G \in \mathcal{G}(\mathcal{X}_0, \mathcal{X}_1)$
  (c) 23 Sept. 2024
(8) p. 340, line 20 from above, the proof of Theorem 309
  (a) equal to 1 so that
  (b) equal to $1 + \varepsilon$ so that
  (c) 23 Sept. 2024
(9) p. 340, line 21 from above
  (a) $\mathcal{Y}_0^*$
  (b) $\mathcal{Y}_0{}^*$
  (c) 12 Dec. 2024
(10) p. 340, line 21 from above
  (a) $\mathcal{Y}_1^*$
  (b) $\mathcal{Y}_1{}^*$
  (c) 12 Dec. 2024
(11) p. 340, line 22 from above, the proof of Theorem 309
  (a) $y^*(\theta)(x)$
  (b) $y^*(x)$
  (c) 23 Sept. 2024
(12) p. 340, line 23 from above, the proof of Theorem 309
  (a) If we use the Poisson integral expression and the change of variables, then we obtain Theorem 309.
  (b) To conclude the proof, we need to show $|y^*(x)| \le 1$. To this end, we have only to prove $|g_j(z)| \le 1$ for all $z \in \bar{S}$. Instead of using g_j, we can consider $2^{j+k}(G^*(z + 2^{-j}i)(G(z+2^{-k}i) - G(z)) - G^*(z)(G(z+2^{-k}i) - G(z)))$ and we have only to show

that $|2^{j+k}(G^*(z+2^{-j}i)(G(z+2^{-k}i)-G(z))-G^*(z)(G(z+2^{-k}i)-G(z)))| \le 1$ for all $z \in \bar{S}$. Thus, by the three-line lemma we may let $z \in \partial S$. In this case, using $\|G\|_{\mathcal{G}(\mathcal{X}_0,\mathcal{X}_1)} < 1-\varepsilon$ and $\|G^*\|_{\mathcal{G}((\mathcal{Y}_0)^*,(\mathcal{Y}_1)^*)} < 1+\varepsilon$, we obtain $|2^{j+k}(G^*(z+2^{-j}i)(G(z+2^{-k}i)-G(z))-G^*(z)(G(z+2^{-k}i)-G(z)))| \le 1-\varepsilon^2$. Thus, we obtain Theorem 309.
   (c) 23 Sept. 2024

(13) p. 340, line 9 from below, three lines above Theorem 310
   (a) ball Banach spaces
   (b) complex ball Banach spaces
   (c) 12 Dec. 2024

(14) p. 340 line 5 from below, Theorem 310
   (a) Banach function spaces
   (b) complex Banach function spaces over $(X, \mathcal{B}, \mu)$
   (c) 8 Dec. 2024

(15) p. 341 lines 13–14 from below, the proof of Lemma 311
   (a) Take out

   Define $F_0 \equiv F\chi_E$, $F_1 \equiv F - F_0$, $G_0 \equiv G\chi_E$ and $G_1 \equiv G - G_0$.

   right after (19.26).
   (b) 8 Dec. 2024

(16) p. 341 line 12 from below, (19.27), the proof of Lemma 311
   (a) $\left\|\frac{G_k(z+w)-G_k(z)}{w} - F_k(z)\right\|_{E_0(\mu)} \le C_\varepsilon|w|\,\|f\|_{E_k(\mu)} \le C_\varepsilon|w|$
   (b) $\left\|\frac{G_k(z+w)-G_k(z)}{w} - F_k(z)\right\|_{E_k(\mu)} \le C_\varepsilon|w|\,\|f_k\|_{E_k(\mu)}$
   (c) 8 Dec. 2024

(17) p. 341 line 8 from below, the proof of Lemma 311
   (a) By using
   (b) Let $t_1, t_2 \in \mathbb{R}$ and $j = 0.1$. By using
   (c) 8 Dec. 2024

(18) p. 341 line 6 from below, the proof of Lemma 311
   (a) $\|f\|_{E_j(\mu)}$
   (b) $\|f_j\|_{E_j(\mu)}$
   (c) 8 Dec. 2024

(19) p. 341, line 4 from below
   (a) three-line theorem
   (b) three-line lemma
   (c) 21 Aug. 2024

(20) p. 341 line 2 from below, Lemma 312
   (a) spaces,
   (b) spaces over a measure space $(X, \mathcal{B}, \mu)$,
   (c) 8 Dec. 2024

(21) p. 341 line 1 from below, Lemma 312
   (a) $\sup_{j=0,1}\sup_{z\in j+i\mathbb{R}}\|F(z)\|_{E_j(\mu)}$
   (b) $\|F\|_{\mathcal{F}(E_0(\mu),E_1(\mu))}$
   (c) 17 Dec. 2024

(22) p. 342, line 1 from above, the proof of Lemma 312
   (a) We claim
   (b) Let μ_0, μ_1 be defined as in (19.13). We claim
   (c) 8 Dec. 2024

(23) p. 342, line 9 from above, the proof of Lemma 312
   (a) bounded
   (b) μ-

   (c) 23 Sept. 2024
(24) p. 342, line 10 from above, the proof of Lemma 312
   (a) dx
   (b) d$\mu(x)$
   (c) 23 Sept. 2024
(25) p. 342 line 13 from above, the proof of Lemma 312
   (a) Lemma 300
   (b) Theorem 301
   (c) 19 Dec. 2024
(26) p. 342, line 15 from above, the proof of Lemma 312 (twice)
   (a) dx
   (b) d$\mu(x)$
   (c) 23 Sept. 2024
(27) p. 342, line 16 from above, the proof of Lemma 312
   (a) add d$\mu(x)$ between large) and large }.
   (b) 23 Sept. 2024
(28) p. 342, line 16 from above, the proof of Lemma 312
   (a) $F_E(1+it;x)$
   (b) $F(1+it;x)$
   (c) 23 Sept. 2024
(29) p. 342, line 17 from above, the proof of Lemma 312
   (a) follows
   (b) follows since E is arbitrary and $a^{1-\theta}b^\theta \le (1-\theta)a+\theta b$ for all $a,b \ge 0$.
   (c) 8 Dec. 2024
(30) p. 342 lines 3 and 2 from below, (19.28), Example 103
   (a) $\sup\limits_{j=0,1}$
   (b) $\max\limits_{j=0,1}$
   (c) 17 Dec. 2024
(31) p. 343, line 17 from above, Remark 14
   (a) Banach spaces
   (b) complex Banach spaces
   (c) 12 Dec. 2024
(32) p. 343, line 12 from below, Example 104
   (a) $L^p(\mu) = L^{p_0}(\mu)^{1-\theta}L^{p_1}(\mu)^\theta$
   (b) $L^p(\mu) = [L^{p_0}(\mu), L^{p_1}(\mu)]_\theta$
   (c) 17 Dec. 2024
(33) p. 345, line 4 from above
   (a) Banach spaces
   (b) complex Banach spaces
   (c) 12 Dec. 2024
(34) p. 345, line 11 from above, Example 105
   (a) Then if
   (b) If
   (c) 19 Dec. 2024
(35) p. 345, line 12 from above, Example 105
   (a) $E_Q^k \equiv L^q(Q)$
   (b) $E_Q^k \equiv L^{q_k}(Q)$
   (c) 12 Dec. 2024
(36) p. 345, line 11 from above, Example 105
   (a) Then if
   (b) If

(c) 12 Dec. 2024

(37) p. 345, line 15 from below, Theorem 313
  (a) Banach spaces
  (b) complex Banach spaces
  (c) 12 Dec. 2024

(38) p. 345, line 12 from above, Example 105
  (a) $L^q(Q)$
  (b) $L^{q_k}(Q)$
  (c) 12 Dec. 2024

(39) p. 345, line 7 from below, the proof of Theorem 313
  (a) $|f| \le |f^0|^{1-\theta}|f_1|^\theta$
  (b) $|f| \le |f^0|^{1-\theta}|f^1|^\theta$
  (c) 12 Dec. 2024

(40) p. 345, line 1 from below
  (a) $k \in \mathbb{N}$
  (b) $k = 0, 1$
  (c) 17 Dec. 2024

(41) p. 346, line 2 from above
  (a) Let $\varphi : (0,\infty) \to (0,\infty)$ be a function and $1 \le q < \infty$.
  (b) Let $\varphi : (0,\infty) \to (0,\infty)$ be a function and $1 \le q < \infty$. Here to understand the situation more deeply, we consider generalized Morrey spaces.
  (c) 23 Sept. 2024

(42) p. 346 lines 12–13 from above, Theorem 314
  (a)
  (b) Suppose that $\theta, q_0, q_1, q, \varphi_0, \varphi_1$ and φ are the same as in Lemma **??**.
  (c) Let $\theta \in (0,1)$, $1 \le q_0 < \infty$, $1 \le q_1 < \infty$, $\varphi_0 \in \mathcal{G}_{q_0}$ and $\varphi_1 \in \mathcal{G}_{q_1}$. Assume $q_0 \ne q_1$ and ${\varphi_0}^{q_0} = {\varphi_1}^{q_1}$.
  (d) 19 Dec. 2024

(43) p. 346, lines 15 and 16 from above
  (a) $\overline{\mathcal{M}}^\varphi_q$
  (b) $\overline{\mathcal{M}}^\varphi_q$
  (c) 17 Dec. 2024

(44) p. 346, line 16 from above
  (a) $\overline{\mathcal{M}}^p_q$
  (b) $\overline{\mathcal{M}}^p_q$
  (c) 17 Dec. 2024

(45) p. 347, line 2 from above, Theorem 316
  (a) Assume $q_0 \ne q_1$.
  (b) Assume $q_0 \ne q_1$ and ${\varphi_0}^{q_0} = {\varphi_1}^{q_1}$.
  (c) 18 Dec. 2024

(46) p. 347, line 5 from below, (19.7), the proof of Theorem 316
  (a) $|f - g_\varepsilon|$
  (b) $|f - \chi_{\{a \le |f|\}} g_\varepsilon|$
  (c) 12 Dec. 2024

(47) p. 348, line 5 from above, (19.10), the proof of Theorem 316
  (a) $\|\chi_{[0,a)}(|f|)f\|_{\mathcal{M}^\varphi_q} \le a^{\frac{q-q_1}{q}} \|f\|_{\mathcal{M}^{\varphi_1}_{q_1}}$
  (b) $\|\chi_{[0,a)}(|g|)g\|_{\mathcal{M}^\varphi_q} \le a^{\frac{q-q_1}{q}} \|g\|_{\mathcal{M}^{\varphi_1}_{q_1}}{}^{\frac{q_1}{q}}$
  (c) 12 Dec. 2024

1.36. **Pages 350–359.**

(1) p. 350, line 2 from above, the proof of Lemma 319
  (a) $f = \chi_{\{a<|f|<a^{-1}\}} f$
  (b) $f = \chi_{\{a\leq|f|\leq a^{-1}\}} f$
  (c) 12 Dec. 2024
(2) p. 350 line 11 from below, (19.13), the proof of Lemma 319
  (a) $\left(\frac{q}{q_1} - \frac{q}{q_0}\right)$
  (b) $\max\left(\frac{q}{q_0}, \frac{q}{q_1}\right)$
  (c) 18 Dec. 2024
(3) p. 350, line 10 from below
  (a) mean value theorem
  (b) fundamental theorem on calculus
  (c) 23 Sept. 2024
(4) p. 350, line 6 from below, the proof of Lemma 319
  (a) $\|\chi_{\{a\leq|f|\leq 1\}} + \chi_{(1,A]}(|f|)\|$
  (b) $\|\chi_{[a,1]}(|f|) + \chi_{(1,A]}(|f|)\|$
  (c) 12 Dec. 2024
(5) p. 350, line 1 from below, the proof of Lemma 319
  (a) $|f|^{\frac{q}{q_j}} = A^{\frac{q}{q_j}} \chi_{[a,A]}(|f|)$
  (b) $|f|^{\frac{q}{q_j}} \leq A^{\frac{q}{q_j}} \chi_{[a,A]}(|f|)$
  (c) 12 Dec. 2024
(6) p. 351 line 4 from above, the proof of Lemma 319
  (a) $\left(\frac{q}{q_1} - \frac{q}{q_0}\right)$
  (b) $\max\left(\frac{q}{q_0}, \frac{q}{q_1}\right)$
  (c) 18 Dec. 2024
(7) p. 351, line 7 from above, the proof of Lemma 319
  (a) $F(\theta) = f$
  (b) $F(\theta) = {f_0}^{1-\theta} {f_1}^{\theta}$
  (c) 12 Dec. 2024
(8) p. 351, line 15 from below, Corollary 320
  (a) $\chi_{[0,a]}$
  (b) $\chi_{[0,a)}$
  (c) 12 Dec. 2024
(9) p. 351, line 1 from below, (19.14), the proof of Lemma 321
  (a) $|f|^{\frac{qz}{q_0}+\frac{q(1-z)}{q_1}}$
  (b) $|f|^{\frac{q(1-z)}{q_0}+\frac{qz}{q_1}}$
  (c) 12 Dec. 2024
(10) p. 352, line 14 from below
  (a) paremeters
  (b) parameters and the functions φ, φ_0 and φ_1
  (c) 23 Sept. 2024
(11) p. 354, line 11 from above, the proof of Lemma 323
  (a) $\varphi^p = {\varphi_0}^{1_0} = {\varphi_1}^{q_1}$
  (b) $\varphi^q = {\varphi_0}^{q_0} = {\varphi_1}^{q_1}$
  (c) 12 Dec. 2024
(12) p. 355 lline 4 from above, Lemma 324

   (a) *304*
   (b) 304
   (c) 25 Dec. 2024
(13) Page 355 line 8 from below,
   (a) Lemma 19.20
   (b) Lemma 304
   (c) 19 Dec. 2024
(14) p. 355, line 2 from below
   (a) We should have added "Assume $q_0 \neq q_1$." in Corollary 326.
   (b) 12 Dec. 2024
(15) p. 356, line 5 from above, Theorem 327
   (a) Suppose that we have parameters p_0, q_0, p_1, q_1, p, and $q \in [1, \infty)$ and $\theta \in (0, 1)$ satisfying

$$p_0 \neq p_1, \quad 1 \leq q_0 \leq p_0 < \infty, \quad 1 \leq q_1 \leq p_1 < \infty, \quad 1 \leq q \leq p < \infty$$

   and

$$\frac{q_0}{p_0} = \frac{q_1}{p_1}, \quad \frac{1}{p} = \frac{1-\theta}{p_0} + \frac{\theta}{p_1}, \quad \frac{1}{q} = \frac{1-\theta}{q_0} + \frac{\theta}{q_1}$$

   (b) Suppose that we have parameters q_0, q_1, and $q \in [1, \infty)$ and $\theta \in (0, 1)$ and functions

$$\varphi_0 \in \mathcal{G}_{q_0}, \quad \varphi_1 \in \mathcal{G}_{q_1}$$

   satisfying

$$\varphi_0{}^{q_0} = \varphi_1{}^{q_1}, \quad \varphi = \varphi_0{}^{1-\theta} \varphi_1{}^{\theta}, \quad \frac{1}{q} = \frac{1-\theta}{q_0} + \frac{\theta}{q_1}, \quad q_0 \neq q_1.$$

   (c) 12 Dec. 2024
(16) p. 356 line 14 from above, two lines above Theorem 328
   (a) the sets of of
   (b) the sets of
   (c) 17 Dec. 2024
(17) p. 356, line 16 from above, Theorem 328
   (a) Suppose that we have parameters p_0, q_0, p_1, q_1, p and $q \in [1, \infty)$ and $\theta \in (0, 1)$
   (b) Suppose that we have parameters q_0, q_1, and $q \in [1, \infty)$ and $\theta \in (0, 1)$ and functions $\varphi_0, \varphi_1, \varphi$
   (c) 12 Dec. 2024
(18) p. 356, line 12 from above
   (a) lattic
   (b) lattice
   (c) 23 Sept. 2024
(19) p. 356, line 18 from above,
   (a) $\mathcal{M}^p_q$
   (b) $\mathcal{M}^\varphi_q$
   (c) 23 Sept. 2024
(20) p. 356, lines 12, 8 and 5 from below, (19.18), (19.19), Lemma 320
   (a) $\widetilde{\mathcal{M}}^p_q$
   (b) $\widetilde{\mathcal{M}}^\varphi_q$
   (c) 12 Dec. 2024
(21) p. 356, line 5 from below, (19.19), Lemma 320
   (a) $\mathcal{M}^{\varphi_1 *}_{q_1}(\mathbb{R}^n)$
   (b) $\overset{*}{\mathcal{M}}{}^{\varphi_1}_{q_1}(\mathbb{R}^n)$
   (c) 23 Sept. 2024
(22) p. 357, line 9 from below

(a) $\sup_{t\in\mathbb{R}}\|e^{-(\theta+it)^2}\chi_{A_R}F(it)\|_{\mathcal{M}^{\varphi_0}_{q_0}}+\sup_{t\in\mathbb{R}}\|e^{-(1-\theta+it)^2}\chi_{A_R}F(1+it)\|_{\mathcal{M}^{\varphi_1}_{q_1}}$

(b) $\left(\sup_{t\in\mathbb{R}}\|e^{-(\theta+it)^2}\chi_{A_R}F(it)\|_{\mathcal{M}^{\varphi_0}_{q_0}}\right)^{1-\theta}\left(\sup_{t\in\mathbb{R}}\|e^{-(1-\theta+it)^2}\chi_{A_R}F(1+it)\|_{\mathcal{M}^{\varphi_1}_{q_1}}\right)^{\theta}$

(c) 23 Sept. 2024

(23) p. 357, line 8 from below
 (a) N, N_0, N_1
 (b) N, N'
 (c) 23 Sept. 2024

(24) p. 357, lines 5 and 6 from below
 (a) $e\sup_{\ell\in[-N,N],N't\in\mathbb{Z}}|e^{-(\theta+it)^2}\chi_{A_R}F(it)\|_{\mathcal{M}^{\varphi_0}_{q_0}}+e\sup_{\ell\in[-N,N],N't\in\mathbb{Z}}\|e^{-(1-\theta+it)^2}\chi_{A_R}F(1+it)\|_{\mathcal{M}^{\varphi_1}_{q_1}}$
 (b)
$$\left(e\sup_{\ell\in[-N,N],N't\in\mathbb{Z}}\|e^{-(\theta+it)^2}\chi_{A_R}F(it)\|_{\mathcal{M}^{\varphi_0}_{q_0}}\right)^{1-\theta}$$
$$\times\left(e\sup_{\ell\in[-N,N],N't\in\mathbb{Z}}\|e^{-(1-\theta+it)^2}\chi_{A_R}F(1+it)\|_{\mathcal{M}^{\varphi_1}_{q_1}}\right)^{\theta}$$
 (c) 23 Sept. 2024

(25) p. 357, lines 3 and 4 from below

(26) take off the description of $G_1(t)$ as well as $F(1+it)\in\overset{*}{\mathcal{M}}{}^{\varphi_1}_{q_1}(\mathbb{R}^n)$ since there does not exist such a decomposition.

(27) p. 357, lines 1 and 2 from below
 (a) $e\sup_{\ell\in[-N,N],N't\in\mathbb{Z}}|e^{-(\theta+it)^2}\chi_{A_R}G_0(t)\|_{\mathcal{M}^{\varphi_0}_{q_0}}$
 $+e\sup_{\ell\in[-N,N],N't\in\mathbb{Z}}\|e^{-(1-\theta+it)^2}\chi_{A_R}G_1(t)\|_{\mathcal{M}^{\varphi_1}_{q_1}}$
 (b) $\left(e\sup_{\ell\in[-N,N],N't\in\mathbb{Z}}|e^{-(\theta+it)^2}\chi_{A_R}G_0(t)\|_{\mathcal{M}^{\varphi_0}_{q_0}}\right)^{1-\theta}$
 $\times\left(e\sup_{\ell\in[-N,N],N't\in\mathbb{Z}}\|e^{-(1-\theta+it)^2}\chi_{A_R}F_1(1+it)\|_{\mathcal{M}^{\varphi_1}_{q_1}}\right)^{\theta}$
 (c) 23 Sept. 2024

(28) Replace p. 358 lines 1–7 with “Note that $A_R\cap\operatorname{supp}(G_0)=\emptyset$”.
 (a) 9 Dec. 2024
 (b) 23 Sept. 2024

(29) p. 358, line 12 from below, the proof of Lemma 330
 (a) $\widetilde{\mathcal{M}}^{\varphi_0}_{q_0(\mathbb{R}^n)}$
 (b) $\widetilde{\mathcal{M}}^{\varphi_0}_{q_0}(\mathbb{R}^n)$
 (c) 22 Dec. 2024

(30) p. 359, line 10 from below
 (a) exhausitively
 (b) exhaustively
 (c) 26 Sept. 2023

(31) p. 359, line 1 from below, Section 19.1.2
 (a) see See
 (b) see
 (c) 22 Dec. 2024

1.37. **Pages 360–369.**

(1) p. 360, line 3 from above, Section 19.1.3
   (a) Gustavsson, Peetre, Berezhnoi, Shestakov, Nilsson [359] and Ovchinikov
   (b) Including Nilsson **, Gustavsson, Peetre, Berezhnoi, Shestakov, and Ovchinnikov
   (c) 12 Dec. 2024
(2) p. 360, line 10 from above, Section 19.1.3
   (a) three-line theorem
   (b) three-line lemma
   (c) 21 Aug. 2024
(3) p. 361, lines 17 and 18 from above, §19.2.1
   (a) B_u^w
   (b) B_w^u
   (c) 17 Dec. 2024
(4) p. 362, line 12 from above, Section 19.2.1
   (a) Ruiz, Vega and Blasco
   (b) Blasco, Ruiz and Vega
   (c) 12 Dec. 2024
(5) p. 362, line 16 from above, Section 19.2.1
   (a) $\mathcal{M}_u^p$
   (b) $\mathcal{M}_q^p$
   (c) 12 Dec. 2024
(6) p. 362, line 17 from above, Section 19.2.1
   (a) Lebegsue
   (b) Lebesgue
   (c) 12 Dec. 2024
(7) p. 362, line 19 from above
   (a) Yuan and Yang
   (b) Yang and Yuan
   (c) 17 Dec. 2024
(8) We should have swapped comments in Section 19.2.1 and 19.2.2, 12 Dec. 2024
(9) p. 363, line 1 from above, §19.2.3
   (a) in the right-hand
   (b) on the right-hand
   (c) 30 Sept. 2024
(10) p. 365, [3]
   (a) Departement
   (b) Department
   (c) 12 Dec. 2024
(11) p. 365, [3]
   (a) Umea
   (b) Umeå
   (c) 12 Dec. 2024
(12) p. 366, [18]
   (a) J. Cerda
   (b) J. Cerdà
   (c) 12 Dec. 2024
(13) p. 366, [21]
   (a) F. Gurbuz
   (b) F. Gürbüz
   (c) 12 Dec. 2024
(14) p. 366, [22]

  (a) J.A. Barcelo
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