Nonlinear Modelling of Labour-employment and Inference by a Simulation test

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1 Introduction

This paper presents a computer-intensive empirical analysis for time-series data of employment level in Japanese manufacturing industry based on the Lambert disequibibrium model which can incorporate the short-side determination between demand and supply as well as frictional unemployment [see Lambert (1988) and as for literature dealing with disequilibrium analysis, see Fair and Jaffee (1972), Maddala and Nelson (1974), Quandt (1982), Ito (1985) and Quandt (1988)].

Econometric studies of Japanese Labour Market based on macroeconomic theory can be seen in Ueda and Yoshikawa(1984) Otake(1988) Teruyama(1993). Ueda and Yoshikawa survey the literature of the labour market from the view of the relations of the macroeconomic theory and empirical Japanese labour market, Japanese supply-side economics, and the dual labour market hypothesis. They maintain that research in respect of the relation of the macroeconomic theory and empirical Japanese labour market has been meagre. Otake compares the labour markets of Japan, U.S., United Kingdom, West Germany, and France by means of a labour demand-supply econometric model, concluding that the real wage of Japan is not elastistic . Teruyama gives an econometric analysis of the Japanese labour markets viewed from enterprise scale on the basis of the Lambert type disequilibrium econometric model as well as VAR model and presents the labor supply and demand function estimation.

This paper extends the Lambert model originally given in a static framework to a time-series model in order to allow for serial dependency in employment fluctuation [see Lambert (1988); see also Teruyama (1993) and Miyakoshi and Tsukuda (1996) who apply the Lambert-type approach]. The extension produces a nonlinear time-series model, the observation series of interest constitutes a nonhomogeneous Markov clain so that mixing properties of innovation process do not transfer to the series in concern. Therefore, in respect of statistical inference, conventional asymptotic test approach based on limiting properties of test statistic does not seem readily available. We evaluate instead p-value by means of simulated data generated under the estimated null and alternative hypotheses. For the purpose of numerical computation of maximun-likelihood estimators, we apply the DUD algothism proposed by Ralston and Jennrich(1978), since the parameter space of our model contains singular points [see also Dennis and Schnabel (1983) for a survey of nonlinear optimization algorithm]. We propose a new convergence criterion based on the performance of the sum of residual in minimizing procedure.

The numerical computatims for this paper were programned by Fortran and conducted by the super-computer SX-4/128H4 of Tohoku University Information Synergy Center.

2 The Lambert model of labour employment

Let l_d and l_s denote labour demand of and supply to a firm and suppose that the employment level l which is realized for the firm is given by

$$l = \min(l_d, l_s). \qquad (1)$$

Consider then a labour market which consists of N firms and let L be their total employment and the total labour demand and supply be LD and LS respectively. Now suppose that the distribution of (l_d, l_s) is able to be approximated by a continuous distribution which has a density function $f(l_d, l_s)$ so that we have the relationships

$$\int_0^1 \int_0^1 f(l_d, l_s) dl_d dl_s = 1,$$

$$LD = \int_0^\infty \int_0^\infty N l_d f(l_d, l_s) dl_d dl_s,$$

$$LS = \int_0^\infty \int_0^\infty N l_s f(l_d, l_s) dl_d dl_s.$$

In view of the assmption (1) of short-side determination of employment level, the realized total employment level is given by

$$L = \int_0^\infty \left[\int_{l_d}^\infty N l_d f(l_d, l_s) dl_s \right] dl_d + \int_0^\infty \left[\int_{l_s}^\infty N l_s f(l_d, l_s) dl_d \right] dl_s, \tag{2}$$

whence we have the relationship

$$L \leq \min(LD, LS).$$

Suppose then that l_{d_i} and l_{s_i} for firm i has stochastic representation:

$$\log(l_{d_i}) = \mu_d + \varepsilon_{d_i} \quad \text{and} \quad \log(l_{s_i}) = \mu_s + \varepsilon_{s_i}$$

where μ_d, μ_s are positive constants, $(\varepsilon_{d_i}, \varepsilon_{s_i}), i = 1, \cdots, N,$ are independently normally distributed such that

$$\begin{pmatrix} \varepsilon_{d_i} \\ \varepsilon_{s_i} \end{pmatrix} \sim N \left[\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma_{dd} & \sigma_{sd} \\ \sigma_{ds} & \sigma_{ss} \end{pmatrix} \right]. \tag{3}$$

In tems of aggregated demand LD and supply LS, we have

$$\log(LD) = \log(N l_d) = \mu_d + \log N + \varepsilon_d,$$

$$\log(LS) = \log(N l_s) = \mu_s + \log N + \varepsilon_s.$$

The equation (2) is expressed as

$$L = \int_0^\infty \left[\int_{\mu_s-\mu_d+\varepsilon_s}^\infty \exp(\mu_s + \log N + \varepsilon_s) n(\varepsilon_d, \varepsilon_s) d\varepsilon_s\right] d\varepsilon_d + \int_0^\infty \left[\int_{\mu_d-\mu_s+\varepsilon_d}^\infty \exp(\mu_d + \log N + \varepsilon_d) n(\varepsilon_d, \varepsilon_s) d\varepsilon_d\right] d\varepsilon_s \quad (4)$$

where $n(\varepsilon_d, \varepsilon_s)$ denotes the Gaussian density function for (3). Then Lambert (1988) showed that (4) is given by an approximated form

$$L = [LD^{-\rho} + LS^{-\rho}]^{-1/\rho}. \quad (5)$$

The parameter ρ is positive and related to the degree of frictional unemployment in such way that larger ρ indicate lesser frictional unemployment so $\rho = \infty$ implies no frictional unemployment; namely, we have

$$\lim_{\rho\to\infty}[LD^{-\rho} + LS^{-\rho}]^{-1/\rho} = \min(LD, LS).$$

In this paper we employ the model of employment in the Japanese manufacturing industry which is constituted of the following set of equations:

$$\log L_t = -\frac{1}{\rho}\log[LD_t^{-\rho} + LS_t^{-\rho}] + u_t, \quad (6)$$

$$LD_t = d_1 + d_2 PI_t + d_3(Y/WPI)_t, \quad (7)$$

$$LS_t = s_1 + s_2(Y/CPI)_t + s_3 L_{t-1} \quad (8)$$

where the $u_t, t = 1, \cdots, T$, are i.i.d.$N(0, \sigma^2)$ random variables; PI_t, WPI_t and CPI_t indicate production index of the manufacturing industry, wholesale price index and consumer price respectively and these three are assumed exogenous variables. Note that the number of employees L_t is observable whereas LD_t and LS_t are unobservable latent variables. As for observation sets, we use monthly data over the period January, 1983 to Decenber, 1995 for one case, and January, 1983 to December, 1990 for the other.The data source for L is the number of employees in the industry of manufacturing in Monthly Report on the Labour Force Survey by Statistics Bureau; Management and Coordination Agency.

Y denotes contractual cash earning in total cash earnings for scheduled hours of the manufacturing industry. The data sets for contractual cash earning, total cash earning, and scheduled work hoursd are due to Policy Planning and Research Department, Ministers Secretariat, Ministry of Labour. The WPI denotes overall wholesale price index and the data set is due to Research and Statistics Department of Bank of Japan. The CPI denotes consumer price index of general index (not excluding fresh food) by Statistics Bureau; Management and Coordination Agency. The variate PI denotes index of industrial production (production weights) of manufacturing industry (with 1990 year base) by Research and Statistics Department; Minister's Secretariat; Ministry of International Trade and Industry.

3 Estimation procedure

Suppose that the number of observations is T; then, the log-likelihood based on the model (6), (7), (8) is given by

$$l_T(\rho, d, s, d^2)$$
$$= -\left(\frac{T}{2}\right)\log(2\pi) - \left(\frac{T}{2}\right)\log\sigma^2 - \frac{\sum_{t=1}^{T}\{\log L_t - \rho^{-1}\log(LD_t^{-\rho} + LS_t^{-\rho})\}^2}{2\sigma^2}$$

where LD_t and LS_t are determined by (7) and (8) respectively, and the initial value L_0 is assumed to be a given constant. For the sake of simplicity, set $y_t = \log L_t, \theta \equiv (\rho, d', s')'$ and set $g_t(\theta) = \rho^{-1}\log(LD_t^{-\rho} + LS_t^{-\rho})$. Then the maximum likelihood (ML) estimator of θ is the minimizor of

$$Q(\theta) \equiv \sum_{t=1}^{T}(y_t - g_t(\theta))^2 \tag{9}$$

and denoted by $\tilde{\theta}$; the ML estimator of σ^2 is given by $\tilde{\sigma}^2 = Q(\tilde{\theta})/T$.

In dealing with the minimization of (9), we employ the DUD (doesn't use derivative) by Ralston and Jennrich (1978) which is an algorithm for minimization of $Q(\theta)$ without use of differenciation. Let k be the dimension of θ, namely $k = 7$ in our case; then the algorithm proceeds as this:

- Choose a set of $(k+1)$ initial values of θ; say, $\theta^{(1)}, \cdots, \theta^{(k+1)}$ which are chosen as the first $(k+1)$ minimizors of $Q(\theta)$ $(Q(\theta^{(1)}) \geq Q(\theta^{(2)}) \geq \cdots \geq Q(\theta^{(k+1)}))$ searched for lattice points.

- Let Δg_t the k-vector whose j-th element is $g_t(\theta^{(j)}) - g_t(\theta^{(k+1)})$ and minimize,

$$\bar{Q}(\alpha) = \sum_{t=1}^{T} [y_t - \{g_t(\theta^{(k+1)}) + (\Delta g_t)'\alpha\}]^2$$

  with respect to k-vector α.

- Let $\Delta\Theta$ be the $k \times k$ matrix whose i-th column is equal to $\theta^{(i)} - \theta^{(k+1)}$, and set

$$\theta^{(k+2)} = \theta^{(k+1)} + \Delta\Theta\alpha.$$

- Setting $\theta^{(2)}, \cdots, \theta^{(k+2)}$ as the new $(k+1)$ initial values, repeat the preceding steps.

The iteration is regarded as converged when two criteria are satisfied: One is that we reach at $\theta^{(k+1)}$ for which

$$\frac{|Q(\theta^{(K+1)}) - Q(\theta^{(K)})|}{Q(\theta^{(K)})} < \varepsilon$$

for a small ε. The other is that the sum of the residuals

$$\sum_{t=1}^{T} \{y_t - g_t(\theta^{(K+1)})\} \tag{10}$$

is regarded as sufficiently small. For a linear regression model with intercept, if $\theta^{(K+1)}$ is the minimizor of (9), the sum (10) equals 0 evidently, but it does not necessarily hold in case of nonlinear regression. But for the model (6)-(8), the right-hand side of (6) is represented as

$$\begin{aligned} g_t(\theta) &= -\frac{1}{\rho} \log[LD_t^{-\rho} + LS_t^{-\rho}] \\ &= \log d_1 + \log \left[\left\{ 1 + \frac{d_2}{d_1} PI_t + \frac{d_3}{d_1} \left(\frac{Y}{WPI} \right)_t \right\}^{-\rho} \right. \\ &\quad \left. + \left\{ \frac{s_1}{d_1} + \frac{s_2}{d_1} \left(\frac{Y}{CPI} \right)_t + \frac{s_3}{d_1} L_{t-1} \right\}^{-\rho} \right]^{-1/\rho}, \end{aligned}$$

which is representable after reparametrisation as

$$d_1^* + \log \left[\left\{ 1 + d_2^* PI_t + d_3^* \left(\frac{Y}{WPI} \right)_t \right\}^{-\rho} + \left\{ s_1^* + s_2^* \left(\frac{Y}{CPI} \right)_t + s_3^* L_{t-1} \right\}^{-\rho} \right]^{-\rho}.$$

Set $\theta^* = (\rho, d^*, s^*)$; then minimization of $Q(\theta^*)$ with respect to d_1^* produces the first-order condition

$$\sum_{t=1}^{T} \{ y_t - g_t(\theta^*) \} = 0$$

which is representable in terms of the original parameter, as

$$\sum_{t=1}^{T} \{ y_t - g_t(\theta) \} = 0$$

for a minimizing θ. Hence follows the condition (10).

In application of the DUD procedure to our least-square problem (9) for instance, we must prepare 8 initial sets of ρ and the LD and LS coefficient values. Although it is desirable that those initial values are consistent estimates, there does not seem feasible consistent estimates available in our nonlinear set-up. So, insead, we choose those initial values by lattice-point search. As for coefficient values, we determine the range of the lattice-points in which we look for the initial values as follows:

(1) Regress L on LD and LS respectively and estimate the coefficients by the OLS method.

(2) Frame the lattice so as to locate the estimated coefficients in the center.

(3) Delimit the range of the lattice-points around the estimated coefficients by setting the number of lattice lines to be 7 to 17 such that each of the coefficients satisfies $0 < LS, LD < 2L$.

For each of the lattice-points, the sum of square (9) is evaluated and first 8 minimizors are chosen as the initial values. This procedure of lattice search requires a lot of computation time to deal with and so we used the Super-Computer SX-4/128H4.

4 Testing the presence of frictional unemployment

In the framwork of our disequilibrium model, $\rho = +\infty$ implies that the employment is determined by the short side between demand and supply, whereas $\rho < \infty$ implies the existence of frictional unemployment. So we set the null hypothesis H_0 as $\rho = +\infty$ while the alternative H_1 is that $\rho < \infty$. The parameter of interest is then ρ and the rest constitute nuisance parameters and so denote $\eta = (d', s')'$ so that $\theta = (\rho, \eta')'$. Denote by $\hat{\eta}, \hat{\sigma}^2$ the ML estimators of η, σ^2 under H_0 and denote by $\tilde{\theta} = (\tilde{\rho}, \tilde{\eta}')'$ the ML estimator of θ under H_1. The test is then conducted by the following procedure:

- Generate $(\log L_t)^*, t = 1, \cdots, T$, according to (6) by setting $\rho = \infty, \eta = \hat{\eta}$ by means of i.i.d. normal random numbers $(0, \hat{\sigma}^2), u_1^*, \cdots, u_T^*$.
- Estimate $\tilde{\rho}^*$ based on the simulated data set $(\log L_t)^*, t = 1, \cdots, T$ generated by the null model with $(\hat{\eta}, \hat{\sigma}^2)$ as its parameters.
- Repeat the generation of $\tilde{\rho}^*$ N times and estimate the p-value by $\sharp(\tilde{\rho}^* \leq \tilde{\rho})/N$.

In order to see whether this simulation-based test has sufficient power to discern H_1 from H_0, we generate simulated values $\tilde{\rho}^*$ under H_1.

The test results are summarized as follows:

- The estimation results are exhibited in Table 1. As far as sign condition of the estimated coefficients is concerned, we find that all estimated coefficients are appropriately estimated. By comparing the residual sums of squares under the null and alternative hypothesis, we can see that residual sum of squares under the alternative hypothesis is smaller than that of null hypothesis in both case 1 and 2. Hence our estimation procedure itself does not seem to involve serious problem.
- The distributions of $\tilde{\rho}^*$ under the null and alternative hypotheses are very distinctly discerned as are seen in Figures 1 and 2. The test seems to have reasonable power.

- In both case 1 and 2, the p-values with alternative modifications are all greater than 0.1. Those results imply that the null hypothesis of non-existence of the frictional unemployment cannot be reject under 5 per cent significant level. Namely, they indicate that significant frictional unemployment is not indicated in the observed Japanese manufacturing industry data sets. So the labour-employment level in the Japanese manufacturing industry seems determined by other market disequilibrium factors.

- In view of the Lambert model, if $LD = LS$ and $\rho < \infty$, the attained employment level is

$$[LD^{-\rho} + LS^{-\rho}]^{-1/\rho} = LD \cdot 2^{-1/\rho},$$

  whence the frictional unemployment rate for $LD = LS$ is given by

$$FUR = \frac{LD - LD \cdot 2^{-1/\rho}}{LD} = 1 - 2^{-1/\rho}.$$

  Our estimates of $\tilde{\rho}$ is around 11; if we set $\tilde{\rho} = 11$, the corresponding FUR estimate is about 0.06. The test result implies that this estimate is not regarded as significant.

5 Some Remarks

The paper dealt with a time-series version of Lambert's disequilibrium model of labour employment, and proposed a simulation-based test of the presence of frictional unemployment in the Japanese manufacturing industry. The test indicates significant frictional unemployment in the time-series data and also the test is shown to have sufficient discerning power.

The analysis, is based on the assumption that the wage rate is pretermined in the determination of employment level. That assumption is not quite pertinent, leaving the determination of the wage rate completely out of investigation. Incorporating explicitly the wage adjustment process into our model is a subject which remains open. For the purpose of modelling a joint process of employment and wage, a nonlinear error-correction model for the joint process of employment and wage rate would be another direction of investigation which might be regarded as a nonlinear reduced-form approach in contrast to Kunitomo and Sato (1996) who propose a nonlinear structual equation approach (see Johansen (1995) for linear error-correction models).

Another aspect of the paper which requires further scrutiny is how to take into account the sampling variation of the simulated p-value estimate

$$p(\hat{\eta}, \hat{\sigma}^2) = \sharp(\hat{\rho}^* \leq \tilde{\rho})/N$$

due to the use of the M.L. estimates $\hat{\eta}, \hat{\sigma}^2$ instead of the true η, σ^2. No pertinent asymptotic theory is relied upon in evaluating that variation in view of nonhomogenous character of our Markov chain, difficutly in checking conditions for stability, and the small number of available observations. In order to deal with this sample-variation problem, we propose a mixed use of a cross-validation method with the simulation method which was expounded in Section 4.

A possible subsample method is as follows: Delete one y_s at a time from the sample set $(y_1, \cdots, y_T)$ and apply the least-square estimation (9) to the remaining observations so that the sum of squared residuals is given by

$$\begin{aligned} R(\eta) &= \sum_{t=2}^{T}(y_t - f_t(\eta))^2 \quad \text{if} \quad s = 1 \\ &= \sum_{t=1, t\neq s, t\neq s+1}^{T}(y_t - f_t(\eta))^2 \quad \text{if} \quad s \neq 1, T, \\ &= \sum_{t=1}^{T-1}(y_t - f_t(\eta))^2 \quad \text{if} \quad s = T \end{aligned}$$

where $f_t(\eta) = \min(LD_t, LS_t)$. The subsample procedure produces T estimates of η and σ^2 which are denoted by $(\hat{\eta}_t^+, (\hat{\sigma}_t^2)^+), t = 1, \cdots, T$, where $(\hat{\sigma}_t^2)^+ = R(\hat{\eta}_t^+)/(T-1)$ if $t = 1, T$ and $(\hat{\sigma}_t^2)^+ = R(\hat{\eta}_t^+)/(T-2)$, otherwise. With $\tilde{\rho}$ fixed as estimated for the original sample, T p-values $p(\hat{\eta}_t^+, (\hat{\sigma}_t^2)^+), t = 1, \cdots, T$, are then produced by means of the Monte

Carlo method of Section 4. The empirical distribution of those T p-values gives us some information concerning the null distribution of $p(\tilde{\eta}, \tilde{\sigma}^2)$. We have alternative ways of p-value estimation:

(1) The median of $\{p(\hat{\eta}_t^+, (\hat{\sigma}_t^2)^+), t = 1, \cdots, T\}$.

(2) To be conservative, we might use the $(1-\alpha)$ percentile of the emprircal distribution, setting α equal to, say, 0.05 or 0.01.

(3) Unless the original $\tilde{\rho}$ belongs to the extreme tails of the empirical distribution, report $p(\hat{\eta}, \hat{\sigma}^2)$ together with $(1-\alpha)$ confidence-level interval constructed from the empirical distribution.

The methods (1) and (2) are point estimation whereas (3) presents a confidence statement.

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