Option Pricing Performance under Stochastic Volatility in Japanese Security Market

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Abstract

This article compares pricing performances of two representative option pricing models under stochastic volatility, *i.e.*, *log-volatility model* and *square-root volatility model*, by employing Japanese Nikkei 225 index options data. We estimate the parameters of volatility process by adopting Monte Carlo filter approach of Kitagawa (1996) and compare the option pricing performances of alternative option pricing models over both in-sample and out-of-sample period. The results show that incorporating stochastic volatility into option pricing model significanly improves pricing performace relative to Black-Scholes model, and in particular, square-root volatility model outperforms log-volatility model.

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Key Words: Option Pricing, Stochastic Volatility, Nikkei 225 Index Option, Monte Carlo Filter

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1 Introduction

The option pricing under stochastic volatility is one of the relatively longstanding topics in finance literature. The empirical observations of underlying return process have called for option pricing model accommodating stochastic structure of the volatility. In option pricing model, the volatility process has been usually imposed as an additional state variable. However, the specification of unobservable volatility process is more or less an empirical issue rather than economic reasoning.[1]

Since the seminal work of Hull and White (1987), two option pricing models under stochastic volatility have been popularized. The first strand of the literature is to specify that the log-volatility follows mean-reverting process. This model has been discussed by Scott (1987), Chesney and Scott (1989), and Melino and Turnbull (1990), etc. Many empirical studies equipped with newly coined techniques have focused on this model since the discrete version can be easily converted into state space form with AR(1) state process. For example, see Harvey, Ruiz and Shephard (1994), Ghysels, Harvey and Renault (1996), and the references therein. Nevertheless, in this case, the closed-form expression of option pricing formula is not available, and consequently, researchers usually resort to Monte Carlo simulation and/or numerical techniques.

The second study is to assume that the squared-volatility obeys square-root process. This model provides a closed-form expression of option value with Fourier inversion involved. Since Heston (1993), theoretical improvements covering jump behavior of underlying asset, for example have been accomplished. See Scott (1997), Bakshi and Chen (1997), Heston and Nandi (2000), and Duffie, Pan and Singleton (2000), for instance. Many financial economists have investigated this type of model based on cross sectional analysis which is the standard practice of extracting information from the market prices of traded options, and concluded

[1]Of course, the endogenous volatility process of asset return can be also derived in the general equilibrium context.

that accommodating stochastic volatility into option pricing contributes to the improvement of pricing performance. In this regard, Bates (1996), Bakshi, Cao and Chen (1997), and Nandi (1998) are notable studies.

We will investigate aforementioned two option pricing models by comparing their pricing performances in Japanese security market. Notwithstanding the abundance of empirical analysis of the so called *stochastic volatility model* (the discrete time equivalent of log-volatility model), the option pricing performance using these estimation results has not extensively explored, except some early period studies such as Scott (1987), Melino and Turnbull (1990). In contrast, the square-root volatility model have been investigated extensively by *implied parameters approach*. It is also generally agreed that the option pricing model incorporating the square-root volatility process enhances the pricing performance relative to Black-Scholes model. This unbalanced amount of empirical results seems to be due to the fact that the square-root volatility model has the closed-from expression of option value. In addition, two models have not been compared yet. These facts motivate this study.

Using the Nikkei 225 index returns data, we estimate the parameters of volatility process by adopting Monte Carlo filter method of Kitagawa (1996). This approach is based on a Monte Carlo method in which successive prediction, filtering, conditional probability density functions are approximated by many of their realizations. This method can be applied to a broad class of nonlinear no-Gaussian state space model. Next, the market price of volatility risk is estimated by using options data and parameters estimates. Then, the parameter estimates together with risk premium of volatility allow us to compare pricing performances of alternative option models. However, as mentioned before, the explicit option pricing formula under log-volatility process is not available. To circumvent this valuation problem, we employ an approximate valuation approach suggested by Kunitomo and Kim (2001).

The plan of this chapter is as follows. Section 2 discusses option pricing under two alternative volatility processes. The econometric methodology is addressed in section 3. Section 4 describes sample data. Section 5 reports the empirical results. Section 6 concludes. Finally, section 7 gives brief appendices.

2 Option Pricing under Stochastic Volatility

2.1 Option Pricing under Log-Volatility Process

Let us consider the economy where there are two primitive assets, *i.e.*, the stock and money market account. The stock price S_t obeys the stochastic differential equation

$$dS_t = \mu(S_t, V_t, t)S_t dt + \sigma_t S_t dW_{1t}, \tag{1}$$

where the volatility σ_t is generated by

$$d\,\log\sigma_t = \kappa(\theta - \log\sigma_t)dt + \delta\,dW_{2t}, \tag{2}$$

or, equivalently

$$d\sigma_t = \kappa(\theta + \frac{1}{2\,\kappa}\delta^2 - \log\sigma_t)\,\sigma_t\,dt + \delta\,\sigma_t\,dW_{2t} \tag{3}$$

for constants κ, θ, and δ. The return process and its volatility are assumed to have a constant correlation *i.e.*, $E[dW_{1t}\,dW_{2t}] = \rho\,dt$. We assume that the interest rate r is constant and the stock generates a constant dividend yield d. The setup of (1) and (2) has been investigated by Wiggins (1987), Scott (1987), Chesney and Scott (1989), Scott (1991), and Melino and Turnbull (1990) in evaluating stock and currency options. [2]

We consider a European call option on the security S with expiration date T, whose price is denoted by $C(S_t, \sigma_t, \tau)$ with $\tau \equiv T - t$ and exercise price K.

[2] In replacement of (2), some researchers assume that the log-'variance' follows mean-reverting process:

$$d\log\sigma_t^2 = \kappa(\theta - \log\sigma_t^2)dt + \delta dW_{2t}. \tag{4}$$

The assumption of (4) seems to be largely motivated by econometric tractability. See, Harvey, Ruiz and Shephard (1994), for example.

¿From the general equilibrium argument (for example, Cox, Ingersoll and Ross (1985)), the fundamental partial differential equation for a European call option pricing function $C(S_t, \sigma_t, \tau)$ becomes

$$\begin{aligned} &\frac{1}{2}\sigma^2 S^2 C_{SS} + \rho\delta\sigma^2 S C_{S\sigma} + \frac{1}{2}\delta^2\sigma^2 C_{\sigma\sigma} + (r-d)SC_S \\ &+\Big[\sigma\,\kappa(\theta + \frac{\delta^2}{2\,\kappa} - \log\sigma) - \lambda^*(\sigma)\Big]C_\sigma - r\,C - C_\tau = 0, \end{aligned} \tag{5}$$

with initial boundary condition $C(S_t, \sigma_t, 0) = \max[S_T - K, 0]$, where the subscripts on C represent partial derivatives with respect to each variables and $\lambda^*(\sigma)$ is the risk premium associated with stochastic volatility. Following Melino and Turnbull (1990), we set $\lambda^* = \lambda\,\delta\,\sigma_t$ for constant λ.

The equation (5) also gives the option value which is represented by

$$C(S_t, \sigma_t, \tau) = \tilde{E}_t[\exp(-r\,\tau)\max[S_T - K, 0]], \tag{6}$$

where $\tilde{E}$ is the risk-adjusted expectations operator and the risk adjustment is embodied in two state variables S and σ:

$$dS_t = (r-d)S_t dt + \sigma_t S_t dW_{1t} \tag{7}$$

and

$$d\,\log\sigma_t = \kappa(\theta^* - \log\sigma_t)dt + \delta\,dW_{2t}, \tag{8}$$

where $\theta^* = \theta - \frac{\lambda\,\delta}{\kappa}$, or, equivalently

$$d\sigma_t = \kappa(\theta^{**} - \log\sigma_t)\sigma_t\,dt + \delta\,\sigma_t\,dW_{2t}, \tag{9}$$

where $\theta^{**} = \theta^* + \frac{\delta^2}{2\,\kappa}$.

For the calculation of option value under log-volatility process, we adopt an approximation approach called small disturbance expansion approach proposed by Kunitomo and Kim (2001), and Kim (2001). It should be noted that in our setting we have $C(S_t, \sigma_t, \tau) = \exp(-d\,\tau)\hat{C}(S_t, \sigma_t, \tau; \hat{r})$ with $\hat{r} \equiv r - d$, since the interest rate r is assumed to be constant. Using (7) and (9), and following

Kunitomo and Kim (2001) provide the expression for option value:

$$\hat{C}(S,\sigma,\tau;\hat{r}) = [S_0\,\Phi(d_1) - K\,\exp(-\hat{r}\,\tau)\Phi(d_2)] + \delta\,S_0\,\phi(d_1)\Big[\frac{a_{12}}{\sqrt{\Sigma}} - \frac{a_{11}}{\Sigma}d_2\Big] + o(\delta), \tag{10}$$

where $\Phi(\cdot)$ is the distribution function of standard normal variable and $\phi(\cdot)$ is its density function. The integrated variance through time to expiration, Σ, is equal to $\int_0^T \bar{\sigma}_t^2 dt$, where

$$\bar{\sigma}_t = \exp[\exp(-\kappa\,t)(\log\sigma_0 - \theta) + \theta].$$

In addition, d_1 is given by

$$d_1 = \frac{1}{\sqrt{\Sigma}}\left[\log\frac{S_0}{K} + \hat{r}\,\tau + \frac{1}{2}\Sigma\right]$$

and $d_2 = d_1 - \sqrt{\Sigma}$. The remaining coefficients a_{11} and a_{12} are given as follows:

$$a_{11} = \rho\int_0^T \bar{\sigma}_t Y_t \int_0^t Y_s^{-1}\bar{\sigma}_s^2\,ds\,dt$$

and

$$a_{12} = -\lambda\int_0^T \bar{\sigma}_t Y_t \int_0^t Y_s^{-1}\bar{\sigma}_s\,ds\,dt,$$

where

$$Y_t = \exp\left[(\log\sigma_0 - \theta)(\exp(-\kappa\,t) - \exp(-\kappa)) + \kappa(1-t)\right].$$

More tractable expressions of Σ, a_{11}, and a_{12} are provided in Appendix.

2.2 Option Pricing under Square-Root Volatility Process

Heston (1993) (in a similar fashion, Bates (1996), Scott (1997), Bakshi, Cao and Chen (1997), and Duffie, Pan and Singleton (2000) in an extended stochastic environment) assumed that the volatility obeys

$$d\,\sigma_t^2 = \kappa(\theta - \sigma_t^2)dt + \delta\,\sigma_t\,dW_{2t}, \tag{11}$$

with $E[dW_{1t}dW_{2t}] = \rho\,dt$. Following Heston (1993), the risk premium on the volatility risk is assumed to be proportional to the conditional variance, *i.e.* $\lambda(\sigma_t^2) = \lambda\,\sigma_t^2$ for constant λ. In this case, the closed form expression of option value can be available. We reproduce the option pricing formula in Appendix for convenience.

3 Econometric Methodology

Our test methodology takes the following steps. In the first step, the parameters of volatility process based on underlying return process are estimated. In the next step, we estimate the parameter of risk premium of volatility using the options data and the estimates of volatility parameters. In the final step, the pricing performances of in-sample and out-of-sample are compared after the pricing error measures have been defined.

3.1 Estimation of Volatility Parameters

First, consider the log-volatility model. We simply discretize the return process (1) and the volatility process (2):

$$R_{n\triangle} \equiv \frac{S_{n\triangle} - S_{(n-1)\triangle}}{S_{(n-1)\triangle}} = \mu(\cdot)\,\triangle + \sigma_{n\triangle}\,\epsilon_{1,n\triangle} \tag{12}$$

and

$$\log \sigma_{n\triangle} = \log \sigma_{(n-1)\triangle} + [\kappa\,\theta - \kappa\,\log \sigma_{(n-1)\triangle}]\triangle + \delta\,\sqrt{\triangle}\,(\rho\epsilon_{1,n\triangle} + \sqrt{1-\rho^2}\,\epsilon_{2,n\triangle}), \tag{13}$$

where $\epsilon_{1,n\triangle}$ and $\epsilon_{2,n\triangle}$ are two independent standard normal variables, $\triangle$ denotes time interval, and n is the positive integer.[3] We set the trade days in a year to be 250 days and therefore $\triangle = 1/250$. We should note that more natural approximations rather than (12) and (13) can be possible because the solutions of original SDEs are available. However, because the solution of square-root volatility is not known, the simple scheme (13) will be used for comparison purpose.

In principle, the specification of $\mu(\cdot)$ is problematic, because the option price (which is in itself not the function of $\mu(\cdot)$) is affected by way of parameters of

[3]To be precise, the $\sigma_{n\triangle}$ in (12) should be expressed by $\sigma_{(n-1)\triangle}$ due to the Euler-Maruyama approximation. Taylor (1994) referred to (12) and (13) as Contemporaneous Autoregressive Random Variance Model (in short, CARV), and one-lag volatility version of (12) and (13) as Lagged ARV (LARV). Scott (1987) used CARV, whereas Chesney and Scott (1989) used LARV, for example.

volatility process which is, in turn, influenced by the specification of the drift of return process. This issue is also spelled out in detail in the next section. We newly define the observation process as $y_{n\triangle} \equiv R_{n\triangle} - \mu(\cdot)\triangle$. We also set $x_{n\triangle} \equiv \log \sigma_{n\triangle}$.

Let Ψ denote $(\kappa, \theta, \delta, \rho)$. We estimate Ψ by Monte Carlo filter/smoother approach developed by Kitagawa (1996). In this method, each distribution is expressed by many of its realizations, and the trajectory of each particle in successive prediction stages is simulated by using assumed model. In the filtering stage, the resampling with a weight proportional to the likelihood is performed to get a set of particles that represents the filter distribution.

If we define $Y_{n'\triangle}$ as the set of observations $\{y_{1\triangle}, \cdots, y_{n'\triangle}\}$, the conditional density $p(x_{n\triangle}|Y_{n'\triangle})$ is called the predictor, the filter, and the smoother, respectively corresponding to the three distinct cases , $n > n'$, $n = n'$, and $n < n'$. Monte Carlo filter/smoother approximate the distributions by empirical distributions determined by the set of particles. Let N be the number of data observations and m the number of particles. We denote the particles of predictor and filter by $p^{(j)}_{n\triangle}$ and $f^{(j)}_{n\triangle}$ for each day n and $j = 1, \cdots, m$.

Monte Carlo filtering can be conducted by adopting the following 2 steps.

1. Generate a random number $f^{(j)}_{0\triangle} \sim N(\theta, \frac{\delta^2}{2\kappa})$ for $j = 1, \cdots, m$, where $N(\cdot, \cdot)$ is normal distribution function.

2. Repeat the following steps for $n = 1, \cdots, N$.

   (a) Generate two independent standard normal variables $\epsilon^{(j)}_{1,n\triangle}$ and $\epsilon^{(j)}_{2,n\triangle}$ for $j = 1, \cdots, m$.

   (b) Compute $p^{(j)}_{n\triangle} = f^{(j)}_{(n-1)\triangle} + [\kappa\theta - \kappa f^{(j)}_{(n-1)\triangle}]\triangle + \delta\sqrt{\triangle}\,(\rho\,\epsilon_{1,n\triangle} + \sqrt{1-\rho^2}\,\epsilon_{2,n\triangle})$ for $j = 1, \cdots, m$.

   (c) Compute $a^{(j)}_{n\triangle} = \phi(y_{n\triangle}\exp(-p^{(j)}_{n\triangle})) \cdot |\exp(-p^{(j)}_{n\triangle})|$ for $j = 1, \cdots, m$, where $\phi(\cdot)$ is the standard normal density function.

(d) Generate $f_{n\triangle}^{(j)} \sim (\sum_{i=1}^{m} a_{n\triangle}^{(i)})^{-1} \sum_{i=1}^{m} a_{n\triangle}^{(i)} I(x, p_{n\triangle}^{(i)})$ for $j = 1, \cdots, m$ by the resampling of $p_{n\triangle}^{(1)}, \cdots, p_{n\triangle}^{(m)}$.

In step 1, we set the initial filter to follow the steady state distribution of OU process.

The maximum likelihood estimates of parameters Ψ can be estimated by maximizing the log-likelihood $l(\Psi)$:

$$l(\Psi) = \sum_{n=1}^{N} \log p(y_n|Y_{n-1}) \cong \sum_{n=1}^{N} \log(\sum_{j=1}^{m} a_n^{(j)}) - N \log m. \tag{14}$$

We obtain the maximum likelihood estimates by a grid search.

The estimation of square-root volatility model can be done similarly. The square-root volatility (11) is also discretized as

$$\sigma_{n\triangle}^2 = \sigma_{(n-1)\triangle}^2 + [\kappa\,\theta - \kappa\,\sigma_{(n-1)\triangle}^2]\triangle + \delta\sigma_{(n-1)\triangle}\sqrt{\triangle}\,(\rho\epsilon_{1,n\triangle} + \sqrt{1-\rho^2}\,\epsilon_{2,n\triangle}). \tag{15}$$

We set $x_{n\triangle} \equiv \sigma_{n\triangle}^2$. The monte carlo filtering are modified as follows.

1. Generate a random number $f_{0\triangle}^{(j)} \sim Ga(\frac{2\kappa\theta}{\delta^2}, \frac{2\kappa}{\delta^2})$ for $j = 1, \cdots, m$, where $Ga(\cdot, \cdot)$ is gamma distribution function.

2. Repeat the following steps for $n = 1, \cdots, N$.

   (a) Not changed

   (b) Compute $p_{n\triangle}^{(j)} = f_{(n-1)\triangle}^{(j)} + [\kappa\theta - \kappa f_{(n-1)\triangle}^{(j)}]\triangle + \delta\sqrt{f_{(n-1)\triangle}^{(j)}}\sqrt{\triangle}\,(\rho\,\epsilon_{1,n\triangle} + \sqrt{1-\rho^2}\,\epsilon_{2,n\triangle})$ for $j = 1, \cdots, m$.

   (c) Compute $a_{n\triangle}^{(j)} = \phi(y_{n\triangle}/\sqrt{p_{n\triangle}^{(j)}}) \cdot |1/\sqrt{p_{n\triangle}^{(j)}}|$ for $j = 1, \cdots, m$, where $\phi(\cdot)$ is the standard normal density function.

   (d) Not changed

In step 1, we set the initial filter to follow the steady state distribution of square-root process. To compare the goodness of the fit of two candidate models, the Akaike's Information Criterion (AIC), defined by $AIC = -2 \cdot l(\hat{\Psi}) + 2 \cdot \#(\Psi)$ is evaluated. We use routines **ran2** and **gamdev** in Press *et. al.* (1992) as random uniform and normal deviates, respectively and **GAMMA(S)** in Dagpunar (1988) as random gamma deviate.

3.2 Estimation of Risk Premium

For the estimation of risk premium of volatility λ, options data over the estimation period should be utilized. To this end, we simply adopt the nonlinear least squares regression whose estimator is obtained by solving the problem:

$$\lambda = \arg\min_{\lambda} \sum_{n=1}^{N} \sum_{i=1}^{M_n} \left(\frac{C(i_n; \hat{\Psi}, \hat{\sigma}_n)}{S(i_n)} - \frac{C(i_n)}{S(i_n)} \right)^2, \tag{16}$$

where $C(i_n)$ (res.$S(i_n)$) is the option price (res. stock price) observed at time $n\triangle$. We assume that the error term is *i.i.d.* random variable with mean 0. It is expected that both observed and theoretical option price normalized by stock price ensure this assumption of the error term. The standard error of λ is calculated based on (4.3.21) in Amemiya (1985). The nonlinear least squares estimator applied to (16) is obtained using at-the-money options data.

3.3 Pricing Performance Measure

We provide three pricing performance measures over the test period. The first and second one are yen-basis pricing errors, and the last one is relative error.

First, using the estimates of Ψ, λ and σ_n for $n = 1, ..., N$, we calculate Mean Absolute Error of option pricing (in short, MAE) defined by

$$MAE = \frac{1}{\sum_{n=1}^{N} M_n} \sum_{n=1}^{N} \sum_{i=1}^{M_n} |C(i_n) - C(i_n; \hat{\Psi}, \hat{\lambda}, \hat{\sigma}_n)|, \tag{17}$$

where $C(i_n)$ is the ith option price observed at time n and M_n is the number of observations at time n.

Second, we also provide Root Mean Squared Error of option pricing (in short, RMSE) defined by

$$RMSE = \sqrt{\frac{1}{\sum_{n=1}^{N} M_n} \sum_{n=1}^{N} \sum_{i=1}^{M_n} \left[C(i_n) - C(i_n; \hat{\Psi}, \hat{\lambda}, \hat{\sigma}_n) \right]^2}, \tag{18}$$

Finally, Root Mean Squared Error Relative to option price (in short, RMSER) is given as follows:

$$RMSER = \sqrt{\frac{1}{\sum_{n=1}^{N} M_n} \sum_{n=1}^{N} \sum_{i=1}^{M_n} \left[\frac{C(i_n) - C(i_n; \hat{\Psi}, \hat{\lambda}, \hat{\sigma}_n)}{C(i_n)} \right]^2}. \tag{19}$$

In calculating (17), (18), and (19), over out-of-sample period, we execute Monte Carlo filter to obtain the volatility estimates by utilizing the estimates of volatility parameters. For comparison purpose, we also set up 40 and 20 trade day historical volatilities, over test period.

4 Data

As sample data, we employ daily closing Nikkei 225 index and its option contracts written on the index. The source of our data is the Osaka Security Exchange. The sample covers the time period from January 4, 1991 until June 30, 1998 (Entire Period). The parameters of option pricing models are estimated using the data over January 4, 1991 to December 30, 1997 (Estimation Period). We set aside the six months period around the last day of estimation period, December 30, 1997 for evaluating pricing error. We call the test period before (respectively, after) December 30, 1997 in-sample period (respectively, out-of-sample period).

As the proxy for the unobservable short rate, one month CD rate is adopted. The dividend yield data is taken from the predicted average dividend yield data (*Yoso-kijun Heikin Rimawari*, which is announced by *Nihon Keizai Shimbun* on every trading day).

Table 1, Table 2 and Table 3 exhibit the descriptive statistics of underlying return series. The salient feature of statistics is that the rate of return process shows second or third order autocorrelation although higher order autocorrelations are not significant. Since the continuous time option pricing model admits no autocorrelation of rate of return, autocorrelations are filtered off by AR(3) estimates. That is, we use the disturbance series of AR(3) model as rate of return series to be estimated.

¿From Table 3, we observe that Ljung-Box Qs of the resultant return series drop significantly. This procedure can be justified by the fact that option pricing formulas are determined irrespective of the form of return process drift function. However we also should notice that aforementioned AR(3)-adjustment procedure is only one candidate strategy for removing autocorrelation, and option pricing formulas could be indirectly affected by the specification of the drift by way of the parameter estimates of the volatility process.[4]

The mean and standard deviation of original (respectively, AR(3)-adjusted) return series are -0.0916 (respectively, 0.0000) and 0.2262 (respectively, 0.2254) in annual basis. See also Figure 1.

Table 1: Descriptive Statistics of Nikkei 225 index Rate of Return

Raw implies the rate of return on Nikkei 225 index from January 7, 1991 through December 30, 1997. Adjusted stands for the autocorrelation-adjusted disturbance series as the result of AR(3) estimation.

	Sample	Mean(%)	Std. Dev.	Skewness	Kurtosis	Min	Max
Raw	1726	-0.0366	0.0143	0.1032	5.6044	-0.0633	0.0737
Adjusted	1723	0.0000	0.0143	0.0744	5.5475	-0.0643	0.0734

Table 2: Parameter estimates of AR(3) model

$R_n = a_0 + a_1 R_{n-1} + a_2 R_{n-2} + a_3 R_{n-3} + \epsilon_n$ is estimated. The estimates are of percentage unit. s.e. implies White's heteroscedasticity-consistent standard error.

	a_0	a_1	a_2	a_3
estimates	-0.0372	-3.8721	-5.7017	0.7738
s.e.	0.0340	3.1848	3.1132	3.1321

[4] Lo and Wang (1995) specified the drift function as the trending O-U process and investigated the effect of this specification on the Black and Scholes value. Hafner and Herwartz (2001) also utilized this trend reversion process to capture the implication of autoregressive dynamics on the option value under stochastic volatility.

Table 3: Autocorrelation of Rate of Returns

Raw implies the rate of return on Nikkei 225 index from January 7, 1991 through December 30, 1997. Adjusted stands for the autocorrelation-adjusted disturbance series as the result of AR(3) estimation.

Lag	Autocorrelation		Ljung-Box Q		$\chi^2_{0.05}$(Lag)
	Raw	Adjusted	Raw	Adjusted	
1	-0.036	-0.0001	2.203	0.0000	3.841
2	-0.056	0.0009	7.590	0.0013	5.991
3	0.012	0.0024	7.830	0.0112	7.815
4	0.016	0.0143	8.277	0.3654	9.488
5	0.011	0.0086	8.499	0.4920	11.071
6	-0.010	-0.0055	8.672	0.5453	12.592
7	-0.006	-0.0012	8.735	0.5477	14.067
8	0.008	0.0069	8.849	0.6294	15.507
9	0.013	0.0131	9.150	0.9262	16.919
10	-0.003	-0.0011	9.162	0.9281	18.307
11	0.017	0.0163	9.670	1.3907	19.675
12	0.015	0.0158	10.083	1.8267	21.026

For the test period, we divide options data into several categories according to either moneyness and/or time to expiration. At-the-money sample is assumed to satisfy $0.97 \leq S/K \leq 1.03$. Out-of-the-money (respectively, in-the-money) sample is set to satisfy $S/K < 0.97$ (respectively, $1.03 < S/K$). The longest time to maturity of Nikkei 225 index option is four months in calendar day. It is relatively short in comparison with those of US and other European countries. Hence we divide the entire samples into short and medium term options according to time to expiration. The short term option has maturity time less or equal to 60 days. The medium term option takes 60 days to four months to mature. In addition, options whose price is less or equal to 5 yen are excised because these options have minor impacts on pricing errors.

5 Pricing Performance Results

5.1 The Estimates of Volatility Process

The estimates of volatility parameters are given in Table 4. It is worth while to remark some features of the results. Firstly, the estimates of θ imply that the long-term level of volatility of log-volatility model (respectively, square-root volatility model) is 22.3% (respectively, 20.0%), which is close to the sample counterpart. Secondly, the correlations of stock return and the volatility process in both model are negative, which is consistent with other empirical findings. Finally, from the AIC criteria, the log-volatility model shows only slightly better goodness of the fit than the square-root volatility model. [5]

The estimated volatility level, *per se* is critical inputs in pricing options. The two stochastic volatilities of option pricing models over the entire period including out-of-sample period are shown in Figure 2 and Figure 3. Meanwhile, investors in

[5] In this study, we do not discuss the reliability of estimates because our main concern is to examine the pricing performances. However, smoothing scheme of self-organizing state space model proposed by Kitagawa (1998) may give a guidance for this issue, for example.

Table 4: The Estimates of Volatility Parameters

The parameters of volatility, $\Psi \equiv (\kappa, \theta, \delta, \rho)$, are estimated by using the autocorrelation-corrected rate of return on Nikkei 225 stock index running from January 10, 1991 through December 30, 1997, 1723 time series observations.

Model	κ	θ	δ	ρ	Log L	AIC
Log-Volatility	0.54	-1.50	0.99	-0.20	5047.7897	-10087.5794
Square-Volatility	3.40	0.04	0.45	-0.10	5046.8223	-10085.6446

the Black-Scholes world generally use the historical volatility as the estimate of unique unobservable volatility parameter. It is generally argued that the implied volatility is also commonly used as the estimates of the volatility and pricing bias of Black-Scholes pricing formula is smaller than the case of historical volatility. This chapter focuses on the option pricing performances based on time series data. In this respect, we do not consider the benchmark Black-Scholes value based on the implied volatility using cross-sectional options data. For the implied parameters approach, see Bakshi, Cao and Chen (1997), and Nandi (1998), for example. In general, there is no established rule as to the time span which investors should take into account in estimating historical volatility. Therefore, we provide 40 and 20 trade day historical volatility to accommodate the ambiguity of historical volatility time span. The filtered volatility and historical volatility over in-sample and out-of-sample are depicted in Figure 4 and Figure 5. These results tell us that 40 trade days historical volatility is underestimated relative to the volatilities of stochastic volatility over in-sample period, while 20 trade days historical volatility is overestimated relative to the volatilities of stochastic volatility over out-of-sample period.

The remaining input for option pricing, the market price of volatility risk is given in Table 5. The risk premium of log-volatility (respectively, square-root volatility) is -0.2406 (respectively, -1.2146) and significant. Remind that negative values of λ induce higher option prices. The risk premium is estimated using the estimates of volatility parameters, the filtered volatility and options prices over July 1, 1992 to December 30, 1997. As options data, we used at-the-money samples which satisfy $0.98 \leq S_t/K \leq 1.2$.[6]

Table 5: The Estimates of Risk Premium

Based on nonlinear least squares regression, the risk premium λ is estimated by utilizing the estimates ψ and employing the options data from July 1, 1992 through December 30, 1997, 6,183 number of observations. SSR represents sum of squared residuals and s.e. denotes the standard error following (4.2.23) of Amemiya (1985). For the estimation of λ, Nikkei 225 index options with $0.98 \leq S_t/K \leq 1.2$ are used.

Model	λ	s. e.	SSR
Log-Volatility	-0.2406	0.0293	0.2820
Square-Volatility	-1.2146	0.0554	0.1962

5.2 In-Sample Performance

We have set the in-sample period to be July 1, 1997 to December 30, 1997, the last six calendar days of parameter estimation period. Table 6 provides the pricing performances of stochastic volatility option pricing models and Black-Scholes model.

The main results can be described as follows. Firstly, incorporating two stochastic volatility structures into option pricing models largely improves the pricing performance of the original Black and Scholes model. For 2558 total samples, MAEs of log and square-root volatility model are 82.774 and 67.351 while those of Black and Scholes model with 40 and 20 trade day historical volatility are 105.292 and 123.105, respectively. Similarly, RMSEs (respectively, RMSERs) of stochastic volatility model are 126.746 and 102.501 (respectively, 0.482 and

[6]The Nikkei 225 index option market has started as the near-American style option market on June 12, 1989 and completely shifted to European style option market on June 12, 1992. This fact is one of reasons why our options data begins from July 1, 1992.

0.383), while those of Black and Scholes model are 153.341 and 193.306 (respectively, 0.645 and 1.007). Furthermore, this results hold irrespective of moneyness and time to maturity.

Secondly, among option pricing models under two stochastic volatilities, the one under square-root volatility shows better performance. However, remind that for the goodness of the fit of return process, the log-volatility model is slightly better than the square-root volatility model.

Finally, the Black and Scholes model based on 20 trade days historical volatility shows the worst performance. This implies that Black and Scholes model is very sensitive to the time span of historical volatility.

To be summarized, it can be said that incorporating stochastic volatility into option pricing model significantly improves pricing performance relative to Black-Scholes model, and in particular, square-root volatility model outperforms log-volatility model.

5.3 Out-of-Sample Performance

We have set the out-of-sample period to be January 5, 1998 to June 30, 1998, the first six calendar days out of parameter estimation period. Table 7 provides the pricing performances of stochastic volatility option pricing models and Black-Scholes model.

The main results are similar to the case of in-sample period with some minor differences entailed. Firstly, incorporating two stochastic volatility structures into option pricing models also largely improves the pricing performance of the original Black and Scholes model. For 2568 total samples, MAEs of two stochastic volatility model are 98.346 and 87.047 while Black and Scholes model with 40 and 20 trade day historical volatility are 115.181 and 110.096, respectively. The values of RMSE and RMSER show the similar results. These results also hold irrespective of moneyness and time to maturity.

Secondly, the option pricing model under square-root volatility also shows the best performance.

Thirdly, the Black and Scholes model based on 40 trade days historical volatility shows the worst performance. This is the opposite result of in-sample period.

Finally, as it might be expected, the size of pricing errors of out-of-sample period are greater than those of in-sample period.

5.4 The Effect of Risk Premium on Pricing Performance

As seen before, the estimates of market price of volatility risk have significant negative values. We consider the effect of risk premium on the pricing performances. For this purpose, we calculate the pricing errors by setting the risk premium to be zero, $\lambda = 0$. Table 8 provides the pricing performances of two option pricing models under stochastic volatility, over in-sample and out-of-sample period.

For in-sample period, the pricing errors of two option models under stochastic volatility have dropped in a small magnitude. For total sample over in-sample period, MAEs of log-volatility and square-root volatility model with negative risk premiums are 82.774 and 67.351, while those with zero risk premium become 81.454 and 64.866, respectively. It seems that the reduction in pricing errors under the constraint $\lambda = 0$ is hard to be reconciled with investors behavior.

In contrast, the pricing errors of two stochastic volatility models have increased over out-of-sample period. In particular, the deterioration of pricing performance of square-root volatility model is noticeable. For total sample over out-of-sample period, MAEs of log-volatility and square-root volatility model with negative risk premiums are 98.346 and 87.047, while those with zero risk premium become 101.713 and 95.898, respectively.

To sum up, the non-zero market price of volatility risk is an important factor from the viewpoint of investors in the sense that incorporating risk premium contributes to performance improvement at least, over out-of-sample period.

Table 6: Option Pricing Performance: In-Sample Period

MAE, RMSE and RMSER are mean absolute error, root mean squared error and root mean squared error relative to option price, which are defined by (17), (18) and (19), respectively. Total is entire option samples over in-sample period *i.e.*, from July 1, 1997 through December 30, 1997. Short (respectively, Medium) is option sample with time to maturity smaller than 60 calendar days (respectively, over 60 calendar days to four months). ALL is entire sample under Total, Short, and Medium. ATM is at-the-money sample which satisfies $0.97 \leq S/K \leq 1.03$. OTM is out-of-the-money sample which satisfies $S/K < 0.97$. ITM is in-the-money sample which satisfies $1.03 < S/K$. Sample is the number of observed call option prices. BS40 (respectively, BS20)is the original Black-Scholes option pricing model based on 40 (respectively, 20) trade day historical volatility. log-volatility (respectively, square-root volatility) is the option pricing model under log-volatility (respectively, square-root volatility).

Sample			BS40			BS20			Log-Volatility			Square-Root Volatility		
Mat.	Mon.	Obs.	MAE	RMSE	RMSER	MAE	RMSE	RMSER	MAE	RMSE	RMSER	MAE	RMSE	RMSER
	ALL	2558	105.292	153.341	0.645	123.105	193.306	1.007	82.774	126.746	0.482	67.351	102.501	0.383
	ATM	794	112.626	155.654	0.263	130.363	194.367	0.316	86.181	124.947	0.227	68.625	98.096	0.189
Total	OTM	1181	88.093	137.516	0.920	108.650	186.310	1.455	63.892	104.043	0.679	46.837	71.035	0.536
	ITM	583	130.146	178.507	0.137	142.504	205.392	0.155	116.384	165.213	0.128	107.170	150.901	0.117
	ALL	1743	91.250	135.213	0.727	100.931	155.472	1.163	71.962	113.014	0.544	61.504	99.756	0.433
	ATM	499	101.258	138.390	0.285	106.717	152.686	0.326	74.254	103.972	0.244	59.896	88.628	0.210
Short	OTM	781	64.183	101.850	1.057	78.432	133.934	1.714	46.466	76.461	0.782	35.231	55.239	0.618
	ITM	463	126.121	175.177	0.138	132.646	188.709	0.148	112.497	163.000	0.129	107.554	154.433	0.120
	ALL	815	135.324	186.282	0.419	170.529	256.101	0.536	105.899	152.007	0.311	79.855	108.136	0.244
	ATM	295	131.857	181.150	0.222	170.361	249.495	0.297	106.357	154.060	0.193	83.390	112.308	0.145
Med.	OTM	400	134.776	188.626	0.562	167.651	259.733	0.714	97.915	143.338	0.407	69.499	94.554	0.321
	ITM	120	145.676	190.811	0.134	180.538	259.909	0.178	131.385	173.487	0.125	105.687	136.416	0.100

Table 7: Option Pricing Performance: Out-of-Sample Period

MAE, RMSE and RMSER are mean absolute error, root mean squared error and root mean squared error relative to option price, which are defined by (17), (18) and (19), respectively. Total is entire option samples over out-of-sample period *i.e.*, from January 5, 1998 through June 30, 1998. Short (respectively, Medium) is option sample with time to maturity smaller than 60 calendar days (respectively, over 60 calendar days to four months). ALL is entire sample under Total, Short, and Medium. ATM is at-the-money sample which satisfies $0.97 \leq S/K \leq 1.03$. OTM is out-of-the-money sample which satisfies $S/K < 0.97$. ITM is in-the-money sample which satisfies $1.03 < S/K$. Sample is the number of observed call option prices. BS40 (respectively, BS20)is the original Black-Scholes option pricing model based on 40 (respectively, 20) trade day historical volatility. log-volatility (respectively, square-root volatility) is the option pricing model under log-volatility (respectively, square-root volatility).

Sample			BS40			BS20			Log-Volatility			Square-Root Volatility		
Mat.	Mon.	Obs.	MAE	RMSE	RMSER	MAE	RMSE	RMSER	MAE	RMSE	RMSER	MAE	RMSE	RMSER
	ALL	2568	115.181	169.427	0.612	110.096	156.145	0.456	98.346	143.639	0.438	87.047	132.814	0.336
	ATM	754	127.063	175.052	0.302	125.469	163.388	0.266	108.985	145.242	0.233	95.275	130.716	0.202
Total	OTM	1048	84.521	134.976	0.913	79.863	114.429	0.665	68.701	101.104	0.645	54.025	86.500	0.484
	ITM	766	145.431	202.845	0.153	136.327	193.768	0.142	128.431	185.522	0.138	124.129	179.111	0.131
	ALL	1731	102.380	153.966	0.715	91.483	135.221	0.513	83.690	130.467	0.498	74.332	121.146	0.381
	ATM	477	108.896	146.645	0.326	98.211	123.582	0.277	84.481	112.776	0.237	72.332	98.200	0.207
Short	OTM	675	73.414	116.688	1.103	60.925	88.535	0.776	51.738	79.800	0.761	39.574	62.998	0.573
	ITM	579	130.782	193.082	0.147	121.566	181.506	0.135	120.288	181.624	0.136	116.499	176.936	0.130
	ALL	837	141.653	197.601	0.301	148.588	192.326	0.305	128.655	167.626	0.275	113.343	154.168	0.213
	ATM	277	158.348	215.359	0.253	172.410	215.329	0.248	151.180	188.469	0.225	134.782	172.930	0.194
Med.	OTM	373	104.623	162.933	0.377	114.133	150.344	0.386	99.399	131.135	0.348	80.171	117.645	0.254
	ITM	187	190.787	230.468	0.170	182.028	227.583	0.162	153.643	197.104	0.144	147.753	185.685	0.133

Table 8: Option Pricing Performance When $\lambda = 0$

MAE, RMSE and RMSER are mean absolute error, root mean squared error and root mean squared error relative to option price, which are defined by (17), (18) and (19), respectively. Total is entire option samples over out-of-sample period *i.e.*, from January 5, 1998 through June 30, 1998. Short (respectively, Medium) is option sample with time to maturity smaller than 60 calendar days (respectively, over 60 calendar days to four months). ALL is entire sample under Total, Short, and Medium. ATM is at-the-money sample which satisfies $0.97 \leq S/K \leq 1.03$. OTM is out-of-the-money sample which satisfies $S/K < 0.97$. ITM is in-the-money sample which satisfies $1.03 < S/K$. Sample is the number of observed call option prices. Log-Vol (In) (respectively, Sq-Vol (In)) is option pricing model under Log-Volatility (respectively, Square-Root volatility) model over in-sample period, July 1, 1997 to December 30, 1997. Similarly, Log-Vol (Out) (respectively, Sq-Vol (Out)) is option pricing model under Log-Volatility (respectively, Square-Root volatility) model over out-of-sample period, January 5, 1998 to June 30, 1998.

Sample		Log-Vol (In)			Sq-Vol (In)			Log-Vol (Out)			Sq-Vol (Out)		
Mat.	Mon.	MAE	RMSE	RMSER	MAE	RMSE	RMSER	MAE	RMSE	RMSER	MAE	RMSE	RMSER
Total	ALL	81.454	122.630	0.461	64.866	100.132	0.332	101.713	146.856	0.436	95.898	141.936	0.337
	ATM	87.349	122.173	0.224	70.916	99.703	0.188	114.535	150.996	0.238	109.053	146.605	0.217
	OTM	60.407	96.894	0.647	39.161	61.782	0.457	71.792	104.867	0.642	62.317	96.997	0.480
	ITM	116.060	163.203	0.127	108.696	150.739	0.116	130.027	186.584	0.139	128.894	183.058	0.133
Short	ALL	71.670	111.634	0.523	60.663	99.218	0.380	84.308	130.572	0.491	76.485	122.803	0.371
	ATM	75.333	103.575	0.244	62.242	91.001	0.211	85.830	113.941	0.238	76.386	103.268	0.213
	OTM	45.032	73.114	0.749	31.039	49.848	0.535	52.010	79.020	0.751	40.438	62.540	0.554
	ITM	112.768	162.282	0.129	108.932	154.735	0.120	120.710	181.649	0.136	118.592	178.157	0.131
Med.	ALL	102.315	143.344	0.290	73.854	102.061	0.192	137.708	175.811	0.290	136.046	174.990	0.251
	ATM	107.676	148.419	0.187	85.589	112.906	0.139	163.965	199.263	0.237	165.306	200.350	0.225
	OTM	90.427	131.461	0.376	55.019	80.113	0.241	107.595	139.995	0.370	101.911	139.127	0.306
	ITM	128.761	166.709	0.121	107.788	134.211	0.099	158.877	201.099	0.146	160.792	197.463	0.139

6 Concluding Remarks

Using the Nikkei 225 index return and its options data, we have investigated the pricing performances of two common option pricing models under stochastic volatility. The empirical results have witnessed that incorporating stochastic volatility structure into option pricing model enhances pricing performances in Japanese security market. In particular, accommodating square-root volatility process into option pricing model sharply contributes to pricing error reduction.

As mentioned before, the estimates of unobservable volatility are clearly important for pricing performances. In this sense, comparing our results with the implied parameters approach based on cross sectional analysis using only options data should shed light on our outcomes. These considerations should be included in further research.

7 Appendices

7.1 Some Inputs in Option Pricing Formula under Log-Volatility

Let $Ei(\cdot)$ denote the exponential integral function defined by

$$Ei(z) = -\int_{-z}^{\infty} \frac{\exp(-x)}{x}\, dx.$$

In addition, we set $\gamma \equiv \theta - \log \sigma_0$. Then, Σ_a, a_{11}, and a_{22} are given as follows.

(1) Σ

$$\Sigma = \frac{\exp(2\theta)}{\kappa}[Ei(z_1) - Ei(z_2)],$$

where $z_1 = -2\gamma$ and $z_2 = -2\gamma \exp(-\kappa T)$.

(2) a_{11}

$$a_{11} = \frac{\rho}{2\kappa^2\gamma}\Big\{\sigma_0^3 - \exp(2\theta(1-\exp(-\kappa T)))\Big[\sigma_0^{2\exp(-\kappa T)}(\sigma_0 - \gamma\exp(\theta)Ei(z_3)$$

$$+\gamma\exp(\theta)Ei(z_4)) + 3\gamma\exp(\theta(1+2\exp(-\kappa T)))Ei(z_5)\Big] + 3\gamma\exp(3\theta)Ei(z_6)\Bigg\},$$

where $z_3 = -\gamma\exp(-\kappa T)$, $z_4 = -\gamma$, $z_5 = -3\,\gamma\exp(-\kappa T)$, and $z_6 = -3\,\gamma$.

(3) a_{12}

$$\begin{aligned} a_{12} \quad &= \frac{-\lambda}{2\kappa^2\gamma}\Bigg\{\sigma_0^2 + 2\,\exp(2\theta)\,\gamma\,Ei(z_1) \\ &- \exp(2\theta)[\exp(-2\exp(-\kappa\,T)\,\theta)\,\sigma_0^{2\exp(-\kappa T)} + 2\,\gamma Ei(z_2)]\Bigg\}, \end{aligned}$$

We calculate the exponential integrals by utilizing the following relation:

$$Ei(z) = c + \log|z| + \sum_{n=1}^{\infty}\frac{z^n}{n\cdot n!},$$

where c is Euler's constant.

7.2 Option Pricing Formula under Square-Root Volatility

Given (7) and (11), Heston (1993) derived the European call option value as follows.[7]

$$C(S_t, \sigma_t, \tau) = \exp(-d\,\tau)S_t\,P_1 - \exp(-r\,\tau)K\,P_2.$$

In the above formula, P_j for $j = 1, 2$ are given as follows.

$$P_j(x, v, \tau; \log[K]) = \frac{1}{2} + \frac{1}{\pi}\int_0^{\infty} Re\left[\frac{\exp(-i\psi\log[K])f_j(x, v, \tau;\psi)}{i\psi}\right]d\psi,$$

where

$$\begin{aligned} f_j(x, v, \tau;\psi) &= \exp(C(\tau;\psi) + D(\tau;\psi)\,v + i\psi\,x, \\ C(\tau;\psi) &= r\,\psi i\tau + \frac{\kappa\,\theta}{\delta^2}\left\{(b_j - \rho\,\delta\,\psi i + h)\tau - 2\log\left[\frac{1 - g\exp(h\,\tau)}{1-g}\right]\right\} \\ D(\tau;\psi) &= \frac{b_j - \rho\delta\psi i + h}{\delta^2}\left(\frac{1-\exp(h\,\tau)}{1 - g\exp(h\,\tau)}\right), \\ g &= \frac{b_j - \rho\delta\psi i + h}{b_j - \rho\delta\psi i - h}, \\ h &= \sqrt{(\rho\delta\psi i - b_j)^2 - \delta^2(2u_j\psi i - \psi^2)}, \end{aligned}$$

$u_1 = \frac{1}{2}$, $u_2 = -\frac{1}{2}$, $b_1 = \kappa + \lambda - \rho\,\delta$, $b_2 = \kappa + \lambda$, $v = \sigma_t^2$, and $x = \log[\exp(-d\,\tau)S_t]$.

[7] Heston (1993) assumed no dividend flow.

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Figure 1: Nikkei 225 Index and its Return Process over Estimation Period

(upper) Daily Nikkei 225 stock index running from January 4, 1991 to December 30, 1997 (1727 trade days). (middle) return process starting from January 7, 1991 (1726 series). (lower) autocorrelation-adjusted return process starting from January 10, 1991 (1723 series).

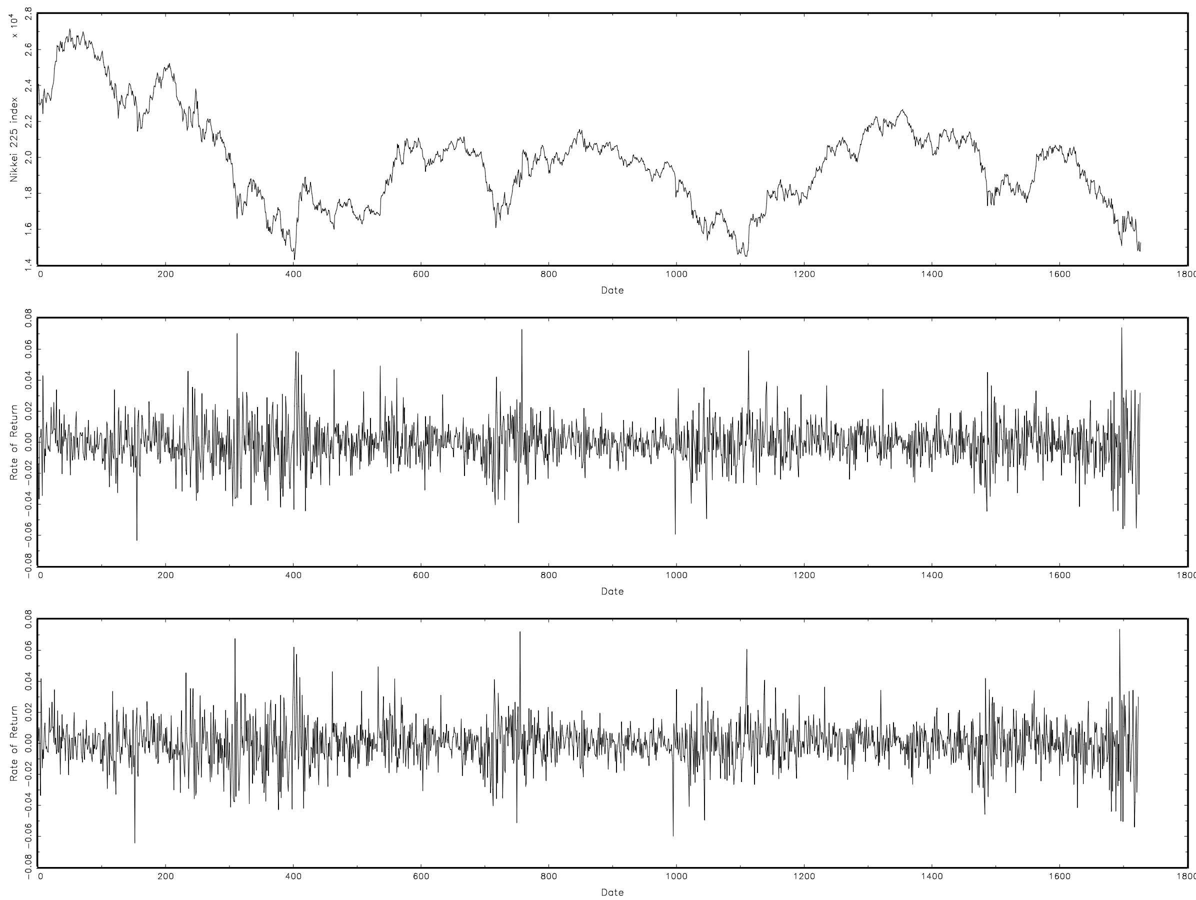

Figure 2: Log-Volatility From January 10, 1991 through June 30, 1998

Filtered log-volatility of Nikkei 225 stock index return runs from January 10, 1991 through June 30, 1998.

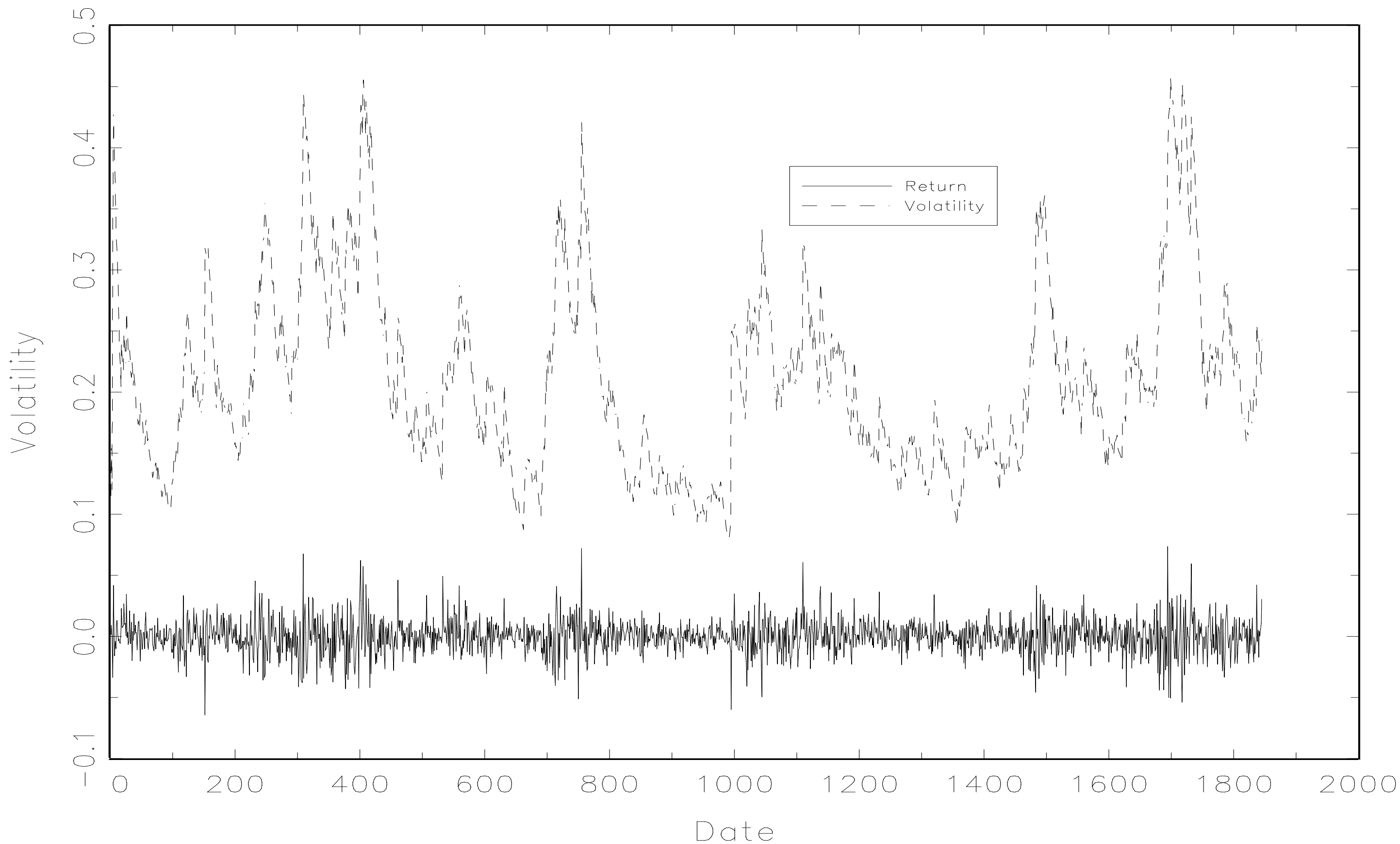

Figure 3: Square-Root Volatility from January 10, 1991 through June 30, 1998

Filtered square-root volatility of Nikkei 225 stock index return runs from January 10, 1991 through June 30, 1998.

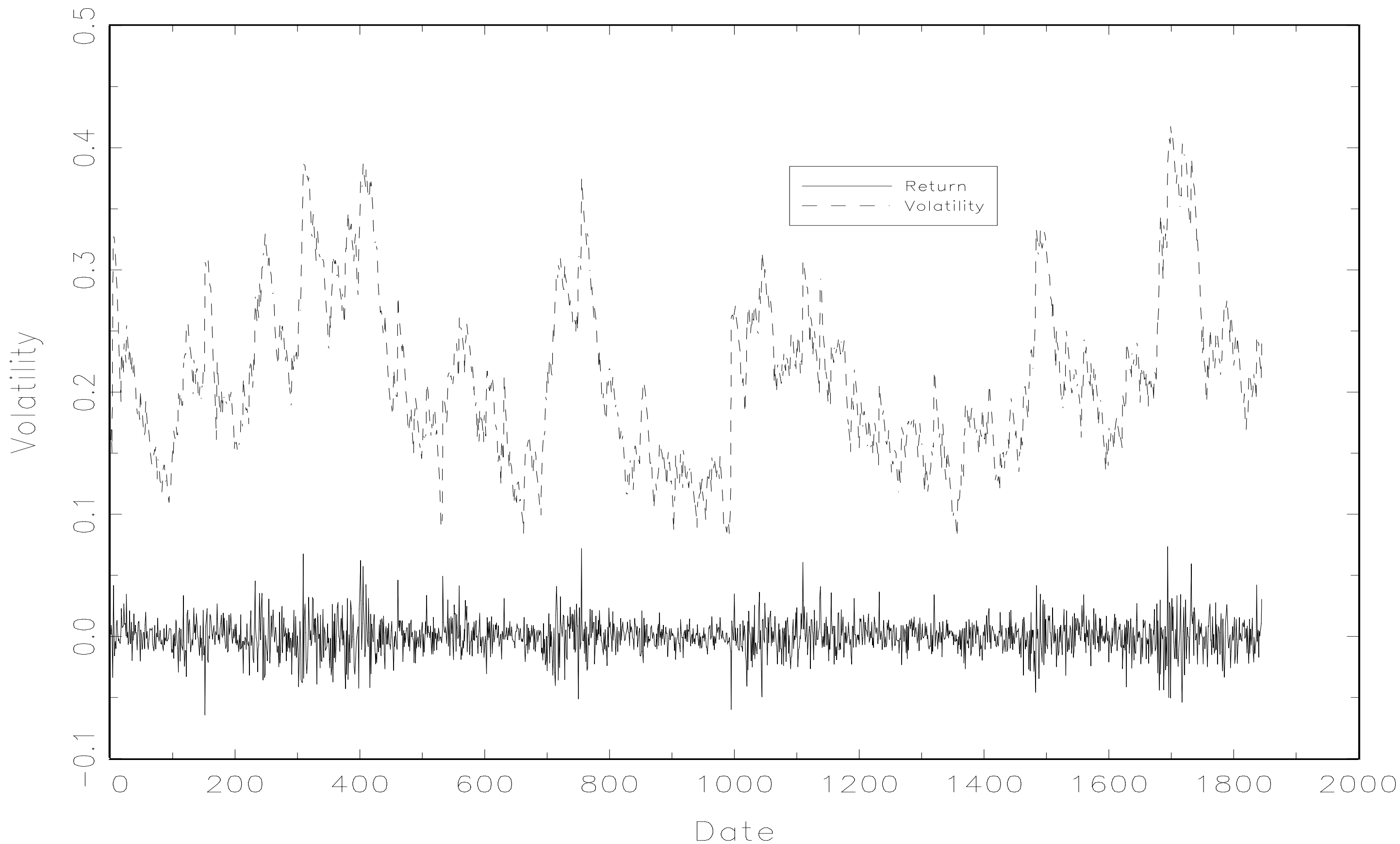

Figure 4: Filtered Volatility and Historical Volatility over In-Sample Period (July 1, 1997 through December 30, 1997)

Filtered volatility are those of log-volatility and square-root volatility model. Historical volatility 40 (respectively, 20) represents historical volatility of 40 (respectively, 20) trade days.

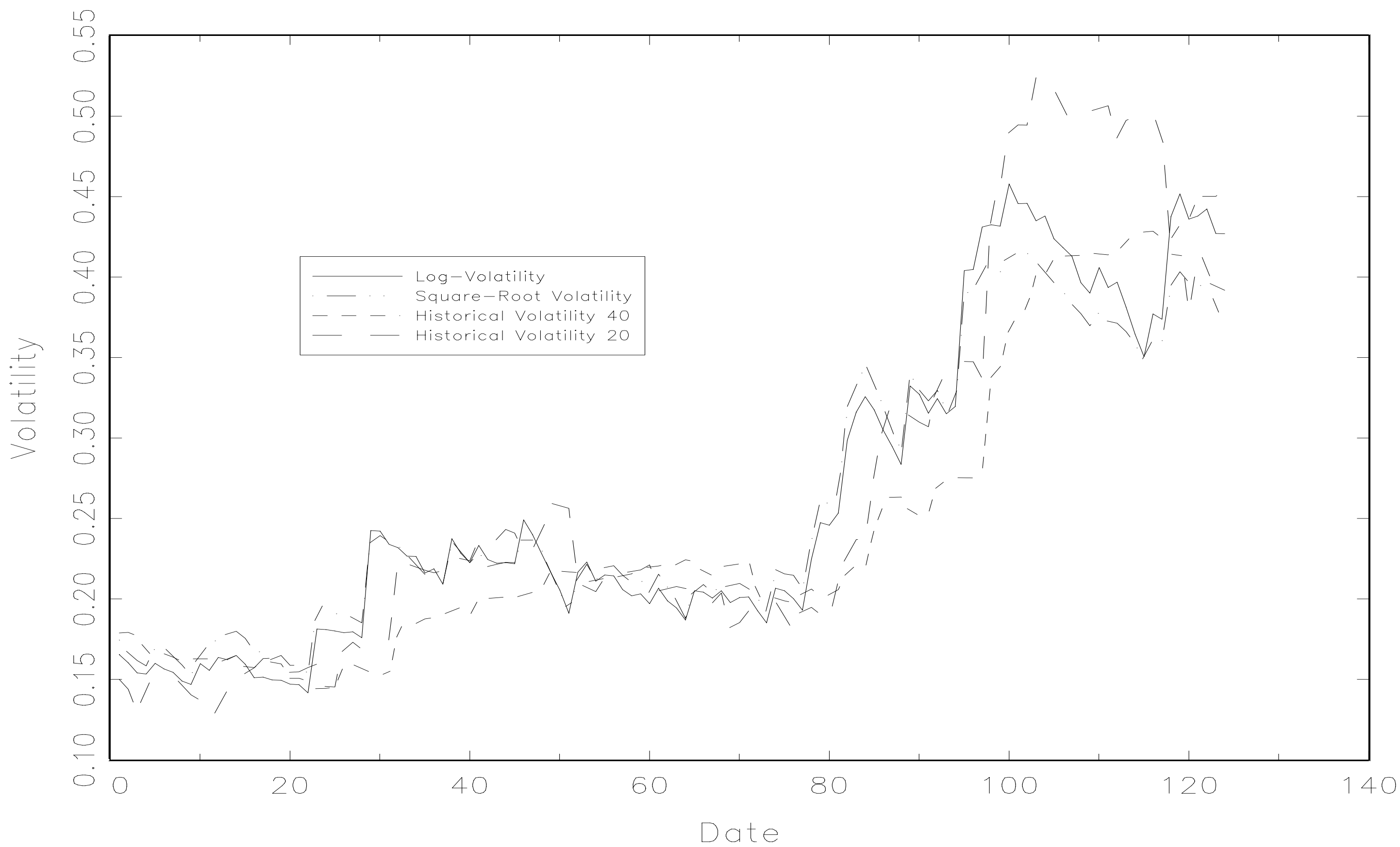

Figure 5: Filtered Volatility and Historical Volatility over Out-of-Sample Period (January 5, 1998 through June 30, 1998)

Filtered volatility are those of log-volatility and square-root volatility model. Historical volatility 40 (respectively, 20) represents historical volatility of 40 (respectively, 20) trade days.