Measuring Business Cycle Turning Points in Japan with a Dynamic Markov Switching Factor Model

Toshiaki Watanabe*
Department of Economics
Tokyo Metropolitan University
twatanab@bcomp.metro-u.ac.jp

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Abstract

In the dynamic factor model proposed by Stock and Watson (1989, 1991), a single unobserved factor common to some macroeconomic variables is defined as a composite index to measure business cycles. Kim and Nelson (1998) extend their model combining with the regime-switching model of Hamilton (1989) so that the mean growth rate of the index may vary depending on whether the economy is in the boom regime or in the recession regime. An advantage of the Kim and Nelson (1998) model is that estimating the model by a Bayesian method produces the posterior probabilities that the economy is in the recession regime, which can be used to date the business cycle turning points. This article estimates the Stock and Watson (1989,1991) and the Kim and Nelson (1998) models using some macroeconomic variables in Japan. The model comparison using Bayes factor does not provide strong evidence that the Kim and Nelson (1998) model is favored over the Stock and Watson (1989,1991) model and no major differences between the composite indices produced by the two models are found. On the other hand, the Kim and Nelson (1998) model produces the estimates of turning points close to the reference dates by the Economic and Social Research Institute in Cabinet Office.

KEY WORDS: Business cycles, Factor model, Gibbs sampling, Marginal Likelihood, Markov switching, Particle filter.

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1 Introduction

How should we measure business cycles? This problem has long attracted the attention of many macroeconomists and econometricians, and several methods have been proposed. A well-known method is the one based on dynamic factor models proposed by Stock and Watson (1989,1991). They define the composite index of coincident economic indicators to measure the state of overall economic activity as a single unobserved factor common to several macroeconomic variables using a dynamic factor model. Because their model can be estimated by the maximum likelihood method via the Kalman filter, their composite index can be estimated by running the Kalman filter or smoother given the maximum likelihood estimates of the parameters.

Kim and Nelson (1998) extend the dynamic factor model of Stock and Watson (1989,1991) so that the mean growth rate of the composite index, may vary depending on whether the economy is in the recession regime or in the boom regime. They specify the mean growth rate of the index using the regime-switching model of Hamilton (1989). One advantage of their model is that it produces not only the composite index but also the probabilities that the economy is in the recession regime, which can be utilized to date the business cycle turning points. It is, however, difficult to evaluate the likelihood in their model, so that they apply a Bayesian method via the Gibbs sampler. Specifically, the model parameters, the latent factor, and the regime are sampled from their posterior distribution using the Gibbs sampler, and simulated draws are used for Bayesian posterior analysis.

This article applies the Kim and Nelson (1998) model to macroeconomic data in Japan to measure business cycles in Japan. While several researchers such as Ohkusa (1992), Mori, Satake, and Ohkusa (1993), and Fukuda and

Onodera (2001) have already applied the Stock and Watson (1989,1991) model to the analysis of business cycles in Japan, there are few who have applied the Kim and Nelson (1998) model. The only exception is Kaufman (2000), who applies the Kim and Nelson (1998) model to eight countries including Japan. While she uses the quarterly data for real GDP, consumption, and investment, we use the monthly data selected from ten macroeconomic variables (see Table 1(A)) used by the Economic Planning Agency (EPA), which was reorganized as Economic and Social Research Institute (ESRI) in Cabinet Office after January 2001, to construct its composite index.

Following Kim and Nelson (1998), we estimate the composite index and the probabilities that the economy is in the recession as well as the model parameters using a Bayesian method via the Gibbs sampler. To evaluate the Kim and Nelson (1998) model, we also analyze whether the regime-shift occurs in the mean growth rate of the composite index by comparing the Kim and Nelson (1998) model with the Stock and Watson (1989,1991) model. Classical test statistics such as the likelihood ratio statistics are not directly applicable to this analysis (see Hansen (1992) and Garcia (1998)). In a Bayesian framework, model comparisons are conducted based on the posterior odds, which is the ratio of the marginal likelihood, which does not cause any problem in analyzing whether the regime-shit occurs or not. We adopt this method and calculate the marginal likelihood following the method proposed by Chib (1995). A diagnostic checking is also conducted.

The model comparison using Bayes factor does not provide strong evidence that the Kim and Nelson (1998) model is favored over the Stock and Watson (1989,1991) model. In addition, no major differences between the composite indices produced by the two models are found. On the other hand, the Kim and Nelson (1998) model produces the estimates of turning points close to the reference dates by the Economic and Social Research Institute in Cabinet Office.

The rest of this article is organized as follows. Section 2 explains the Kim and Nelson (1998) model and a Bayesian method for analyzing this model. Section 3 fits the model to macroeconomic data in Japan and summarizes the results. Conclusions are given in Section 4.

2 Econometric Methodology

2.1 Dynamic Factor Models

Our analysis is based on the dynamic factor models proposed by Stock and Watson (1988,1991) and developed by Kim and Nelson (1998). We start with a brief summary of these models.

Let ΔY_{it} $(i = 1, \ldots, n)$ represent the growth rate of the ith macroeconomic variable defined as the first difference of the log of the ith variable. In dynamic factor models, ΔY_{it} consists of two components: One is the component common to all variables ΔC_t, which is interpreted as the first difference of the composite index C_t, and the other is the idiosyncratic component of the ith variable e_{it}.

$$\Delta Y_{it} = \lambda_i(L)\Delta C_t + e_{it}, \tag{1}$$

where L is the lag operator and $\lambda_i(L) = \lambda_{i0}L + \cdots + \lambda_{ir_i}L^{r_i}$. The idiosyncratic component e_{it} is assumed to follow an autoregressive (AR) process

$$\psi_i(L)e_{it} = \epsilon_{it}, \quad \epsilon_{it} \sim i.i.d.N(0, \sigma_i^2) \tag{2}$$

where $\psi_i(L) = 1 - \psi_{i1}L - \cdots - \psi_{iq_i}L^{q_i}$. The common factor ΔC_t is specified as

$$\phi(L)(\Delta C_t - \mu_{s_t} - \delta) = \nu_t, \quad \nu_t \sim i.i.d.N(0, 1) \tag{3}$$

where $\phi(L) = 1 - \phi_1 L - \cdots - \phi_p L^p$, δ is the long-run growth of the index, and μ_{s_t} is the deviation from that long-run growth. Notice that the variance of ν_t is normalized to unity for identification of the model.

While Stock and Watson (1989,1991) assume that $\mu_{S_t} = 0$, Kim and Nelson (1998) allow it to vary depending on whether the economy is in a recession ($S_t = 0$) or in a boom ($S_t = 1$) as follows.

$$\mu_{s_t} = \mu_0 + \mu_1 S_t, \quad \mu_1 > 0. \tag{4}$$

Kim and Nelson (1998) assume that S_t follows a Markov process with transition probabilities

$$\begin{aligned} P(S_t = 1|S_{t-1} = 1) &= \pi_{11} \\ P(S_t = 0|S_{t-1} = 0) &= \pi_{00}. \end{aligned} \tag{5}$$

We work with the demeaned growth rate of the i variable $\Delta y_{it}(= \Delta Y_{it} - \Delta \overline{Y_{it}})$. Then, equations (1) and (3) can be expressed as follows.

$$\Delta y_{it} = \lambda_i \Delta c_t + e_{it} \tag{6}$$

$$\phi(L)(\Delta c_t - \mu_{s_t}) = \nu_t \tag{7}$$

where $\Delta c_t = \Delta C_t - \delta$.

Let $\Delta y_t = [\Delta y_{1t}, \ldots, \Delta y_{nt}]'$. Then, the above model can be represented as a state space form:

$$\Delta y_t = H\zeta_t, \tag{8}$$

$$\zeta_t = M_{s_t} + F z_t + u_t, \quad u_t \sim i.i.d.N(0, \Sigma_u). \tag{9}$$

In this paper, we assume that $n = 5$, $p = 3$, $r_i = q_i = 1$ ($i = 1, \ldots, n$). Then, ζ_t, H, M_{s_t}, F, and σ_u are given by

$$\zeta_t = [\Delta c_t, \Delta c_{t-1}, \Delta c_{t-2}, e_{1t}, e_{2,t}, e_{3t}, e_{4t}, e_{5,t}]'$$

$$
H = \begin{bmatrix}
\lambda_{10} & \lambda_{11} & 0 & 1 & 0 & 0 & 0 & 0 \\
\lambda_{20} & \lambda_{21} & 0 & 0 & 1 & 0 & 0 & 0 \\
\lambda_{30} & \lambda_{31} & 0 & 0 & 0 & 1 & 0 & 0 \\
\lambda_{40} & \lambda_{41} 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
\lambda_{50} & \lambda_{51} 0 & 0 & 0 & 0 & 0 & 0 & 1
\end{bmatrix}
$$

$$
F = \begin{bmatrix}
\phi_1 & \phi_2 & \phi_3 & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & \psi_1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \psi_2 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & \psi_3 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & \psi_4 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \psi_5
\end{bmatrix}
$$

$$
M_{s_t} = [\phi(L)\mu_{S_t}, 0, 0, 0, 0, 0, 0, 0]'
$$

$$
u_t = [\nu_t, 0, 0, \epsilon_{1t}, \epsilon_{2t}, \epsilon_{3t}, \epsilon_{4t}, \epsilon_{5t}]'
$$

$$
\Sigma_u = \begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & \sigma_1^2 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \sigma_2^2 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & \sigma_3^2 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & \sigma_4^2 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \sigma_5^2
\end{bmatrix}
$$

where $\phi(L)\mu_{s_t} = \mu_{S_t} - \phi_1\mu_{S_{t-1}} - \phi_2\mu_{S_{t-2}} - \phi_2\mu_{S_{t-3}}$.

In the Stock and Watson (1988,1991) model where μ_{s_t} is constant, the model that consists of equations (8) and (9) becomes a usual linear-Gaussian state space model, whose likelihood can be evaluated by executing the Kalman filter. Therefore, the Stock and Watson (1988,1991) model can be estimated by the maximum likelihood method. It is not true for the Kim and Nelson (1998) model in which μ_{s_t} changes depending on S_t following a Markov process.

2.2 Estimation Method

As mentioned, while the Stock and Watson (1989,1991) model can be estimated by the maximum likelihood method via the Kalman filter, it is not so for the Kim and Nelson (1998) model. Following Kim and Nelson (1998), we apply a Bayesian method via the Gibbs sampler.

The Gibbs sampler is a Monte Carlo method for sampling from joint distributions using conditional distributions. It is a convenient tool in Bayesian inference when it is difficult to obtain the joint posterior distributions.

To see how the Gibbs sampler works, let us consider the problem of sampling k (possibly vector-valued) random variables $(\theta_1, \theta_2, \ldots, \theta_k)$ from the joint density $f(\theta_1, \theta_2, \ldots, \theta_k)$. Suppose that, for all $i = 1, 2, \cdots, k$, it is possible to generate θ_i from conditional distribution $f(\theta_i | \{\theta_j\}_{j \neq i})$ by some methods. Starting from an arbitrary set of initial value $(\theta_1^{(0)}, \theta_2^{(0)}, \ldots, \theta_k^{(0)})$, we draw $\theta_1^{(1)}$ from $f(\theta_1 | \theta_2^{(0)}, \theta_3^{(0)}, \ldots, \theta_k^{(0)})$, $\theta_2^{(1)}$ from $f(\theta_2 | \theta_1^{(1)}, \theta_3^{(0)}, \ldots, \theta_k^{(0)})$, and so on up to $\theta_1^{(k)}$ from $f(\theta_k | \theta_1^{(1)}, \theta_2^{(1)}, \ldots, \theta_{k-1}^{(1)})$. Let us call this procedure one iteration. After l such iterations, we obtain $(\theta_1^{(l)}, \theta_2^{(l)}, \ldots, \theta_k^{(l)})$. Under mild conditions, it converges in distribution to be a set of random variables from $f(\theta_1, \theta_2, \ldots, \theta_k)$ as $l \to \infty$.

For a sufficiently large M, $(\theta_1^{(l)}, \theta_2^{(l)}, \ldots, \theta_k^{(l)})$ $(l = M+1, M+2, \cdots, M+N)$ can approximately be regarded as a sample from $f(\theta_1, \theta_2, \ldots, \theta_k)$ although they are serially correlated and not i.i.d. sample. Hence, the first M draws are discarded and the last N draws are used for posterior inference. For instance, the expectation of a function of the parameters, $g(\theta_1, \theta_2, \ldots, \theta_k)$, is estimated by the sample average

$$E[g(\theta_1, \theta_2, \ldots, \theta_k)] = \frac{1}{N} \sum_{l=M+1}^{M+N} g(\theta_1^{(l)}, \theta_2^{(l)}, \ldots, \theta_k^{(l)}). \tag{10}$$

The unknown parameters in the Kim and Nelson (1998) model that consists of equations (1)–(5) are: λ_{ij} $(i = 1, \ldots, n; j = 1, \ldots, r_i)$, ψ_{ij} $(i =$

$1,\ldots,n; j=1,\ldots,q_i)$, σ_i^2 $(i=1,\ldots,n)$, ϕ_j $(j=1,\ldots,p)$, μ_0,μ_1, π_{00}, and π_{11} . As well as these values, latent variables $\Delta\tilde{c}_T=[\Delta c_1,\ldots,\Delta c_T]$ and $\tilde{S}_T=[S_1,\ldots,S_T]$ are also treated as if they were unknown parameters. Then, we sample from the following conditional distributions sequentially.

$$f(\lambda_{i1},\ldots,\lambda_{ir_i}|\theta_{/(\lambda_{i1},\ldots,\lambda_{ir_i})},\Delta\tilde{c}_T,\tilde{S}_T,\Delta\tilde{y}_T)\quad (i=1,\ldots,n) \tag{11}$$

$$f(\psi_{i1},\ldots,\psi_{iq_i}|\theta_{/(\psi_{i1},\ldots,\psi_{iq_i})},\Delta\tilde{c}_T,\tilde{S}_T,\Delta\tilde{y}_T)\quad (i=1,\ldots,n) \tag{12}$$

$$f(\sigma_i^2|\theta_{/\sigma_i^2},\Delta\tilde{c}_T,\tilde{S}_T,\Delta\tilde{y}_T)\quad (i=1,\ldots,n) \tag{13}$$

$$f(\phi_1,\ldots,\phi_p|\theta_{/(\phi_1,\ldots,\phi_p)},\Delta\tilde{c}_T,\tilde{S}_T,\Delta\tilde{y}_T) \tag{14}$$

$$f(\mu_0,\mu_1|\theta_{/(\mu_0,\mu_1)},\Delta\tilde{c}_T,\tilde{S}_T,\Delta\tilde{y}_T) \tag{15}$$

$$f(\pi_{00}|\theta_{/\pi_{00}},\Delta\tilde{c}_T,\tilde{S}_T,\Delta\tilde{y}_T) \tag{16}$$

$$f(\pi_{11}|\theta_{/\pi_{11}},\Delta\tilde{c}_T,\tilde{S}_T,\Delta\tilde{y}_T) \tag{17}$$

$$f(\tilde{S}_T|\theta,\Delta\tilde{c}_T,\Delta\tilde{y}_T) \tag{18}$$

$$f(\Delta\tilde{c}_T|\theta,\tilde{S}_T,\Delta\tilde{y}_T) \tag{19}$$

where $\Delta\tilde{y}_T=[(\tilde{y}_{11},\ldots,\tilde{y}_{n1})',\ldots,(\tilde{y}_{1T},\ldots,\tilde{y}_{nT})']$, θ is the set of all parameters, and $\theta_{/\omega}$ is the set of all parameters except ω.

For the unknown parameters, we adopt the following priors.

$$(\lambda_{i1},\ldots,\lambda_{ir_i}\sim N(0,I_{r_i}),\quad i=1,\ldots,n$$

$$(\psi_{i1},\ldots\psi_{iq_i})'\sim N(0,I_{q_i})I_{S(\psi_i)},\quad i=1,\ldots,n$$

$$(\phi_1,\ldots,\phi_p)'\sim N(0,I_p)I_{S(\phi)}$$

$$(\mu_0,\mu_1)'\sim N(0,I_2)I[\mu_1<0,\mu_2>0],$$

$$\sigma_i^2\sim IG(1/2,1/2).$$

$$\pi_{00}\sim beta(18,2),\quad \pi_{11}\sim beta(18,2)$$

where $I[\cdot]$ is the indicator function that takes one if the condition in the bracket is satisfied and zero otherwise, and $I_{S(\psi_i)}$ (or $I_{S(\phi)}$) is the indicator function that takes one if the roots of the polynomial $\psi_i(L)$ (or $\phi(L)$) lie outside the unit circle and zero otherwise. Under these priors, the conditional distributions (11)–(17) may simply be calculated, and it is easy to sample from those distributions (see Kim and Nelson (1998, 1999)).

Sampling $\tilde{S}_T$ from (18) can be conducted using the Hamilton (1989) filter. Running the Hamilton (1989) filter produces $p(S_t|\Delta\tilde{c}_t)$ and $p(S_t|\Delta\tilde{c}_{t-1})$ for $t = 1, \ldots, T$. Then, after generating S_T from $p(S_T|\Delta\tilde{c}_T)$, we can proceed backwards in time. Specifically, given S_{t+1}, s_t is generated using the probability

$$p(S_t|\Delta\tilde{c}_t, S_{t+1}) = \frac{p(S_{t+1}|S_t)p(S_t|\Delta\tilde{c}_t)}{p(S_{t+1}|\Delta\tilde{c}_t)}$$

where $p(S_{t+1}|S_t)$ is the transition probability, and $p(S_t|\Delta\tilde{c}_t)$ and $p(S_{t+1}|\Delta\tilde{c}_t)$ are obtained from the Hamilton (1989) filter.

Once $\tilde{S}_T$ are given, the Kim and Nelson (1998) model can be represented by a linear Gaussian state space model. Therefore, it is straightforward to sample $\Delta\tilde{c}_T$ from (19) using the Kalman filter and smoother. Another state space representation of the Kim and Nelson (1998) model is possible. Suppose that $n = 5$, $p = 3$, and $r_i = q_i = 1$ $(i = 1, \ldots, n)$ again. Let $\Delta y^*_{it} = \Delta y_{it} - \psi_{i1}\Delta y_{i,t-1}$ and $\Delta y^*_t = [\Delta y^*_{1t}, \ldots, \Delta y^*_{nt}]'$. Then, the Kim and Nelson (1998) model may be represented as

$$\begin{aligned} \Delta y_t &= \Lambda z_t + \epsilon_t, \quad \epsilon_t \sim i.i.d.N(0, \Sigma_\epsilon) && (20) \\ z_t &= M_{s_t} + \Phi z_t + u_t, \quad v_t \sim i.i.d.N(0, \Sigma_v). && (21) \end{aligned}$$

Then, z_t, Λ, M_{s_t}, Φ, Σ_ϵ, and Σ_v are given by

$$z_t = [\Delta c_t, \Delta c_{t-1}, \Delta c_{t-2}]'$$

$$\Lambda = \begin{bmatrix} \lambda_{10} & -\lambda_{10}\psi_{11} + \lambda_{11} & -\lambda_{11}\psi_{11} \\ \lambda_{20} & -\lambda_{20}\psi_{21} + \lambda_{21} & -\lambda_{21}\psi_{21} \\ \lambda_{30} & -\lambda_{30}\psi_{31} + \lambda_{31} & -\lambda_{31}\psi_{31} \\ \lambda_{40} & -\lambda_{40}\psi_{41} + \lambda_{41} & -\lambda_{41}\psi_{41} \\ \lambda_{50} & -\lambda_{50}\psi_{51} + \lambda_{51} & -\lambda_{51}\psi_{51} \end{bmatrix}$$

$$\Phi = \begin{bmatrix} \phi_1 & \phi_2 & \phi_3 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

$$M_{s_t} = [\phi(L)\mu_{S_t}, 0, 0]'$$

$$v_t = [\nu_t, 0, 0]'$$

$$\Sigma_\epsilon = \begin{bmatrix} \sigma_1^2 & 0 & 0 & 0 & 0 \\ 0 & \sigma_2^2 & 0 & 0 & 0 \\ 0 & 0 & \sigma_3^2 & 0 & 0 \\ 0 & 0 & 0 & \sigma_4^2 & 0 \\ 0 & 0 & 0 & 0 & \sigma_5^2 \end{bmatrix}$$

$$\Sigma_v = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

where $\phi(L)\mu_{s_t} = \mu_{S_t} - \phi_1\mu_{S_{t-1}} - \phi_2\mu_{S_{t-2}} - \phi_3\mu_{S_{t-3}}$. Following Kim and Nelson (1998), we use this state space model instead of equations (8) and (9) to sample $\Delta\tilde{c}_T$.

Once $\Delta\tilde{c}_T$ is obtained, they can be transformed into the composite index $\tilde{C}_T = [C_1, \ldots, C_T]$ as

$$C_t = \Delta c_t + C_{t-1} + \delta$$

where δ is the long-run growth rate of the index, which can be estimated using the steady-state Kalman gain obtained from applying the Kalman filter to the state space model that consists of equations (8) and (9) (see Kim and Nelson (1988,1999)).

2.3 Model Comparison

Model comparison in a Bayesian framework can be performed using posterior odds ratio. Posterior odds ratio between model i, M_i, and model j, M_j, is given by

$$
\begin{aligned}
POR &= \frac{f(M_i|\Delta\tilde{y}_T)}{f(M_j|\Delta\tilde{y}_T)} \\
&= \frac{f(\Delta\tilde{y}_T|M_i)}{f(\Delta\tilde{y}_T|M_j)} \frac{f(M_i)}{f(M_j)},
\end{aligned}
$$

where $\frac{f(\Delta\tilde{y}_T|M_i)}{f(\Delta\tilde{y}_T|M_j)}$ and $\frac{f(M_i)}{f(M_j)}$ are called Bayes factor and prior odds respectively.

As is the usual practice, we set the prior odds to be 1, so that the posterior odds ratio is equal to the Bayes factor. To evaluate the Bayes factor, which is the ratio of the marginal likelihoods, we follow the basic marginal likelihood identity in Chib (1995). The log of the marginal likelihood of model M_i can be written as

$$
\begin{aligned}
&\log f(\Delta\tilde{y}_T|M_i) \\
&\quad = \log f(\Delta\tilde{y}_T|M_i,\theta_i) + \log f(\theta_i|M_i) - \log f(\theta_i|M_i,\Delta\tilde{y}_T), \qquad (22)
\end{aligned}
$$

where θ_i is the set of unknown parameters for model M_i, $\ln f(\Delta\tilde{y}_T|M_i,\theta_i)$ is the likelihood, $\ln f(\theta_i|M_i)$ is the prior density, and $\ln f(\theta_i|M_i,\Delta\tilde{y}_T)$ is the posterior density.

The above identity holds for any value of θ_i, but following Chib (1995), we set θ_i at its posterior mean calculated using the MCMC draws. It is straightforward to evaluate the prior density. The posterior density is evaluated using the method proposed by Chib (1995), and the likelihood is evaluated using the particle filter (see Pitt and Shephard (1999)), which will be explained in the next subsection.

2.4 Particle Filter

Consider the state space model that consists of equations (20) and (21). To evaluate the likelihood and perform diagnostic checks, we need to sample from the filtering density $f(z_t,S_t|\Delta\tilde{y}_t)$. We use the particle filter proposed by Pitt and Shephard (1999) for this sampling.

First, note that the filter density $f(z_t, S_t|\Delta\tilde{y}_{t-1})$ is proportional to

$$f(\Delta y_t|z_t, S_t)f(z_t, S_t|\Delta\tilde{y}_{t-1}) = f(\Delta y_t|z_t)f(z_t, S_t|\Delta\tilde{y}_{t-1}). \tag{23}$$

The second term can be expressed by an integral:

$$f(z_t, S_t|\Delta\tilde{y}_{t-1}) = \int f(z_t, S_t|z_{t-1}, S_{t-1})f(z_{t-1}, S_{t-1}|\Delta\tilde{y}_{t-1})dz_{t-1}dS_{t-1}. \tag{24}$$

Suppose that we have M candidate points $\left\{z_{t-1}^{(m)}, S_{t-1}^{(m)}\right\}$ $(m = 1, \ldots, M)$ sampled from the density $f(z_{t-1}, S_{t-1}|\tilde{y}_{t-1})$, then the latter integral could be estimated by the average

$$f(z_{t-1}, S_{t-1}|\Delta\tilde{y}_{t-1}) \approx \frac{1}{M}\sum_{m=1}^{M} f(z_t, S_t|z_{t-1}^{(m)}, S_{t-1}^{(m)}). \tag{25}$$

We sample from the joint density $f(z_t, S_t, m|\Delta\tilde{y}_t)$, where m is an index on the mixture in (25),

$$\begin{aligned} f(z_t, S_t, m|\Delta\tilde{y}_t) &\propto f(\Delta y_t|z_t)f(z_t, S_t|z_{t-1}^{(m)}, S_{t-1}^{(m)}) \\ &\propto f(\Delta y_t|z_t)f(z_t|S_t, z_{t-1}^{(m)})p(S_t|S_{t-1}^{(m)}) \\ &= \eta_{S_t,m} f(z_t|S_t, z_{t-1}^{(m)}, \Delta y_t). \end{aligned} \tag{26}$$

By first selecting the indices (S_t, m) with probability proportional to $\eta_{S_t,m}$ and then sampling from $f(z_t|S_t, z_{t-1}^{(m)}, \Delta y_t)$, which is $N(\mu_{t|t}^{(m)}, \Sigma_{t|t}^{(m)})$ in the present context, provides us with a sample out of the first density $f(z_t, S_t|\Delta\tilde{y}_t)$. The mean $\mu_{t|t}^{(m)}$ and the variance $\Sigma_{t|t}^{(m)}$ are given by

$$\begin{aligned} \mu_{t|t}^{(m)} &= M_{S_t} + \Phi z_{t-1}^{(m)} + \Sigma_v\Lambda'\Sigma_\epsilon^{-1}e_t \\ \Sigma_{t|t}^{(m)} &= \Sigma_v - \Sigma_v\Lambda'\Sigma_\epsilon^{-1}\Lambda\Sigma_v, \end{aligned}$$

where $e_t = \Delta y_t - \Lambda(M_{S_t} + \Phi z_{t-1}^{(m)})$.

Further, given the Gaussian distributions

$$\begin{aligned} y_t|z_t &\sim N(\Lambda z_t, \Sigma_\epsilon) \\ z_t|S_t, z_{t-1}^{(m)} &\sim N(M_{S_t} + \Phi z_{t-1}^{(m)}, \Sigma_v) \end{aligned}$$

the first stage weight $\eta_{S_t,m}$ are derived with the use of:

$$\begin{aligned} f(\Delta y_t|z_t)f(z_t|S_t, z_{t-1}^{(m)}) &= f(\Delta y_t, z_t|S_t, z_{t-1}^{(m)}) \\ &= f(z_t|S_t, z_{t-1}^{(m)}, \Delta y_t)f(\Delta y_t|z_{t-1}^{(m)}). \end{aligned} \tag{27}$$

Thus,

$$\begin{aligned} \eta_{S_t,m} &= f(\Delta y_t|z_{t-1}^{(m)})p(S_t|S_t^{(m)}) \\ &\propto |\Sigma_\epsilon|^{-1/2} \exp\left(-\frac{1}{2}e_t'\Sigma_\epsilon^{-1}e_t\right) p(S_t|S_{t-1}^{(m)}). \end{aligned} \tag{28}$$

2.5 Diagnostics and Likelihood

The diagnostics are based on the one-step ahead prediction density:

$$\begin{aligned} & f(\Delta y_{t+1}|\Delta\tilde{y}_t, \theta) \\ &= \int f(\Delta y_{t+1}|z_{t+1}, S_{t+1}, \Delta\tilde{y}_t, \theta)f(z_{t+1}, S_{t+1}|z_t, S_t, \theta) \\ &\quad f(z_t, S_t|\Delta\tilde{y}_t, \theta)dS_t dz_t dS_{t+1} dz_{t+1} \end{aligned} \tag{29}$$

Given the sampled values $\left\{z_t^{(m)}, S_t^{(m)}\right\}$, we first sample $S_{t+1}^{(m)}$ using the transition probabilty $p(S_{t+1}^{(m)}|S_t)$ and then sample $z_{t+1}^{(m)}$ from

$$z_{t+1}^{(m)}|z_t^{(m)}, S_{t+1}^{(m)} \sim N(M_{S_t} + \Phi z_t^{(m)}, \Sigma_v)$$

Based on these M draws on S_{t+1} and z_{t+1} generated from the one-step ahead prediction density, we calculate the probability that $\Delta y_{i,t+1}$ will be less than the observed value $y_{i,t+1}^o$

$$P(\Delta y_{i,t+1} \leq \Delta y_{i,t+1}^o|\Delta\tilde{y}_t, \theta) \approx u_{i,t+1}^M = \frac{1}{M}\sum_{m=1}^{M} Pr(\Delta y_{i,t+1} \leq \Delta y_{i,t+1}^o|\Delta\tilde{y}_t, \theta)$$

Under the null of a correctly specified model, $u_{i,t}^M$ converges in distribution to independently and identically distributed uniform random variables as $M \to \infty$ (Rosenblatt (1952)). This provides a valid basis for diagnostic checking. These variables can be mapped into the normal distribution, by using the inverse of the normal distribution function $n_{i,t}^M = F^{-1}(u_{i,t}^M)$ to give a standard sequence of independent and identically distributed normal variables.

The M draws on S_{t+1} and z_{t+1} generated from the one-step ahead prediction density can also be used to obtain the one-step ahead prediction density as follows.

$$f(\Delta y_{t+1}|\Delta \tilde{y}_t, \theta) = \frac{1}{M}\sum_{m=1}^{M} f(\Delta y_{t+1}|z_{t+1}^{(m)}, S_{t+1}^{(m)})$$

Using these values, we can evaluate the likelihood.

3 Empirical Results

3.1 Data Description

Economic and Social Research Institute (ESRI) uses eleven macroeconomic variables to construct its Coincident Index. (For definitions of these eleven variables, see Table 1(A).) Among them, "Business Profit" (ZBOAS) is quarterly data and the other ten variables are monthly data. We obtained the raw data for these ten variables from 1975:1 to 2000:12 and transformed them into seasonally adjusted ones by the Census-X11 method. The use of all ten variables to estimate the Stock and Watson (1989,1991) and the Kim and Nelson (1998) model is, however, computationally costly. Hence, our analysis is based on the following two datasets, both of which consist of five variables selected by Fukuda and Onodera (2001).

Dataset 1: (1) IIP95P (2) SCI95 (3) ESRAO (4) HWINMF (5) CELL9.

Dataset 2: (1) IIP95P (2) SMSALE (3) HWINMF (4) IIP95O (5) IIP95M

The both datasets were selected based on the principle not only to use variables related to production but also use variables related to trade sales and labor market. On one hand, dataset 1 includes "Index of Wholesale Sales" (SCI95) as a trade sales variable nd "Ratio of Job Offers to Applicants" (ESRAO) and "Index of Non-Scheduled Hours Worked" (HWINMF) as labor market variables. On the other hand, dataset 2 includes "Sales of Small and Medium Size Companies" (SMSALE) as a trade sales variable and HWINF as a labor market related variable. These two datasets, however, differ in the sense that dataset 1 includes variables that are less correlated with 'each other while all variables except HWINMF in dataset 2 dataset 2 are highly correlated with each other. Table 1 (B) reports the contemporaneous correlation of the growth rate of the ten variables, which shows that "Index of Industrial Production" (IIP95P) has large positive correlations with "Index of Raw Materials Consumption (IIP95M), "Index of Operating Rate" (IIP95O), and "Sales of Small and Medium Size Companies" (SMSALE). In addition, Table 2 (C) shows the serial correlation of the growth rate and serial correlation of these ten variables. It shows that two labor market variables, that is, HWINMF and ESRAO, have positive serial correlations and the other variables have negative serial correlations. Dataset 1 includes the both of these two variables while dataset 2 includes only HWINMF.

3.2 Estimation Details

Following Fukuda and Onodera (2001), we set $p = 3$ and $q_i = 1$ $(i = 1, \dots, 5)$ for the both datasets. While Fukuda and Onodera (2001) set $r_i = 0$, we set it equal to 1.

For parameter estimation, we conduct the MCMC simulation with 12000 iterations for each model. The first 2000 draws are discarded and then the next 10000 are recorded. Using these 10000 draws for each of the parameters, we calculate the posterior means, the standard errors of the posterior means, the 95% intervals, and the convergence diagnostic (CD) statistics proposed by Geweke (1992). The posterior means are computed by averaging the simulated draws. The standard errors of the posterior means are computed using a Parzen window with a bandwidth of 1000. The 95% intervals are calculated using the 2.5th and 97.5th percentiles of the simulated draws. Geweke (1992) suggests assessing the convergence of the MCMC by comparing values early in the sequence with those late in the sequence. Let $\theta^{(i)}$ be the ith draw of a parameter in the recorded 10000 draws, and let $\bar{\theta}_A = \frac{1}{n_A}\sum_{i=1}^{n_A}\theta^{(i)}$ and $\bar{\theta}_B = \frac{1}{n_B}\sum_{i=10001-n_B}^{10000}\theta^{(i)}$. Using these values, Geweke (1992) proposes the following statistic called *convergence diagnostics* (CD).

$$\mathrm{CD} = \frac{\bar{\theta}_A - \bar{\theta}_B}{\sqrt{\hat{\sigma}_A^2/n_A + \hat{\sigma}_B^2/n_B}}, \tag{30}$$

where $\sqrt{\hat{\sigma}_A^2/n_A}$ and $\sqrt{\hat{\sigma}_B^2/n_B}$ are standard errors of $\bar{\theta}_A$ and $\bar{\theta}_B$. If the sequence of $\theta^{(i)}$ is stationary, it converges in distribution to the standard normal. We set $n_A = 1000$ and $n_B = 5000$ and compute $\hat{\sigma}_A^2$ and $\hat{\sigma}_B^2$ using Parzen windows with bandwidths of 100 and 500 respectively.

In calculating the marginal likelihood, we set the number of iterations to evaluate the both posterior densities and the likelihood set equal to 2000.

3.3 Estimation Results

Table 2 shows the estimation results for dataset 1. Table 2 (A) and (B) are the results for the Kim and Nelson (1998) model and the Stock and Watson (1989,1991) model respectively. According to the CD values, the null hypothesis that the sequence of 10000 draws is stationary is accepted at the 5% significance level for all parameters in the both models. The log marginal likelihood of the Kim and Nelson (1998) model of -2714.16 is smaller than that of the Stock and Watson (1989,1991) model of -2713.31, indicating that the latter model is favorable over the former model.

Table 2 (C) shows the results of diagnostic checking based on variables $n_{i,t}^M$ explained in Section 2. The Table shows the mean, the standard deviation, the skewness, the kurtosis, and the Ljung-Box statistics to test the null hypothesis of no serial correlation up to the sixth lag, where the number in brackets show the standard errors. If the model is correctly specified, the asymptotic distribution of $n_{i,t}^M$ is the standard normal. For SCI95, ESRAO, and HWINM, the null hypothesis of no serial correlation is rejected at the 1% level. For all variables, the kurtosis is significantly larger than three.

Figures 1 (A) depicts the CIs estimated by the Kim and Nelson (1998) model and the Stock and Watson (1989,1991) model jointly with that the ESRI's CI. Figure 1 (B) depicts the posterior probability that the economy is in the recession state in each month as inferred from the Kim and Nelson (1998) model. This probability can be calculated simply by averaging 10000 draws of the state S_t sampled from its posterior distribution.

Table 3 shows the results for dataset 2. According to the CD values, the null hypothesis that the sequence of 10000 draws is stationary is accepted at the 5% significance level for all parameters in the both models again. The log marginal likelihood of the Kim and Nelson (1998) model of -2202.10 is slightly larger than that of the Stock and Watson (1989,1991) model of -2203.91, providing evidence, although weak, that the mean growth rate shifts depending on whether the economy is in a recession or in a boom.

Table 3 (C) shows the results of diagnostic checking. Except for HWINMF, the null hypothesis of no serial correlation is rejected at the 1% level. The kurtosis is still significantly larger than three for all variables, indicating that a more leptokurtic distribution such as the Student t distribution may be required for the error terms.[1]

[1] See Geweke (1993), Fernández and Steel (1998), and Watanabe (2000) for the Bayesian treatment when the error terms follow the Student t distribution.

Figures 2 (A) depicts the CIs estimated by the Kim and Nelson (1998) model and the Stock and Watson (1989,1991) model jointly with that the ESRI's CI. Figure 2 (B) depicts the posterior probability that the economy is in the recession state in each month as inferred from the Kim and Nelson (1998) model. In contrast to the probability based on dataset 1, it moves in a wider range between 0% and 100%, compared to Figure 1 (B).

We further estimate the Kim and Nelson model by using the following dataset.

Dataset 3: (1) IIP95P (2) SCI95 (3) ESRAO (4) HWINMF.

This dataset is the one in which CELL 6 is excluded from dataset 1. These four variables are used to construct the Nikkei Business Index because they correspond to the four variables used by the Department of Commerce (DOC) to construct its composite index: industrial production, total personal income less transfer payments in 1987 dollars, employees on nonagricultural payrolls, and total manufacturing and trade sales in 1987 dollars. We only report the posterior probability of a recession, which is depicted in Figure 3. Unlike datasets 1 and 2, the posterior probability moves in a narrow range around 50%, so that it cannnot be used to date the business cycle turning points. This may be attributed to the fact that the four variables in dataset 3 are weakly correlated with each other.

In dataset 2, the null hyposesis of no serial correlation in the diagnostic statistic is rejected for HWINMF. This may be attributed to the fact that HWINMF has positive serial correlation while all other variables in dataset 2 have negative serial correlation and HWINF has weakly correlated with other variables. Hence, we also analyze the following dataset.

Dataset 4: (1) IIP95P (2) SMSALE (3) IIP95O (4) IIP95M

The posterior probabilities of a recession calculated by fitting the Kim and Nelson model to dataset 4 are depicted in Figure 4.

Following Kaufman (2000), we date the turning points by defining period t as a peak if $P(S_t = 1|Y_T) < 0.5$ and $P(S_t = 1|Y_T) > 0.5$ and a trough if the posterior probability $P(S_t = 1|Y_T) > 0.5$ and $P(S_t = 1|Y_T) < 0.5$. The estimated turning points are shown in Table 4 jointly with the reference date by the ESRI.

4 Conclusions

This article fits the Markov switching dynamic factor model proposed by Kim and Nelson (1998) to some macroeconomic variables in Japan. We find that choice of variables is important when we use this model. This model performs poorly with the weakly correlated data and performs well with the highly correlated data.

In this article, we focus on the in-sample fit of the model. Needless to say, it is worthwhile examining the out-of-sample forecasting ability of this model.

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Table 1. Eleven Variables used to construct by the Economics and Social Research Institute to Construct its Composite Index

(A) Definition of Variables

1	IIP95M	Index of Raw Materials Consumption, Mfg.
2	IIP95O	Index of Operating Rate, Mfg.
3	HWINMF	Index of Non-scheduled Hours Worked, Mfg
4	ESRAO	Ratio of Job Offers to Applicants
5	SDS	Sales of Department Stores
6	CELL9	Electric Power Consumption of Large Users
7	IIP95S	Index of Producers' Shipments, Investment Goods
8	SCI95	Index of Wholesale Sales
9	SMSALE	Sales of Small and Medium Size Companies
10	IIP95P	Index of Industrial Production, Mining and Mfg.
11	ZBOAS	Business Profit, All Industries

Note: ZBOAS is quarterly data and the others are monthly data.

(B) Contemporaneous Correlations of the Growth Rate of the Ten Variables

	IIP95M	IIP95O	HWINMF	ESRAO	SDS	CELL9	IIP95S	SCI95	SMSALE	IIP95P
IIP95M	1.0000									
IIP95O	0.8820	1.0000								
HWINMF	0.3321	0.3152	1.0000							
ESRAO	0.2401	0.2429	0.4157	1.0000						
SDS	-0.0671	-0.0943	-0.0656	-0.0054	1.0000					
CELL9	0.6507	0.6196	0.2549	0.2059	-0.0467	1.0000				
IIP95S	0.5250	0.5810	0.2088	0.1738	0.0298	0.4408	1.0000			
SCI95	0.5038	0.5059	0.1158	0.0974	0.3845	0.4562	0.4444	1.0000		
SMSALE	0.6843	0.6632	0.2728	0.2288	0.0348	0.5221	0.6334	0.6070	1.0000	
IIP95P	0.8673	0.8872	0.2524	0.2364	-0.0727	0.6822	0.6624	0.6096	0.7756	1.0000

(C) Serial Correlations of the Growth Rate of Ten Variables

Variabels	IIP95M	IIP95O	HWINMF	ESRAO	SDS	CELL9	IIP95S	SCI95	SMSALE	IIP95P
correlation	-0.3492	-0.4002	0.4176	0.5574	-0.5628	-0.2286	-0.4604	-0.3561	-0.3550	-0.4227

TABLE 2. Estimation Results for Dataset 1

(A) Kim and Nelson Model

Marginal Likelihood = -2714.16

Parameter	Mean	Standard Error	95% Interval	CD
ΔC_t				
π_{00}	0.9045	0.0023	[0.7578,0.9807]	0.64
π_{11}	0.9108	0.0038	[0.7648,0.9815]	-0.89
ϕ_1	-0.0489	0.0164	[-0.3711,0.2305]	1.05
ϕ_2	0.1034	0.0080	[-0.1170,0.2604]	0.87
ϕ_3	0.3572	0.0056	[0.1795,0.4989]	1.60
μ_0	-0.3460	0.0246	[-0.9584,-0.0144]	1.55
μ_1	0.5424	0.0405	[0.0156,1.4138]	-1.59
y_{1t}				
λ_{10}	1.0739	0.0026	[0.9301,1.2141]	1.50
λ_{11}	-0.4708	0.0059	[-0.6210,-0.3100]	-0.77
ψ_1	-0.3803	0.0030	[-0.5192,-0.2328]	-0.89
σ_1^2	0.5208	0.0066	[0.3256,0.7779]	-0.36
y_{2t}				
λ_{20}	0.9507	0.0028	[0.7782,1.1313]	1.11
λ_{21}	-0.4521	0.0055	[-0.6406,-0.2628]	-1.24
ψ_2	-0.3075	0.0010	[-0.4232,-0.1922]	-0.98
σ_2^2	2.0372	0.0031	[1.6911,2.4346]	-0.98
y_{3t}				
λ_{30}	0.4653	0.0031	[0.2540,0.6779]	1.11
λ_{31}	0.3589	0.0013	[0.1601,0.5556]	0.69
ψ_3	0.4102	0.0013	[0.2863,0.5319]	-0.82
σ_3^2	3.1206	0.0054	[2.6375,3.6731]	-1.20
y_{4t}				
λ_{40}	0.5369	0.0022	[0.3850,0.6914]	1.22
λ_{41}	0.4272	0.0025	[0.2679,0.5886]	0.92
ψ_4	0.1370	0.0025	[-0.0049,0.2846]	-1.02
σ_4^2	1.6903	0.0051	[1.4014,2.0190]	-0.66
y_{5t}				
λ_{50}	0.7666	0.0014	[0.6472,0.8872]	0.86
λ_{51}	-0.2366	0.0060	[-0.3932,-0.0800]	-1.06
ψ_5	-0.2060	0.0013	[-0.3359,-0.0773]	-0.15
σ_5^2	0.7474	0.0042	[0.6030,0.9132]	0.85

Note: y_{1t}, y_{2t}, y_{3t}, y_{4t}, y_{5t}represent IIP95P, SCI95, ESRAO, HWINMF, and CELL9 respectively. The first 2000 draws are discarded and then the next 10000 are used for calculating the posterior means, the standard errors of the posterior means, 95% interval, and the convergence diagnostic (CD) statistics proposed by Geweke (1992). The posterior means are computed by averaging the simulated draws. The standard errors of the posterior means are computed using a Parzen window with a bandwidth of 1000. The 95% intervals are calculated using the 2.5th and 97.5th percentiles of the simulated draws. The CD is computed using equation (30), where we set $n_A = 1000$ and $n_B = 5000$ and compute $\hat{\sigma}_A^2$ and $\hat{\sigma}_B^2$ using a Parzen window with bandwidths of 100 and 500 respectively.

(B) Stock and Watson Model

Marginal Likelihood = -2713.31

Parameter	Mean	Standard Error	95% Interval	CD
ΔC_t				
ϕ_1	0.0164	0.0044	[-0.1725,0.2087]	-0.93
ϕ_2	0.1526	0.0008	[0.0362,0.2683]	-0.28
ϕ_3	0.3904	0.0006	[0.2716,0.5026]	0.46
y_{1t}				
λ_{10}	1.304	0.0035	[0.9832,1.2702]	1.65
λ_{11}	-0.5063	0.0026	[-0.6642,-0.3023]	0.93
ψ_1	-0.3893	0.0019	[-0.5293,-0.2335]	0.80
σ_1^2	0.4775	0.0057	[0.2632,0.7090]	-1.28
y_{2t}				
λ_{20}	0.9936	0.0013	[0.8220,1.1708]	0.90
λ_{21}	-0.4840	0.0028	[0.4776,0.9502]	0.75
ψ_2	-0.3025	0.0007	[-0.4179,-0.1868]	1.05
σ_2^2	2.0207	0.0025	[1.6806,2.4113]	-0.73
y_{3t}				
λ_{30}	0.4735	0.0015	[0.2629,0.6904]	-0.23
λ_{31}	0.3372	0.0010	[0.1386,0.5358]	0.85
ψ_3	0.4481	0.0013	[0.3224,0.5679]	0.81
σ_3^2	3.0486	0.0030	[2.5843,3.5953]	1.43
y_{4t}				
λ_{40}	0.5615	0.0014	[0.4061,0.7167]	1.21
λ_{41}	0.4258	0.0023	[0.2692,0.5835]	1.10
ψ_4	0.1391	0.0009	[0.0027,0.2802]	-0.12
σ_4^2	1.6958	0.0030	[1.4137,2.0224]	1.43
y_{5t}				
λ_{50}	0.8001	0.0015	[0.6848,0.9222]	0.47
λ_{51}	-0.2616	0.0030	[-0.4082,-0.1138]	1.03
ψ_5	-0.1975	0.0010	[-0.3266,-0.0666]	1.19
σ_5^2	0.7397	0.0028	[0.5969,0.9018]	1.55

Note: y_{1t}, y_{2t}, y_{3t}, y_{4t}, y_{5t}represent IIP95P, SCI95, ESRAO, HWINMF, and CELL9 respectively. The first 2000 draws are discarded and then the next 10000 are used for calculating the posterior means, the standard errors of the posterior means, 95% interval, and the convergence diagnostic (CD) statistics proposed by Geweke (1992). The posterior means are computed by averaging the simulated draws. The standard errors of the posterior means are computed using a Parzen window with a bandwidth of 1000. The 95% intervals are calculated using the 2.5th and 97.5th percentiles of the simulated draws. The CD is computed using equation (30), where we set $n_A = 1000$ and $n_B = 5000$ and compute $\hat{\sigma}_A^2$ and $\hat{\sigma}_B^2$ using a Parzen window with bandwidths of 100 and 500 respectively.

(C) Diagnostic Check for the Kim and Nelson Model

	IIP95P	SCI95	ESRAO	HWINMF	CELL9
Mean	0.0304	0.0178	0.0230	0.0356	0.0287
	(0.0581)	(0.0565)	(0.0570)	(0.0558)	(0.0584)
St. dev.	1.0223	0.9950	1.0027	0.9829	1.0274
Skewness	-0.2690	-0.1454	-0.1355	0.3565	0.1320
	(0.1391)	(0.1391)	(0.1391)	(0.1391)	(0.1391)
Kurtosis	4.0703	7.9968	5.7695	5.0004	4.8018
	(0.2782)	(0.2782)	(0.2782)	(0.2782)	(0.2782)
LB(6)	8.31	18.17	44.17	61.14	13.43

Note: Numbers in bracket are standard errors. LB(6) is the Ljung-Box statistic including six lags. The critical values for LB(6) are: 10.64 (10%), 12.59 (5%), 16.81 (1%).

TABLE 3. Estimation Results for Dataset 2.

(A) Kim and Nelson Model

Marginal Likelihood = -2202.10

Parameter	Mean	Standard Error	95% Interval	CD
ΔC_t				
π_{00}	0.9178	0.0010	[0.8334,0.9698]	-0.42
π_{11}	0.9368	0.0018	[0.8401,0.9779]	0.955
ϕ_1	-0.2761	0.0057	[-0.4595,-0.0597]	-0.00
ϕ_2	0.0008	0.0049	[-0.1641,0.2059]	-0.31
ϕ_3	0.2657	0.0033	[0.1236,0.4273]	-0.29
μ_0	-0.6341	0.0149	[-0.9535,-0.0767]	-0.80
μ_1	1.0687	0.0242	[0.1435,1.5009]	0.97
y_{1t}				
λ_{10}	1.0930	0.0041	[0.9875,1.2135]	-0.79
λ_{11}	-0.4251	0.0042	[-0.5467,-0.2959]	-1.33
ψ_1	-0.4006	0.0010	[-0.5269,-0.2681]	0.34
σ_1^2	0.2649	0.0010	[0.1997,0.3434]	-1.32
y_{2t}				
λ_{20}	0.8629	0.0035	[0.7463,0.9936]	-0.76
λ_{21}	-0.2101	0.0031	[-0.3285,-0.0920]	-1.39
ψ_2	-0.3417	0.0006	[-0.4509,-0.2308]	-0.44
σ_2^2	0.9226	0.0009	[0.7738,1.0903]	-0.78
y_{3t}				
λ_{30}	0.5736	0.0021	[0.4377,0.7180]	-1.05
λ_{31}	0.4670	0.0022	[0.3356,0.6070]	-0.89
ψ_3	0.0848	0.0017	[-0.0478,0.2173]	-0.38
σ_3^2	1.6668	0.0032	[1.4080,1.9668]	-1.01
y_{4t}				
λ_{40}	1.1547	0.0045	[1.0146,1.2890]	-0.85
λ_{41}	-0.4274	0.0045	[-0.5554,-0.2921]	-1.28
ψ_4	-0.4088	0.0014	[-0.5588,-0.2520]	-0.86
σ_4^2	0.3516	0.0008	[0.2717,0.4494]	-0.41
y_{5t}				
λ_{50}	1.1268	0.0043	[1.0180,1.2518]	-1.21
λ_{51}	-0.3209	0.0044	[-0.4478,-0.1810]	-1.39
ψ_5	-0.2604	0.0013	[-0.3971,-0.1215]	-1.09
σ_5^2	0.3233	0.0008	[0.2504,0.4097]	1.26

Note: y_{1t}, y_{2t}, y_{3t}, y_{4t}, y_{5t} represent IIP95P, SMSALE, HWINMF, IIP95O, and IIP95P. The first 2000 draws are discarded and then the next 10000 are used for calculating the posterior means, the standard errors of the posterior means, 95% interval, and the convergence diagnostic (CD) statistics proposed by Geweke (1992). The posterior means are computed by averaging the simulated draws. The standard errors of the posterior means are computed using a Parzen window with a bandwidth of 1000. The 95% intervals are calculated using the 2.5th and 97.5th percentiles of the simulated draws. The CD is computed using equation (30), where we set $n_A = 1000$ and $n_B = 5000$ and compute $\hat{\sigma}_A^2$ and $\hat{\sigma}_B^2$ using a Parzen window with bandwidths of 100 and 500 respectively.

(B) Stock and Watson Model

Marginal Likelihood = -2203.91

Parameter	Mean	Standard Error	95% Interval	CD
ΔC_t				
ϕ_1	-0.0869	0.0028	[-0.2229 , 0.0499]	1.23
ϕ_2	0.1715	0.0012	[0.0627 , 0.2783]	0.71
ϕ_3	0.3785	0.0007	[0.2728 , 0.4830]	-0.91
y_{1t}				
λ_{10}	1.2304	0.0028	[1.1229 , 1.3471]	0.01
λ_{11}	-0.4982	0.0031	[-0.6302 , -0.3704]	-1.24
ψ_1	-0.3874	0.0009	[-0.5164 , -0.2544]	-0.16
σ_1^2	0.2329	0.0007	[0.1743 , 0.3009]	0.00
y_{2t}				
λ_{20}	0.9695	0.0021	[0.8475 , 1.0997]	-0.08
λ_{21}	-0.2596	0.0025	[-0.3910 , -0.1335]	-1.21
ψ_2	-0.3395	0.0007	[-0.4512 , -0.2278]	-1.05
σ_2^2	0.9042	0.0009	[0.7572 , 1.0738]	-1.40
y_{3t}				
λ_{30}	0.6409	0.0014	[0.4902 , 0.7936]	-0.37
λ_{31}	0.5017	0.0018	[0.3556 , 0.6511]	-0.49
ψ_3	0.0814	0.0013	[-0.0476 , 0.2171]	-1.01
σ_3^2	1.6608	0.0026	[1.4044 , 1.9578]	-1.00
y_{4t}				
λ_{40}	1.3007	0.0030	[1.1849 , 1.4245]	-0.06
λ_{41}	-0.5049	0.0033	[-0.6453 , -0.3658]	-1.16
ψ_4	-0.3640	0.0008	[-0.4894 , -0.2363]	-0.78
σ_4^2	0.3063	0.0007	[0.2389 , 0.3831]	0.07
y_{5t}				
λ_{50}	1.2589	0.0030	[1.1434 , 1.3777]	-0.01
λ_{51}	-0.3726	0.0010	[-0.5166 , -0.2282]	-1.24
ψ_5	-0.2816	0.0013	[-0.4095 , -0.1515]	-1.42
σ_5^2	0.3159	0.0009	[0.2479 , 0.3945]	0.58

Note: y_{1t}, y_{2t}, y_{3t}, y_{4t}, y_{5t} represent IIP95P, SMSALE, HWINMF, IIP95O, and IIP95P. The first 2000 draws are discarded and then the next 10000 are used for calculating the posterior means, the standard errors of the posterior means, 95% interval, and the convergence diagnostic (CD) statistics proposed by Geweke (1992). The posterior means are computed by averaging the simulated draws. The standard errors of the posterior means are computed using a Parzen window with a bandwidth of 1000. The 95% intervals are calculated using the 2.5th and 97.5th percentiles of the simulated draws. The CD is computed using equation (30), where we set $n_A = 1000$ and $n_B = 5000$ and compute $\hat{\sigma}_A^2$ and $\hat{\sigma}_B^2$ using a Parzen window with bandwidths of 100 and 500 respectively.

(C) Diagnostic Check for the Kim and Nelson (1997) Model

	IIP95P	SMSALE	HWINMF	IIP95O	IIP95P
Mean	0.0129	0.0107	0.0263	0.0210	0.0160
	(0.0590)	(0.0571)	(0.0559)	(0.0591)	(0.0586)
St. dev.	1.0384	1.0051	0.9843	1.0406	1.0318
Skewness	-0.3275	0.0174	0.3323	-0.2062	-0.2058
	(0.1391)	(0.1391)	(0.1391)	(0.1391)	(0.1391)
Kurtosis	4.2933	4.4246	4.7422	3.5241	3.6938
	(0.2782)	(0.2782)	(0.2782)	(0.2787)	(0.2787)
LB(6)	4.93	11.92	69.36	10.82	4.94

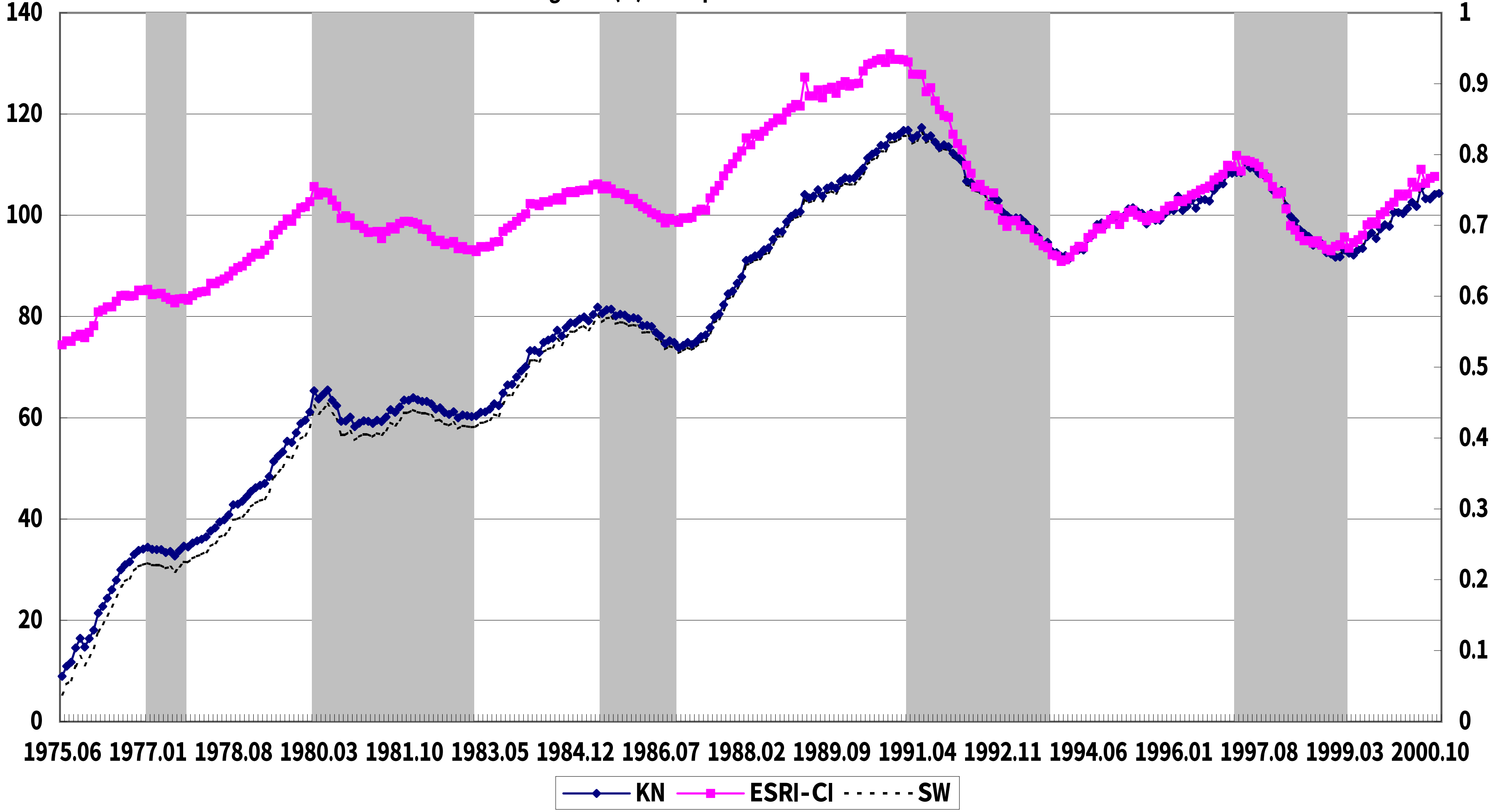
Note: Numbers in bracket are standard errors. LB(6) is the Ljung-Box statistic including six lags. The critical values for LB(6) are: 10.64 (10%), 12.59 (5%), 16.81 (1%).

Table 4 ESRI Business cycle turning points versus our turning points based on Kim and Nelson's posterior probability.

	ESRI	K&N(Data1)	K&N(Data2)	K&N(Data4)
P	1975.03			
T	1977.01			
P				
P	1977.10			
T	1980.02	1980.02	1980.02	1980.02
P		1981.03		1981.06
T		1981.10		1981.10
P	1983.02	1982.12	1982.12	1982.12
T	1985.06	1985.05	1985.05	1985.05
P	1986.11	1986.11	1986.11	1986.11
T	1991.02	1990.12	1990.12	1991.01
P	1993.10	1994.01	1994.01	1994.01
T		1995.03	1995.04	1995.04
P		1995.09	1995.09	1995.09
T	1997.03	1997.03	1997.05	1997.05
P	1999.04	1999.02	1999.01	1999.01
T			2000.08	2000.08

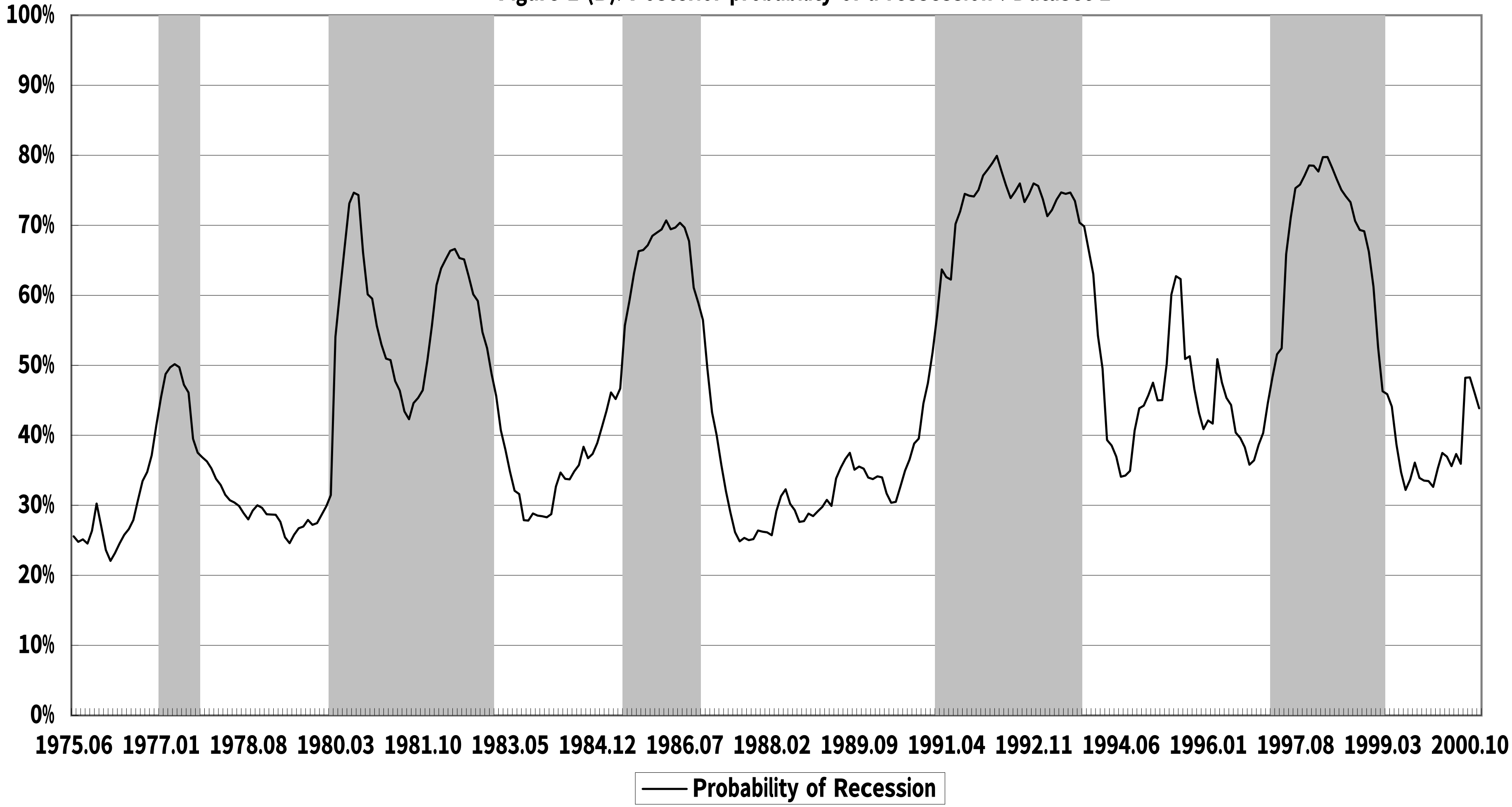
"P"(peak) indicates the date when the posterior probability $P(S_t = 1|y^T) > 0.5$ and $P(S_{t+1} = 1|y^T) < 0.5$. "T"(trough) indicates the date when the posterior probability $P(S_t = 1|y^T) < 0.5$ and $P(S_{t+1} = 1|y^T) > 0.5$.
The column "K&N" is our turning point based on Kim and Neslson model. "ESRI" is the reference date by the Economic and Social Reserch Institution in Cabinet office.

Figure 1(A). Composite Indexes Based On Dataset 1



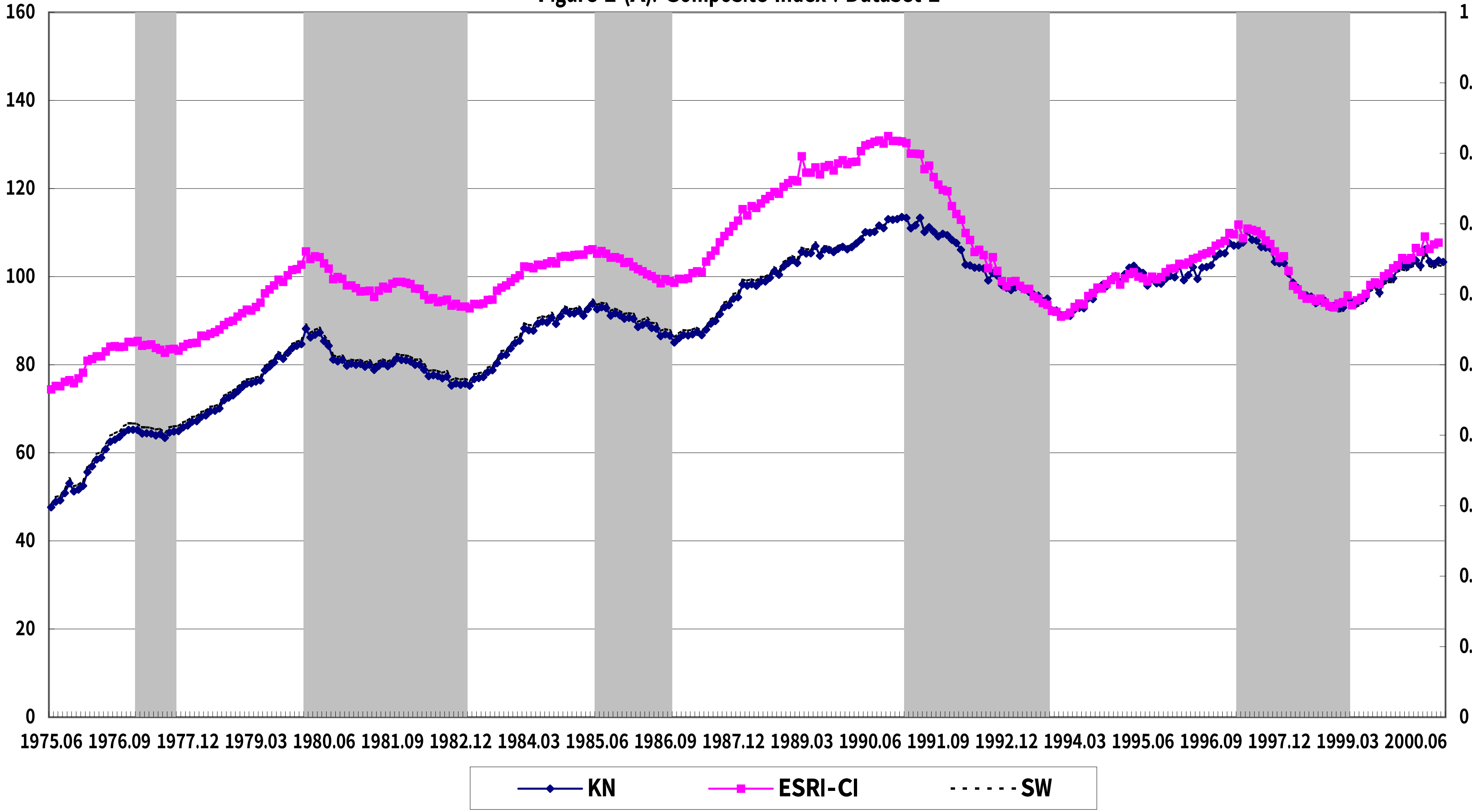
*1. Data set:
IIP95P,SCI95,ESRAO,HWINMF,CELL9
*2. Shaded areas are the contraction periods

Figure 1 (B). Posterior probability of a ressession : Dataset 1



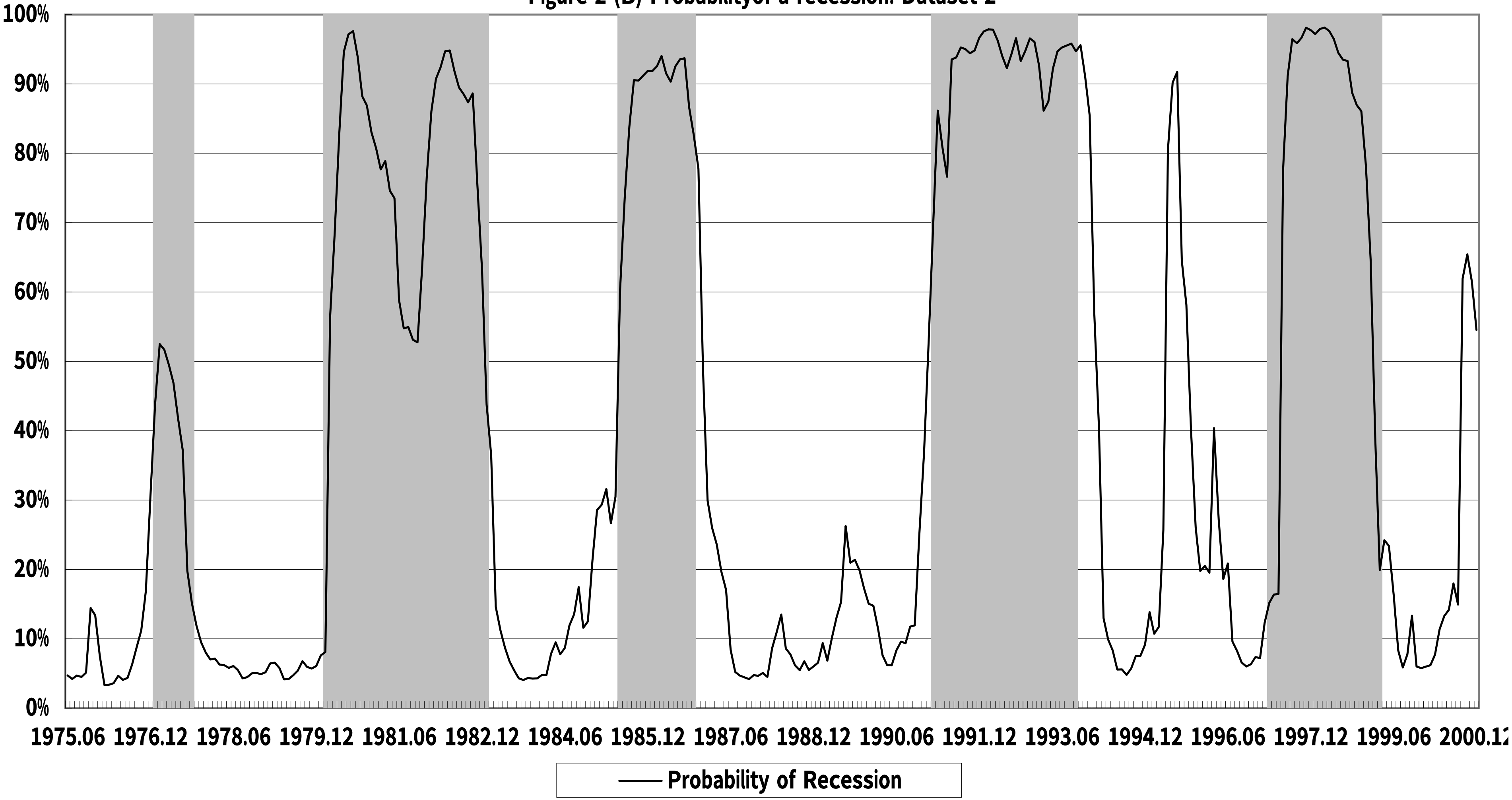
*1. Shaded areas are the contraction periods

Figure 2 (A). Composite Index : Dataset 2



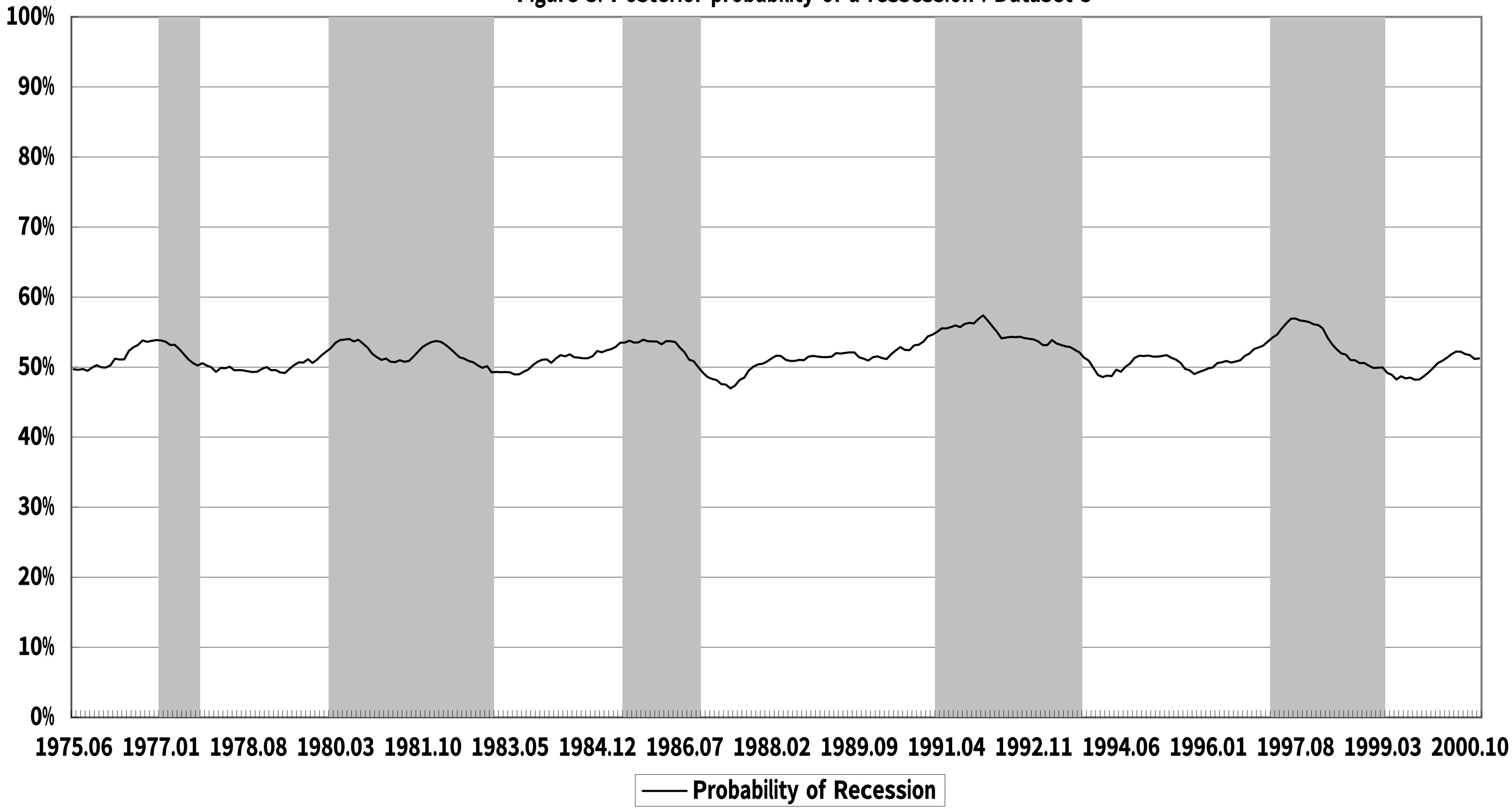
*1. Data set: IIP95P,
SMSALE,HWINMF,IIP95O,IIP95M
*2. Shaded areas are the contraction periods.

Figure 2 (B) Probabilityof a recession: Dataset 2



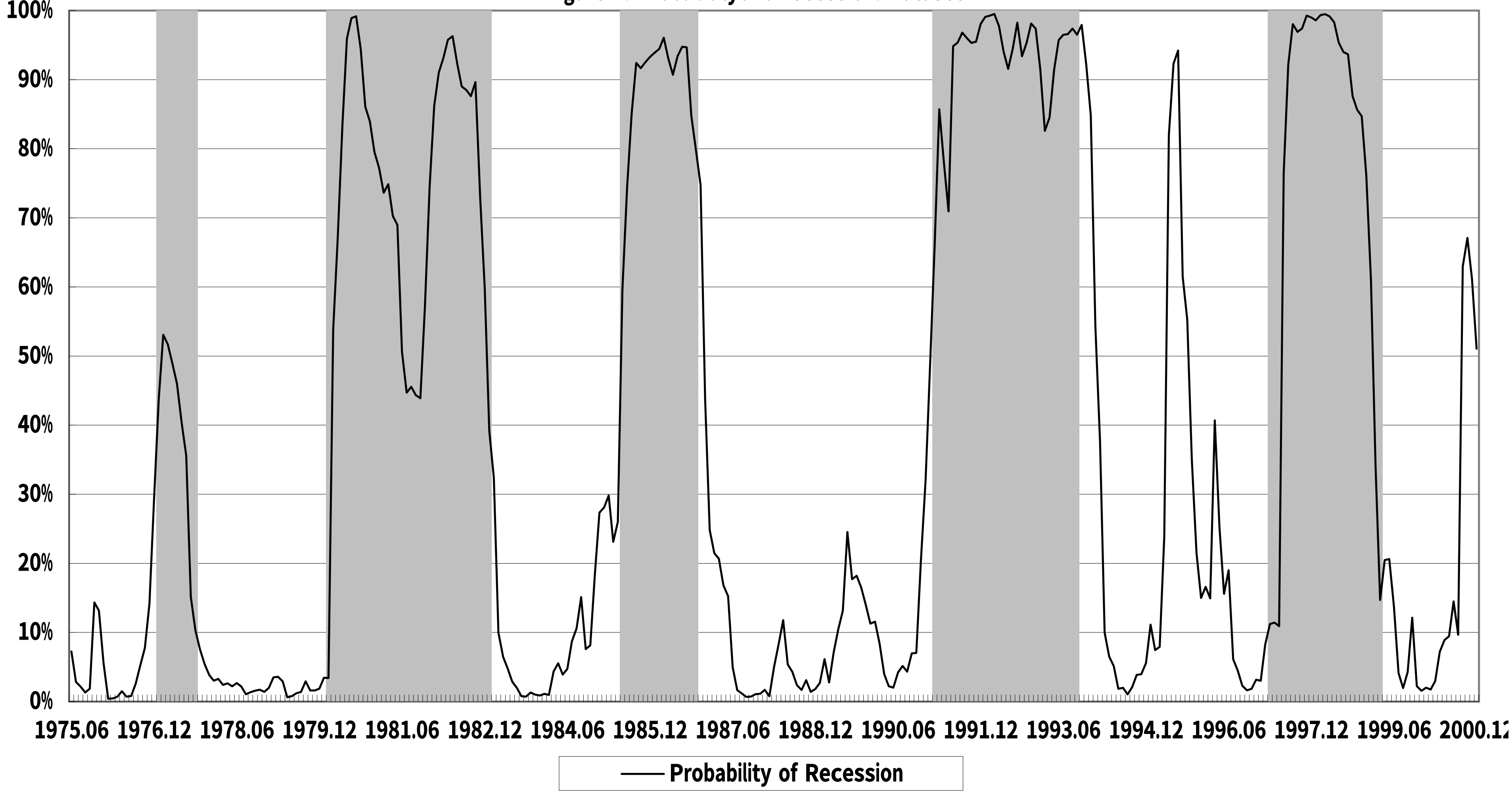
*1. Shaded areas are the contraction periods.

Figure 3. Posterior probability of a ressession : Dataset 3



*1. Shaded areas are the contraction periods

Figure 4. Probabilityof a recession: Dataset 4



*1. Shaded areas are the contraction periods.